

Now we have $\mathbb{1}$, X , P , $(X^2 + P^2)^{1/2}$, $(X^2 - P^2)^{1/2}$
 $(XP + PX)$ in our 'stable' of Hamiltonians,

so we can get arbitrary 2nd order
 Hamiltonians. We also have a

couple of third order Hamiltonians,

$pH + Hp$, $XH + HX$, and one fourth order

Hamiltonian, H^2 .

Let's try to

generate higher order Hamiltonians:

$$x) [H^2, pH + Hp] = H^2 pH + H^3 p - pH^3 - HpH^2$$

$$= H [H, pH + Hp] + [H, pH + Hp] H$$

$$\text{But } [H, pH + Hp] = [H, p]H + H[H, p] = i(XH + HX)$$

$$\text{so } x) = i (HXH + H^2X + HXH + XH^2)$$

$$= i (H^2X + 2HXH + XH^2) \text{ a fifth-order}$$

Hamiltonian.