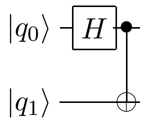
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

MIT 2.111/8.411/6.898/18.435
Quantum Information Science I

November 16, 2010

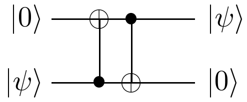
Problem Set #9
(due in class, TUESDAY 23-Nov-10)

P1: (**Feynman's Hamiltonian model for QC**) Read the paper *Quantum Mechanical Computers*, Optics News, February 1985, by Richard Feynman (see course website for link). Following Feynman's prescription, give a Hamiltonian for simulating this discrete quantum circuit,

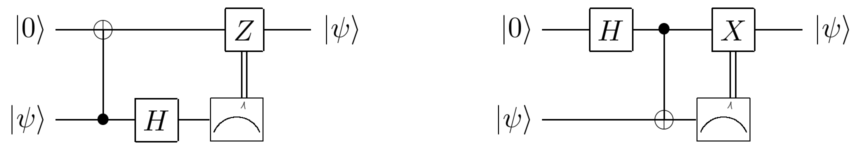


using a system of interacting spins, with one or more spins acting as a clock. Describe your system and explain how your Hamiltonian simulation works.

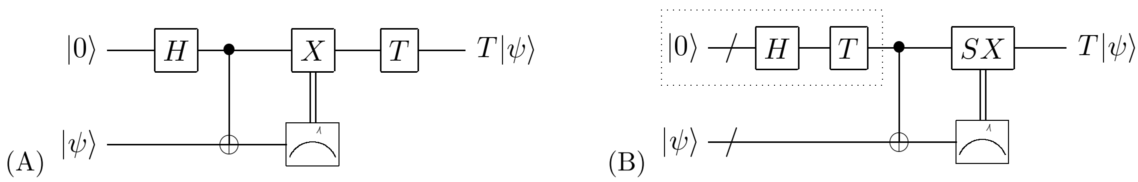
P2: (**Teleportation circuits**) An unknown qubit in the state $|\psi\rangle$ can be swapped with a second qubit which is prepared in the state $|0\rangle$ using only two controlled-NOT gates, with the circuit



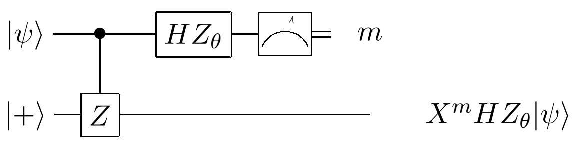
Show that the two circuits below, which use only a single CNOT gate, with measurement and a classically controlled single qubit operation, also accomplish the same task:



P3: (T **gate construction using teleportation**) One way to implement a T gate is to first swap the qubit state $|\psi\rangle$ you wish to transform with some known state $|0\rangle$, then to apply a T gate to the resulting qubit (circuit A, below). Doing this does not seem particularly useful, but actually it leads to something which is! Show that by using the relations $TX = \exp(-i\pi/4)SXT$ and $TU = UT$ (U is the controlled-NOT gate, and T acts on the control qubit) we may obtain circuit B:



P4: (**Z-rotation gate using teleportation**) Let Z_θ denote a single qubit rotation $R_z(\theta)$ about the $\hat{z}$ axis. Prove that the output of this quantum circuit:



is a qubit in the state $X^m HZ_\theta|\psi\rangle$, where m is the measurement result. This is a key idea used in the cluster model of QC. How is this construction related to teleportation?