2.111J/18.435J Quantum Computation Problem Set 6

(Due: Tuesday, November 8, 2005)

1) What are the main criteria for viable quantum computer designs? (For example, read Nielsen and Chuang, Sections 7.1 – 7.2 or google "DiVincenzo Criteria.") Write a line or two about each. List the usual figures of merit used to quantify each criterion. Which ones tend to be in conflict and thus tend to require trade-offs?

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2) Let $\sigma_0 \equiv \mathbb{I} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\sigma_1 \equiv \sigma_x \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_2 \equiv \sigma_y \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, and $\sigma_3 \equiv \sigma_z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

(a) Show that any 2×2 matrix can be written as $\sum_{k=0}^{3} c_k \sigma_k$, where the c_k are complex scalars.

(b) Similarly, show that any 4×4 matrix can be written as $\sum_{k_1=0}^{3} \sum_{k_2=0}^{3} c_{k_1 k_2} \sigma_{k_1} \otimes \sigma_{k_2}$.

(c) Finally, show that any $2^n \times 2^n$ matrix can be written as $\sum_{k_1=0}^{3} \ldots \sum_{k_n=0}^{3} c_{k_1 \ldots k_n} \sigma_{k_1} \otimes \ldots \otimes \sigma_{k_n}$.

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3) Let $\sigma_\pm \equiv \sigma_x \pm i\sigma_y$. Show that $e^{i\omega t \sigma_z/2} \sigma_\pm e^{-i\omega t \sigma_z/2} = e^{\pm i\omega t} \sigma_\pm$.