Department of Mechanical Engineering
Massachusetts Institute of Technology
2.14 Analysis and Design of Feedback Control Systems
Fall 2004
Assignment #3
Distributed Wednesday September 22
Due: Wednesday, September 29 by 5pm in 35-231 Drop Box

Reading:
Chapter 4 and Chapter 10 sections 10.1 &10.2

Problem 1

Show that the Laplace transform of the function

$y(t) = \frac{w_n}{\sqrt{1-z^2}} e^{-z w_n t} \sin(w_d t)$ is given by $Y(s) = \frac{w_n^2}{s^2 + 2zw_n s + w_n^2}$

Hints:

You can use the approach outlined on pp 46-47 of the text, or develop it from the Euler identities:

$$Ae^{jwt} = A\cos(wt) + jA\sin(wt)$$
$$\sin(wt) = \frac{e^{jwt} - e^{-jwt}}{2j}$$
$$\cos(wt) = \frac{e^{jwt} + e^{-jwt}}{2}$$

Problem 2

Nise Problem 4.25

Problem 3

Nise Problem 4.29 -

Derive your answer from the graphs first and then use the Matlab "Step" function to confirm your answer.

Problem 4

Consider the following transfer function, which represents a second order system with a measurement device that acts like a first order system (i.e. a filter)

$$G(s) = G_p(s)G_h(s)$$

$$G_p(s) = \frac{w_n^2}{s^2 + 2zw_n s + w_n^2}$$

$$G_h(s) = \frac{a}{s+a}$$

Assume that w_n=10 and z=0.5 and that a is a design parameter:

a) Using partial fraction expansion, determine the time response of this transfer function to a unit step input.

b) Looking at the response, determine the range of the pole ***a*** such that the step response will be dominantly 2nd order with w_n=10 and z=0.5.

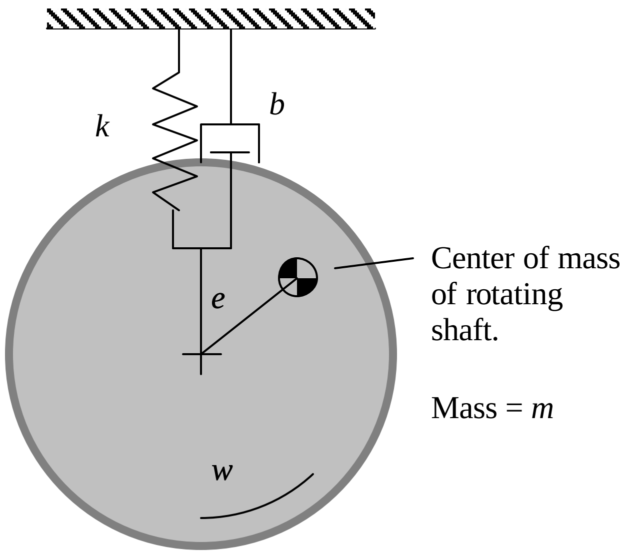
c) Now find the range of ***a*** such that the response will be dominantly 1st order and specify the time constant of this response.

d) Using Matlab, plot three step responses, one for the two rages above and at least one for the intermediate range. Comment on these and in particular on the intermediate one

e) How would you estimate the settling time of the intermediate system?

Problem 5

We are concerned here with the problem of an improperly balanced rotating shaft. This imbalance leads to vibrations on the machine, and we want to use frequency response methods to understand why. Consider the system model shown below:



m= 2 kg
k=200 N/m
b=12 N-sec/m
e=5 cm

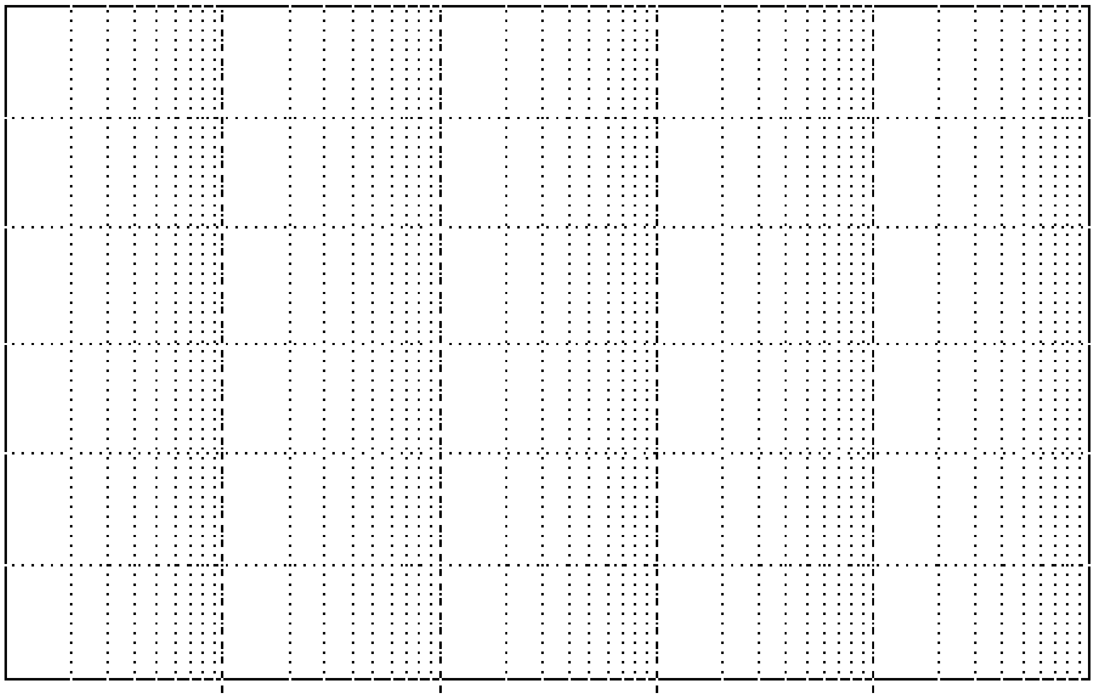
a) Show how we can arrive at a model of the force input to the system that is given by:

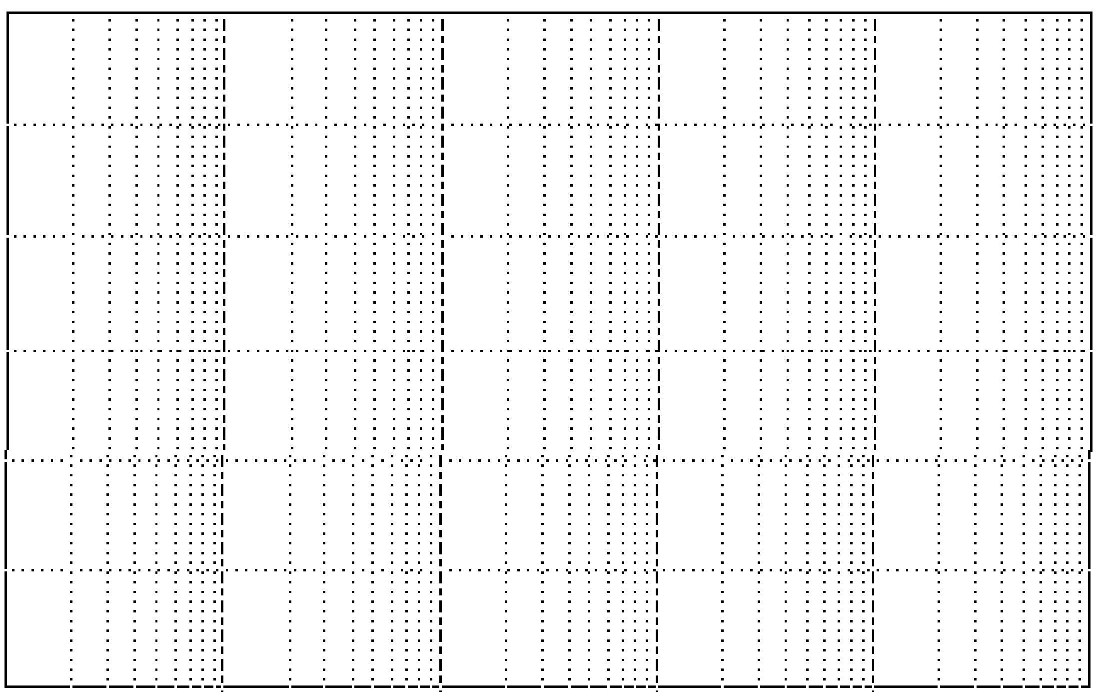
$$F_w(t) = mew^2 \sin(wt)$$

b) For the system as shown derive the transfer function $G(s) = \dfrac{Y(s)}{F_w(s)}$ where Y is the motion of the rotating mass.

c) On the semi-log paper attached, sketch the asymptotic Bode plot of $G(s)$ and then sketch a more exact plot <u>showing all key features</u> on the magnitude and phase diagram. Be as exact as possible. NOTE: For scaling purposes, plot $G(s)$ in units of centimeters, not meters, i.e. plot $G_{cm}(s) = 100 * G(s)$

d) On the same axes as for part c) sketch the <u>magnitude</u> $|F_w(w)|$ for the force input. *Please explain how you arrived at this plot.*





Problem 6
Ship roll stabilization problem - Disturbance analysis

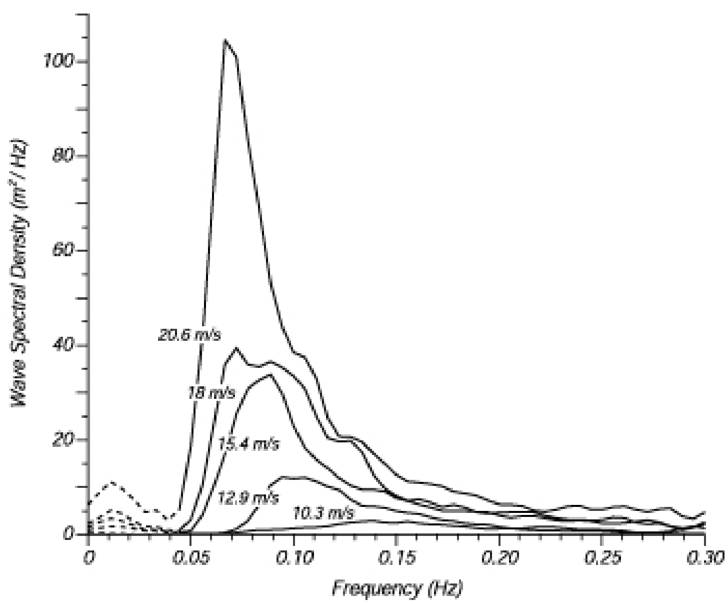
Roll Motion of a ship (right) and lack of roll with stabilizer fins (left)

Assume that your modeling exercise from the last assignment yielded a transfer function for this system

$$\frac{Roll}{Torque} = \frac{q(s)}{T(s)} = \frac{K}{s^2 + 2zw_n s + w_n^2}$$

where K = 1, ζ=0.25 and ω_n = 0.1 rad/sec.

Now consider the data from:
http://oceanworld.tamu.edu/resources/ocng_textbook/chapter16/chapter16_05.htm
which give the typical frequency bands for waves on the open sea:



Wave spectra of a fully developed sea for different wind speeds according to Moskowitz (1964).

NB: This is a power spectral density plot, so you can interpret the vertical axis as the magnitude squared. Also note that the frequency scale is linear and in Hz not rad/sec.

a) How would you model this input to the ship roll dynamics?

b) Plot the Bode Diagram for the ship dynamics

c) Referring to that plot, what is the worst case for the disturbance? (First define what is “bad” and then find the “worst”)

d) If you could re-design the ship and change the roll dynamics for this worst case, what would you do?

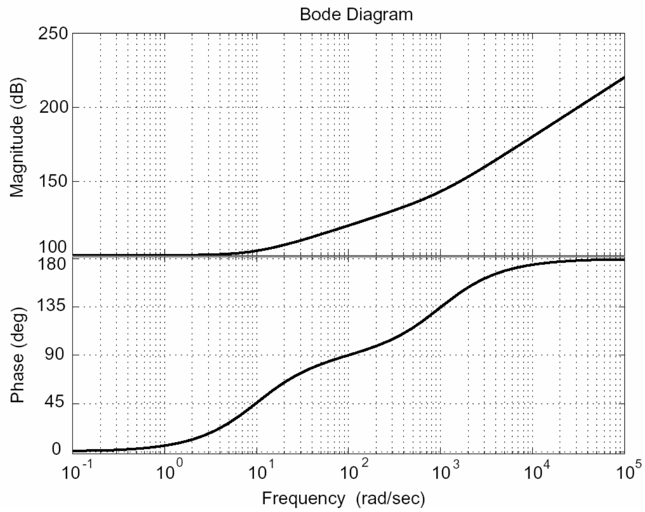
Problem 7

Match the attached bode plots labeled from I to VI with the corresponding transfer function labeled from A to H. Each bode plot has only one corresponding transfer function.

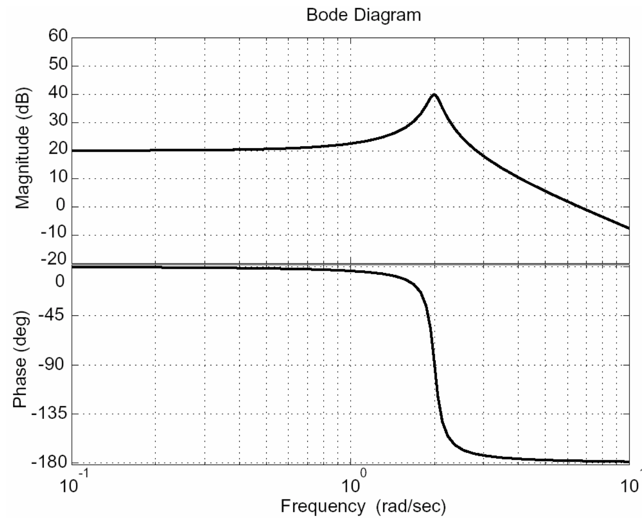
For each choice, write a sentence or two to explain your decision

A) $H(s) = 10\,(s+10)\,(s+1000)$	E) $H(s) = \dfrac{40\,s^2}{(s^2+0.5\,s+4)}$
B) $H(s) = \dfrac{(s+1)}{(s+40)}$	F) $H(s) = \dfrac{s^2}{(s+2)^2}$
C) $H(s) = \dfrac{(s+10)^2}{s\,(s+200)}$	G) $H(s) = \dfrac{40\,s}{(s^2+s+4)}$
D) $H(s) = \dfrac{40}{(s^2+0.2\,s+4)}$	H) $H(s) = \dfrac{0.4\,(s+1)}{(s+40)}$

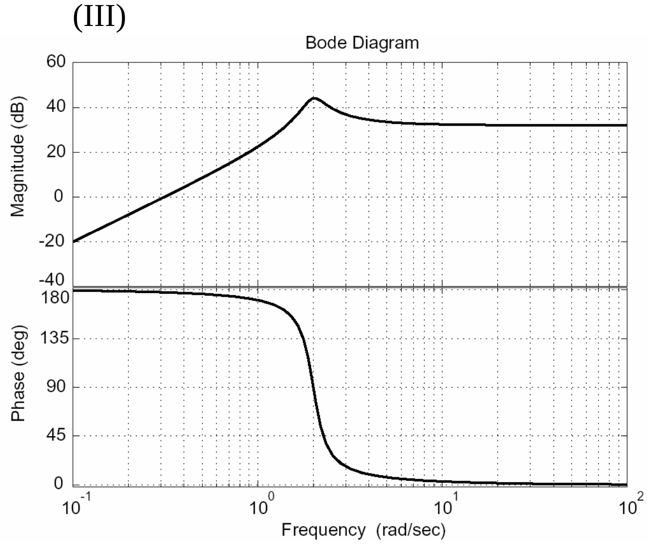
(I)



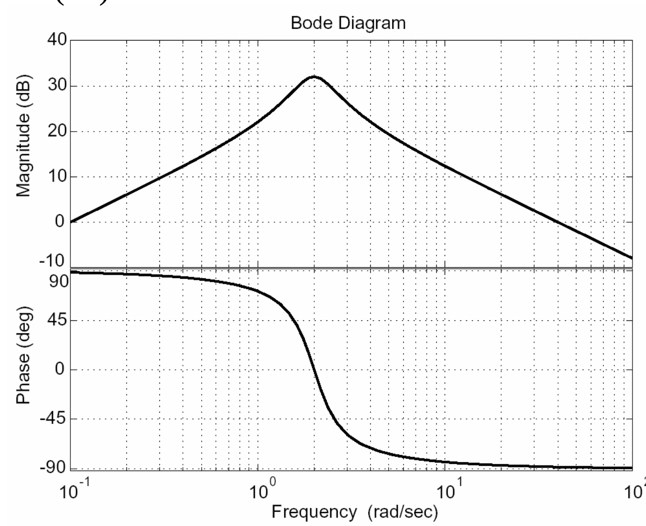
(II)



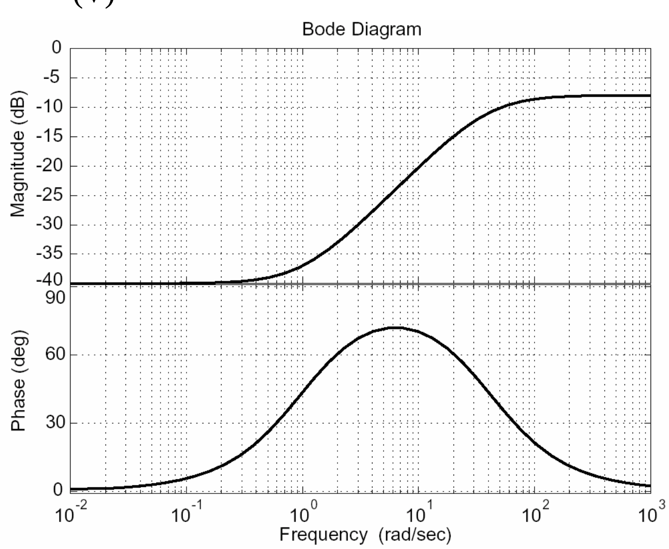
(III)



(IV)



(V)



(VI)