

1) The key here is that we know  $M$ , and  $1 \ll M \ll N$ .

So, rather than applying the operations

$$|0\rangle \rightarrow \frac{1}{\sqrt{N}} \sum_{i=1}^N |i\rangle = |s\rangle,$$

followed by  $\mathbb{1} - 2|\omega\rangle\langle\omega|$  (  $|\omega\rangle \rightarrow -|\omega\rangle$   
 $|\bar{i}+\omega\rangle \rightarrow |\bar{i}\rangle$  )  
 $U_{\omega}$

followed by  $\mathbb{1} - 2|s\rangle\langle s| = U_s$ ,

then repeating  $U_s U_{\omega}$   $O(\sqrt{N})$  times,

apply

$|0\rangle \rightarrow |s\rangle$  followed by

$$\mathbb{1} - 2 \sum_{j=1}^M |\omega_j\rangle\langle\omega_j| \equiv U_{\tilde{\omega}}$$

followed by  $U_s$ , and repeat

$U_s U_{\tilde{\omega}}$   $O\left(\sqrt{\frac{N}{M}}\right)$  times