

iv)  $[H, H^2] = 0$  so that gives nothing new,

$$\begin{aligned} \text{v) } [x, H^2] &= xH^2 - H^2x = (xH - Hx)H + H(xH - Hx) \\ &= [x, H]H + H[x, H] \\ &= i(pH + Hp) \end{aligned}$$

a third order Hamiltonian

vi) similarly  $[p, H^2] = -i(xH + Hx)$   
another third order Hamiltonian

So we're starting to get more.

$$\begin{aligned} \text{vii) } [p, pH + Hp] &= p[p, H] + [p, H]p \\ &= -i(pX + Xp) \end{aligned}$$

a new second order Hamiltonian

viii) Similarly, can get

$$[x, xH + Hx] = i(Xp + pX) \quad \text{Same as vii) up to a - sign.}$$

$$\begin{aligned} \text{ix) } [H, pX + Xp] &= \frac{1}{2} \left( [p^2, xP + Px] + [x^2, xP + Px] \right) \\ &= \frac{1}{2} \left( [p^2, x]P + P[p^2, x] + x[x^2, P] + [x^2, P]x \right) \\ &= \frac{i}{2} \left( -p^2 + x^2 \right), \quad \text{so get } \pm (x^2 - p^2) \end{aligned}$$