

$$3) \quad U_{\text{NOT}} = e^{i\pi/4 \sigma_x^2} e^{+i\pi/4 \sigma_2' \sigma_2^2} e^{-i\pi/4 \sigma_y^2} \quad ?$$

$$= \frac{1}{\sqrt{2}} \left(1 + i\sigma_x^2 \right) \frac{1}{\sqrt{2}} \left(1 + i\sigma_2' \sigma_2^2 \right) \frac{1}{\sqrt{2}} \left(1 - i\sigma_y^2 \right)$$

$$= \frac{1}{2\sqrt{2}} \left(1 + i\sigma_x^2 + i\sigma_2' \sigma_2^2 - i\sigma_y^2 \right. \\ \left. - \sigma_2' (\sigma_x^2 \sigma_2^2) + \sigma_x^2 \sigma_y^2 + \sigma_2' (\sigma_2^2 \sigma_y^2) \right. \\ \left. + i \sigma_2' (\sigma_x^2 \sigma_2^2 \sigma_y^2) \right)$$

$$= \frac{1}{2\sqrt{2}} \left(1 + i\sigma_x^2 + i\sigma_2' \sigma_2^2 - i\sigma_y^2 \right. \\ \left. + i\sigma_2' \sigma_y^2 + i\sigma_2^2 - i\sigma_2' \sigma_x^2 \right. \\ \left. + i\sigma_2' \right)$$

$$= \frac{1}{2} \left(1 + \sigma_2' \right) \otimes \frac{1}{\sqrt{2}} \left(1 + i\sigma_2^2 \right)$$

$$+ \frac{1}{2} \left(1 - \sigma_2' \right) \otimes \frac{1}{\sqrt{2}} \left(\sigma_x^2 - \sigma_y^2 \right)$$

$$= \frac{1}{2} \left(1 + \sigma_2' \right) \otimes e^{i\pi/4 \sigma_2} + \frac{1}{2} \left(1 - \sigma_2' \right) \otimes e^{i\pi/2 (\sigma_x^2 - \sigma_y^2) / \sqrt{2}}$$