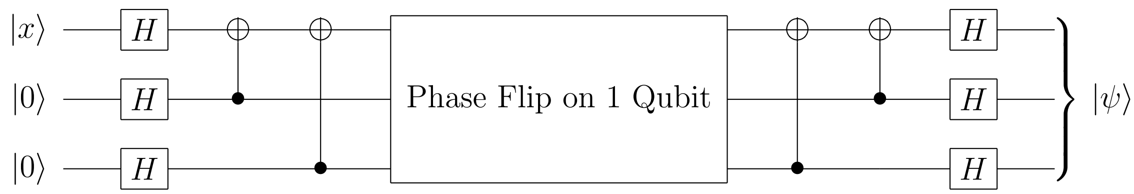
2.111J/18.435J Quantum Computation Problem Set 8

(Due: Tuesday, December 6, 2005)

1) Verify that the following circuit is the appropriate encoder/decoder circuit for the 3 qubit phase flip code.



In other words, exhibit a measurement on the two ancillae in the circuit's output $|\psi\rangle$ that will detect whether a phase flip error occurred and locate the phase flip if it did occur. Then, exhibit an operation based on the above measurement that will correct the phase flip.

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2) Construct an encoding circuit and a decoding circuit for the 9 qubit Shor code

$$|0\rangle \to \frac{1}{2\sqrt{2}}\big(|000\rangle + |111\rangle\big) \otimes \big(|000\rangle + |111\rangle\big) \otimes \big(|000\rangle + |111\rangle\big)$$

$$|1\rangle \to \frac{1}{2\sqrt{2}}\big(|000\rangle - |111\rangle\big) \otimes \big(|000\rangle - |111\rangle\big) \otimes \big(|000\rangle - |111\rangle\big).$$

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3) Verify that the 9 qubit Shor code corrects a bit and/or phase flip on any 1 of the 9 qubits.

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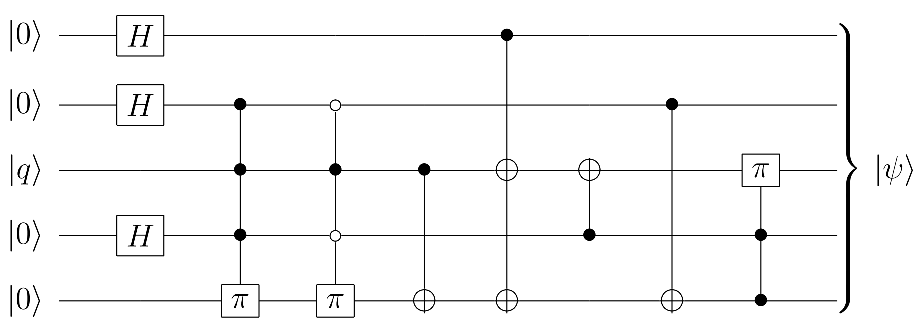
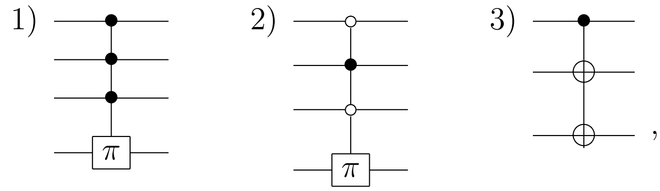
4) What is the output $|\psi\rangle$ of the following circuit, which is the encoder for the Laflamme-Miquel-Paz-Zurek perfect 5 qubit code?



Note the following definitions. The single qubit gates are (in the σ_z eigenbasis):

$$-\boxed{H}- \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}; \qquad -\boxed{\pi}- \;=\; e^{i\pi}\mathbb{I} \;=\; \begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}; \qquad -\oplus- \;=\; \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}.$$

Our more complicated controlled gates, for example



work as follows (in the tensor product σ_z eigenbasis):

(1) does nothing to the first 3 qubits and imparts a -1 phase factor to the 4th qubit if and only if the first 3 qubits are in the state $|111\rangle$.

(2) does nothing to the first 3 qubits and imparts a -1 phase factor to the 4th qubit if and only if the first 3 qubits are in the the state $|010\rangle$.

(3) does nothing to the 1st qubit and applies $\oplus$-gates to both the 2nd and 3rd qubits only if the 1st qubit is in the state $|1\rangle$.

5) Verify that the decoder of the above circuit allows one to correct a bit and/or phase flip on any 1 of the 5 qubits.

6) Consider the code $|0\rangle_{en} = \frac{1}{\sqrt{2}}\big(|01\rangle - |10\rangle\big)$; $|1\rangle_{en} = \frac{1}{\sqrt{2}}\big(|01\rangle + |10\rangle\big)$. Show that an arbitrary superposition of encoded states $\alpha|0\rangle_{en} + \beta|1\rangle_{en}$ is robust against errors of the form $e^{-i\theta\sigma_z/2} \otimes e^{-i\theta\sigma_z/2}$.