MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Quantum Information Science I

October 28, 2010

Problem Set #7
(due in class, 04-Nov-10)

P1: (**Continuous time Grover with noise**) Consider a continuous time Grover algorithm on n qubits employing two Hamiltonians, an oracle Hamiltonian, $H_O = E_O|x\rangle\langle x|$, and a driving Hamiltonian $H_D = E_D\,|\psi\rangle\,\langle\psi|$, where $|x\rangle$ is the target state and $|\psi\rangle$ is state with all the qubits being $|0\rangle + |1\rangle$. Here, we explore the impact of a simple noise model on this algorithm.

(a) Ideally, H_D has the same strength as the oracle H_O. However, we the strength of H_D may fluctuate. Assuming $E_D = E_O(1+\delta)$, calculate the probability of being in the target state at time $\pi\sqrt{2^n}/(2E_O)$.

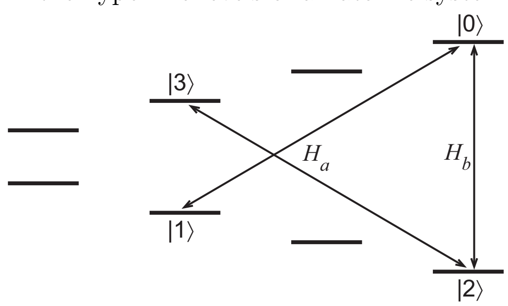
(b) Suppose we run the algorithm on an imperfect quantum computer in which each qubit experiences a δ that has a random value given by the probability distribution $\frac{1}{\sqrt{2\pi}\sigma}\exp(-\delta^2/(2\sigma^2))$.

Calculate how many times one needs to run the algorithm to know what the target state is with probability 2/3, as a function of σ.

P2: (**Compositions of Hamiltonian operations**) Consider a physical system with four energy levels which are addressable, $|0\rangle$, $|1\rangle$, $|2\rangle$, and $|3\rangle$. You are provided with controls which turn one of two Hamiltonians H_b, which couples $\{|0\rangle, |2\rangle\}$, and H_a, which couples $\{|2\rangle, |3\rangle\}$ and $\{|0\rangle, |1\rangle\}$. Specifically,

$$H_a(\alpha) = \begin{bmatrix} 0 & \alpha & 0 & 0 \\ \alpha^* & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha \\ 0 & 0 & \alpha^* & 0 \end{bmatrix} \qquad H_b(\beta) = \begin{bmatrix} 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & 0 \\ \beta^* & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{1}$$

Note that α and β are both complex numbers. These may be visualized as transitions between some subset of energy levels, eg in the hyperfine levels of an atomic system:



Your mission is to perform qubit rotations in the $\{|0\rangle, |1\rangle\}$ qubit subspace, while leaving $\{|2\rangle, |3\rangle\}$ alone.

(a) Suppose H_1 and H_2 are Hamiltonians such that $\mathrm{tr}|H_1| \le \epsilon$ and $\mathrm{tr}|H_2| \le \epsilon$. Prove that $e^{-iH_1}e^{-iH_2}e^{iH_1}e^{iH_2} = e^{-iC} + O(\epsilon^3)$, where $C = i[H_1, H_2] = i(H_1H_2 - H_2H_1)$.

(b) Show our goal is possible in principle, by constructing a set of Hamiltonians from which $H_{01} = \gamma|0\rangle\langle 1| + \gamma^*|1\rangle\langle 0|$ can be generated. Specifically, compute $H_c = i[H_b|_{\beta=1}, H_b|_{\beta=i}]/2$ and $H_d = i[H_a|_{\alpha=i}, H_c]$ and explain how to obtain H_{01} from this, by quantum simulation techniques.

(c) Let $R_x(\theta) = \exp\left[-i\frac{\theta}{2}(|0\rangle\langle 1| + |1\rangle\langle 0|)\right]$ be a rotation about the $\hat{x}$ axis of the $\{|0\rangle, |1\rangle\}$ qubit subspace (it acts as identity on the $\{|2\rangle, |3\rangle\}$ subspace). Give a sequence of individual Hamiltonian evolutions, eg $U = e^{it_1H_a|_{\alpha=1}}e^{it_2H_b|_{\beta=i}}\cdots$, turning on H_a and H_b sequentially (only one Hamiltonian on at a time), with specified values of α, β, and pulse durations, such that $U = R_x(\theta)$ *exactly.*

(d) Do the same for $R_z(\theta) = \exp\left[-i\frac{\theta}{2}(|0\rangle\langle 0| - |1\rangle\langle 1|)\right]$, such that you now have "pulse sequences" for performing arbitrary operations on the $\{|0\rangle, |1\rangle\}$ qubit.