2.111J/18.435J Quantum Computation Problem Set 2
(Due: Tuesday, September 27, 2005)

Notes for Problem Set: Our convention is $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

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1) Prove that if a matrix A is Hermitian (*i.e.,* $A = A^\dagger$ where $A^\dagger$ is the matrix formed by taking the complex conjugate of each element of the transpose of A), then:

(i) All eigenvalues of A are real.

(ii) Eigenvectors corresponding to distinct eigenvalues of A are orthogonal.

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2) Calculate the eigenvalues and normalized eigenvectors of σ_y.

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3) Calculate the eigenvalues and normalized eigenvectors of $\sigma_j = \frac{1}{\sqrt{2}}\sigma_x + \frac{1}{\sqrt{2}}\sigma_z = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$.

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4) Verify that (i) $\sigma_z\sigma_x = i\sigma_y$, (ii) $\sigma_x\sigma_z = -i\sigma_y$, (iii) $\sigma_y\sigma_z = i\sigma_x$, (iv) $\sigma_z\sigma_y = -i\sigma_x$.

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5) Prove that if j_x, j_y, and j_z are real numbers such that $j_x^2 + j_y^2 + j_z^2 = 1$, then $\sigma_j^2 = (j_x\sigma_x + j_y\sigma_y + j_z\sigma_z)^2 = \mathbb{I}$, the identity.

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6) Calculate the 2×2 unitary matrix corresponding to a rotation of π radians counterclockwise around $(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})$. What does this rotation do to $|\uparrow\rangle, |\downarrow\rangle, |\rightarrow\rangle$, and $|\leftarrow\rangle$ (*i.e.*, the eigenvectors of σ_z and σ_x, respectively)?