# Department of Mechanical Engineering

2.14 Analysis and Design of Feedback Control Systems

Introduction to the Operational Amplifier [1]

The integrated-circuit operational-amplifier is the fundamental building block for many electronic circuits, including analog control systems. In this hand-out we examine some of the basic circuits that can be used to implement control systems. We take simplified approaches to the analysis, and this discussion is by no means complete or exhaustive.

What is an operational amplifier (op-amp)? It is simply a very high gain electronic amplifier with a pair of differential inputs. Its functionality comes about through the use of feedback around the amplifier as we will show below.
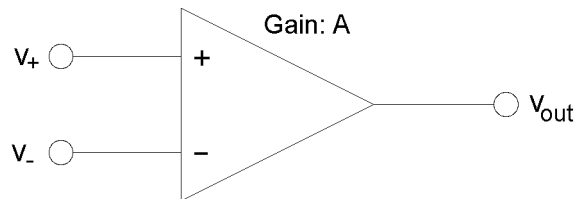
The op-amp has the following characteristics:

- It is basically a "three terminal" amplifier with two inputs and an output. It is a differential amplifier with very high gain

$$v_{out} = A(v_+ - v_-)$$

  where A is typically $10^4 - 10^5$, and the two inputs are known as the *non-inverting* (v_+) and *inverting* (v_-) inputs respectively. In the ideal op-amp we assume that the gain A is infinite.

- In an ideal op-amp no current flows into either input, that is they are voltage-controlled and have infinite input resistance. In a practical op-amp the input current is in the order of pico-amps (10^{-12}) amp, or less.

- The output acts as a voltage source, that is it can be modeled as a Thevenin source with a very low source resistance.

Op-amps come in many forms and with a bewildering array of specifications. They range in cost from a few cents to many dollars, depending on the specs. These specifications include input impedance, input bias current, output offset voltage, external power requiremets, etc. Higher grade amplifiers are known as precision, or instrumentation amplifiers.

In this course we will be relying on the industry standard 741 op-amp wherever possible. Although it was a major step forward in its day, by today's standards it is not a high

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[1]D. Rowell — 9/9/02

performance amplifier. Its major advantage is that it costs about $0.30! If we need higher performnce we will use the OP-07 amp, which is laser trimmed during manufacture to reduce its offset voltage to a very low level.

Op-amps come in a variety of packages. We will be using the common 8 pin DIP (dual in-line package) form. Many amps use a common basic pin-out for this package as shown below:
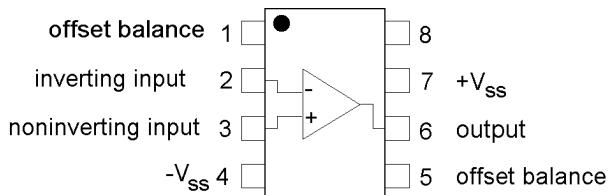
The pins are numbered counter-clockwise from the top left as shown above. (Note that pin 1 is identified by a notch at the top or a dot beside pin 1.) The basic amplifier is connected between pins 2, 3 and 6. The amplifier requires a pair of external supply voltages to operate, these are typically ± 15 volts and are are connected to pins 7 (positive) and 4 (negative). Pins 1 and 5 are usually used for optional external offset nulling circuitry - the actual connection is dependent on the type. We will not use this feature if we can get away without it.

Non-Inverting Configurations

The Unity Gain Non-Inverting amplifier: The simplest configuration of the op-amp is as a unity gain "buffer" amplifier:
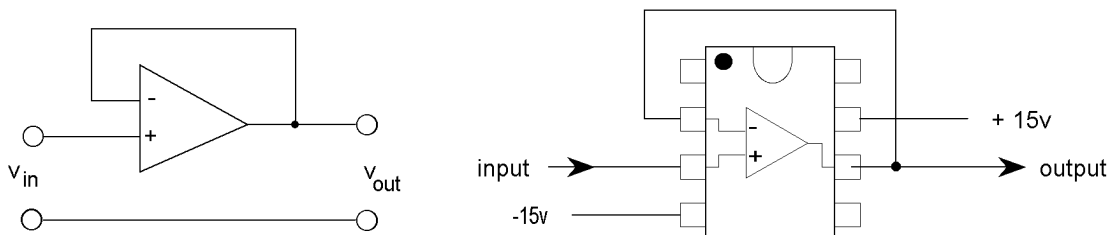
The output is connected directly to the inverting input. Then

$$
\begin{aligned}
v_{out} &= A(v_+ - v_-) \\
&= A(v_+ - v_{out}) \\
&= \frac{A}{1+A} v_+
\end{aligned}
$$

and if as noted above A is typically 10^4 or much higher we may simply say

$$v_{out} = v_+$$

The unity gain amplifier is used commonly to minimize loading on a circuit because it draws no current yet provides a low output resistance output.

The Non-Inverting amplifier with gain: The above circuit may be modified to feed back only a fraction of the output voltage by including a two resistor voltage divider.

2
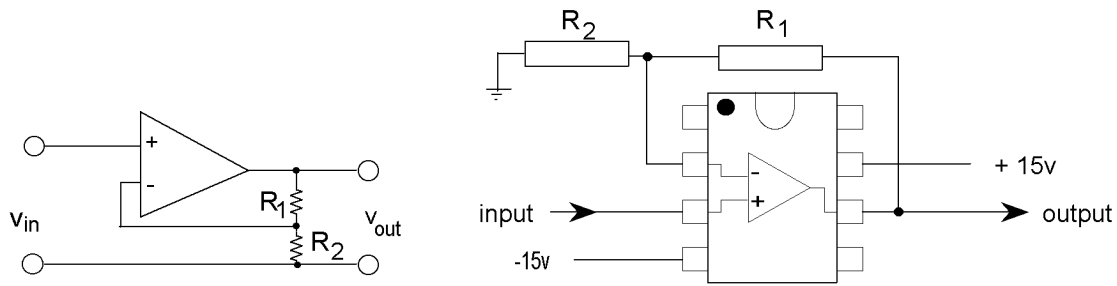
In this case R_1 and R_2 act as a voltage divider, and

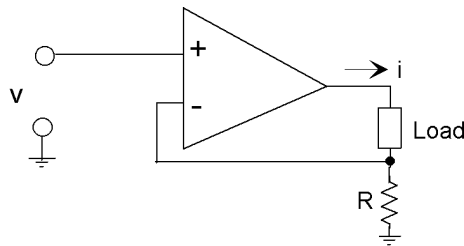$$v_- = \frac{R_2}{R_1 + R_2} v_{out}$$

Then

$$
\begin{aligned}
v_{out} &= A(v_+ - v_{in}) \\
&= A\left(v_{in} - \frac{R_2}{R_1 + R_2} v_{out}\right) \\
&= \frac{A(R_1 + R_2)}{R_1 + R_2 + AR_2} v_{in}
\end{aligned}
$$

and when $A \gg 1$

$$v_{out} = \frac{R_1 + R_2}{R_2} v_{in}$$

For example if $R_1 = 47$ kΩ, and $R_2 = 10$ kΩ, the amplifier voltage gain will be 5.7.

The Voltage-Controlled Current-Source: The above circuit can be modified to produce a current source.

Assume that the op-amp drives a load of unknown resistance. A small resistor R is placed in series with the load. Then the inverting input is $v_- = iR$. At the output of the amplifier

$$
\begin{aligned}
v_{out} &= A\left(v_+ - v_-\right) \\
&= A\left(v_{in} - iR\right)
\end{aligned}
$$

Then

$$i = \frac{1}{R}\left(v_{in} - \frac{1}{A} v_{out}\right)$$
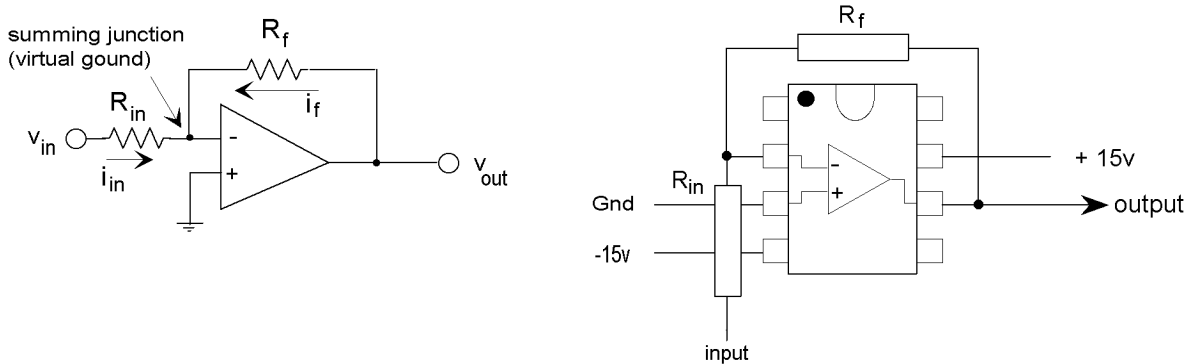
and iff $A \gg 1$

$$i = \frac{1}{R} v_{in}$$

3

which is independent of the load.

Inverting Configurations

The Inverting Amplifier: The inverting amplifier has a configuration as shown below:

The non-inverting input is connected to ground and a pair of resistors, R_{in} and R_f (the feedback resistor) is used to define the gain. To simplify the analysis we make the following assumptions:

(a) The gain A is very high and we let it tend to infinity. Because $v_{out} = A(v_+ - v_-)$ and $v_+ = 0$, this means that $v_- = \lim_{A \to \infty} v_{out}/A = 0$. In other words, the inverting input voltage is so small (μvolts) that we declare it to be a "virtual ground", that is in our simplified analysis we assume $v_- = 0$.

(b) No current flows into the inputs. This allows us to apply Kirchoff's current law at the junction of the resistors.

Apply Kirchoff's current law at the junction of the resistors:

$$i_{in} + i_f = 0$$

Since the junction is at the virtual ground we can simply use Ohm's law:
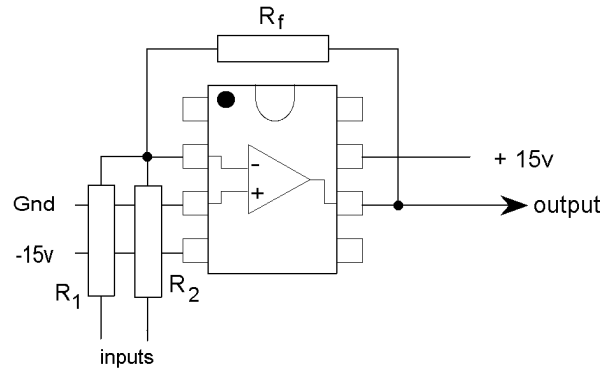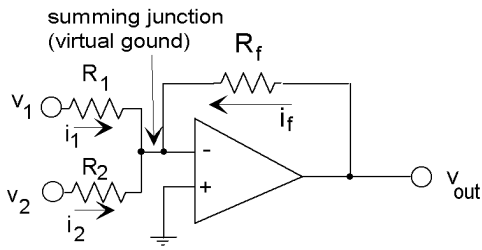
$$\frac{v_{in}}{R_{in}} + \frac{V_{out}}{R_f} = 0$$

or

$$v_{out} = -\frac{R_f}{R_{in}} v_{in}$$

In other words the voltage gain is defined by the ratio of the two resistors.

The Inverting Summer: The inverting amplifier may be extended to include multiple inputs:

As before we assume that the inverting input is at a virtual ground and apply Kirchoff's current law at the summing junction

$$i_1 + i_2 + i_f = 0$$

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{V_{out}}{R_f} = 0$$

or

$$v_{out} = -\frac{R_f}{R_1}v_1 - \frac{R_f}{R_2}v_2$$

which is a weighted sum of the inputs. If $R_1 = R_2 = R_{in}$ the output is a pure sum
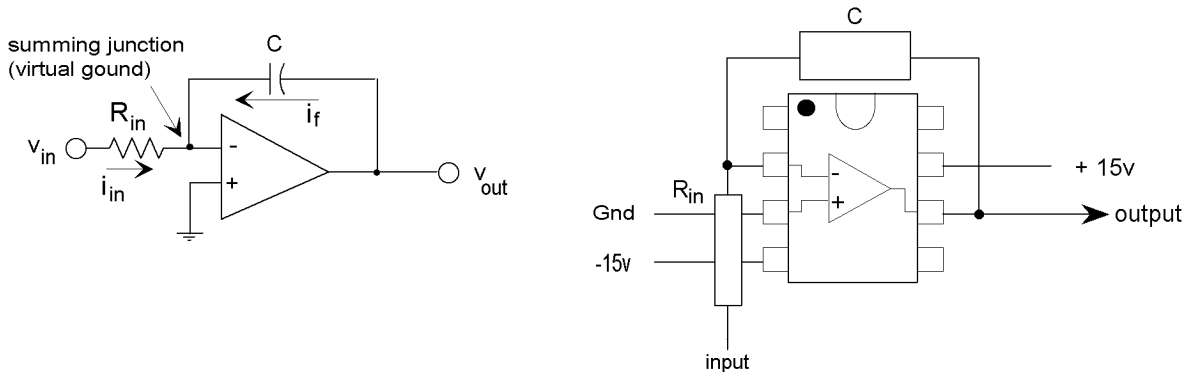
$$v_{out} = -\frac{R_f}{R_{in}}(v_1 + v_2)$$

The summer may be extended to many inputs by simply adding input resistors R_i

$$v_{out} = -\sum_{i=1}^{n}\frac{R_f}{R_i}v_i$$

If all of the input resistors are equal $(R_i = R_{in})$

$$v_{out} = -\frac{R_f}{R_{in}}\sum_{i=1}^{n}v_i$$

5

The Integrator: If the feedback resistor in the inverting amplifier is replaced by a capacitor C the amplifier becomes an integrator.

At the summing junction we apply Kirchoff's current law as before but the feedback current is now defined by the elemental relationship for the capacitor:

$$i_{in} + i_f = 0$$
$$\frac{v_{in}}{R_{in}} + C\frac{dv_{out}}{dt} = 0$$

Then

$$\frac{dv_{out}}{dt} = -\frac{1}{R_{in}C}v_{in}$$

or

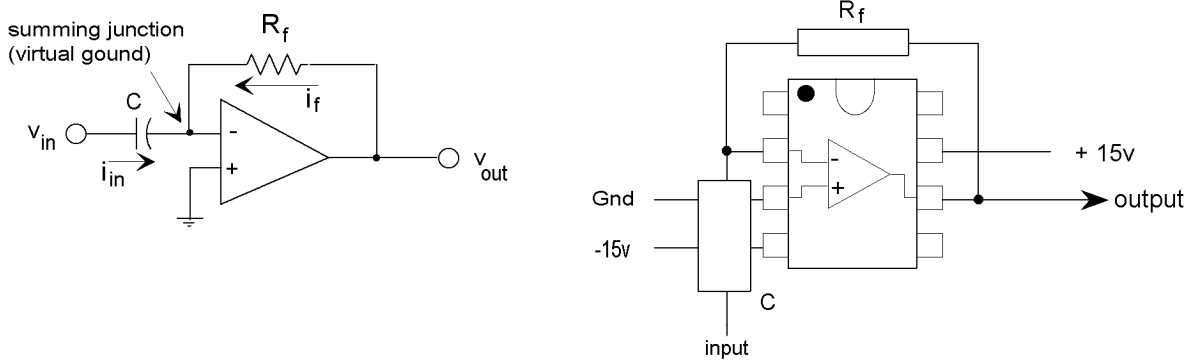$$v_{out} = -\frac{1}{R_{in}C}\int_0^t v_{in}dt + v_{out}(0)$$

As above, the integrator may be extended to a summing configuration by simply adding extra input resistors:

$$v_{out} = -\frac{1}{C}\int_0^t \left(\sum_{i=1}^n \frac{v_i}{R_i}\right) dt + v_{out}(0)$$

and if all input resistors have the same value R

$$v_{out} = -\frac{1}{RC}\int_0^t \left(\sum_{i=1}^n v_i\right) dt + v_{out}(0)$$

The Differentiator: If the input resistor R_{in} in the inverting amplifier is replaced by a capacitor, the circuit becomes a differentiator:

6

At the summing junction we apply Kirchoff's current law as before but the input current is now defined by the elemental relationship for the capacitor:
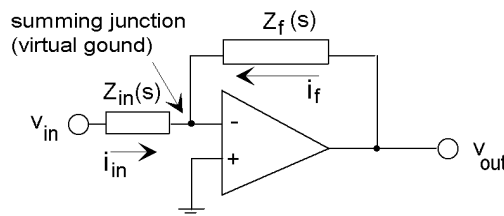
$$i_{in} + i_f = 0$$

$$C\frac{dv_{in}}{dt} + \frac{v_{out}}{R_f} + = 0$$

Then

$$v_{out} = -\frac{1}{R_{in}C}\frac{dv_{in}}{dt}$$

As we have noted in class, pure differentiators are rarely used because of their susceptibility to noise. In practice it is common to put a resistor in series with the capacitor. (See Homework 3).

A General Linear Transfer Function: The inverting configuration may be extended to synthesize many general linear transfer functions by including appropriate networks in the feedback and input paths. Consider the general circuit shown below with elements described by impedances $Z_{in}(s)$ and $Z_f(s)$ in the input and output paths:

At the summing junction

$$i_{in} + i_f = 0$$

$$\frac{V_{in}(s)}{Z_{in}(s)} + \frac{V_{out}(s)}{Z_f(s)} + = 0$$

so that the amplifier becomes a linear system with a transfer function

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = -\frac{Z_f(s)}{Z_{in}(s)}$$

7

This configuration may be used to synthesize lead-lag compensators for control, active filters for signal processing, and many other functions.

The Differential Amplifier
A differential amplifier produces an output that is proportional to the difference between two inputs.
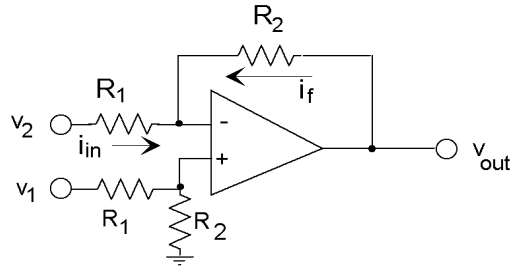
At the non-inverting input

$$v_+ = \frac{R_2}{R_1 + R_2} v_1$$

If the op-amp gain is assumed to be infinite, $v_- = v_+$. The input current at the inverting amplifier is:

$$\begin{aligned} i_{in} &= \frac{v_2 - v_-}{R_1} = \frac{v_2 - v_+}{R_1} \\ &= \frac{1}{R_1} v_2 - \frac{R_2}{R_1(R_1 + R_2)} v_1 \end{aligned}$$

Similarly the feedback current is

$$\begin{aligned} i_f &= \frac{v_{out} - v_-}{R_2} = \frac{v_{out} - v_+}{R_2} \\ &= \frac{1}{R_2} v_{out} - \frac{1}{(R_1 + R_2)} v_1 \end{aligned}$$

Applying Kirchoff's current law at the summinig junction $i_{in} + i_f = 0$, then yields

$$v_{out} = \frac{R_2}{R_1} (v_1 - v_2)$$

so that the output is proportional to the difference between the two inputs.