Long-Baseline Acoustic Navigation for Under-Ice AUV Operations

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Abstract

The recent Arctic GAkkel Vents Expedition (AGAVE) to the Arctic Ocean's Gakkel Ridge (July/August 2007) aboard the Swedish ice-breaker I/B Oden employed autonomous underwater vehicles (AUVs) for water-column and ocean bottom surveys. These surveys were unique among AUV operations to date in requiring geo-referenced navigation in proximity to the seafloor beneath permanent and moving ice cover. We report results for long-baseline (LBL)

acoustic navigation during autonomous under-ice surveys near the seafloor and adaptation of the LBL concept for several typical operational situations including navigation in proximity to the ship during vehicle recoveries. Fixed seafloor transponders were free-fall deployed from the ship for deep positioning. The ship's helicopter collected acoustic travel times from several locations to georeference the transponders' locations, subject to the availability of openings in the ice. Two shallow beacons suspended from the ship provided near-surface spherical navigation in ship-relative coordinates. During routine recoveries, we used this system to navigate the vehicles into open water near the ship before commanding them to surface. In cases where a vehicle was impaired, its position was still determined acoustically through some combination of its acoustic modem, the fixed seafloor transponders, the ship-deployed transponders, and an on-board backup relay transponder. The techniques employed included ranging adapted for a moving origin and hyperbolic navigation.

1 Introduction: The Arctic GAkkel Vents Expedition (AGAVE)

The Arctic GAkkel Vents Expedition (AGAVE) took place aboard the Swedish icebreaker I/B Oden from July 1, 2007 to August 10, 2007 (Fig. 1). A section of the global mid-ocean ridge (MOR) system, the Gakkel Ridge remains relatively unexplored due to its location beneath the permanent drifting Arctic ice pack. Our principal scientific objectives were to characterize the geology, biology, and hydrography of deep sea hydrothermal vent sites on the Gakkel Ridge. It is the slowest spreading of all mid ocean ridges, a fact leading to fundamental but poorly understood differences in crustal formation relative to the rest of the MOR (Dick et al., 2003). Furthermore, the hydrographic isolation of the Arctic Basin in general and of the Gakkel Ridge in particular has important implications for the biogeography of vent-endemic species (Shank et al., 2007). Finally, considerable mystery remains concerning the frequency, magnitude, and distribution of hydrothermal vent sites on the Gakkel Ridge (Edmonds et al., 2003; Edmonds et al., 2007).

To begin to address these issues, the expedition employed a variety of deep-sea assets including a conventional lowered conductivity, temperature, and depth (CTD) probe, a novel towed camera/sampler system and a pair of AUVs, one of which is shown being recovered in Fig. 2. The AUVs performed exploratory water column plume survey work along with high-resolution bathymetric mapping. Coordinated work between the CTD probe, sampler system, and AUVs was necessary to exploit the unique sensing modalities of each asset effectively. Relative to the cable-lowered CTD and sampler system, the AUVs were relatively unconstrained in their motion by the drifting ice. Surveys with towed assets necessarily followed courses set by the ice drift direction and speed. Our mission objectives mirrored previous open ocean AUV-based hydrothermal exploration work (German et al., 2008; Yoerger et al., 2007) where AUVs using geo-referenced navigation have proven to be effective tools for performing exploratory and repeatable surveys over sites hundreds to thousands of meters in extent. Absolute underwater positioning, as opposed to dead reckoning alone, enables repeatable and efficient surveys consistent with an AUV's on-board sensor suite and the intended data gathering task of each dive. These tasks can vary from performing broad area

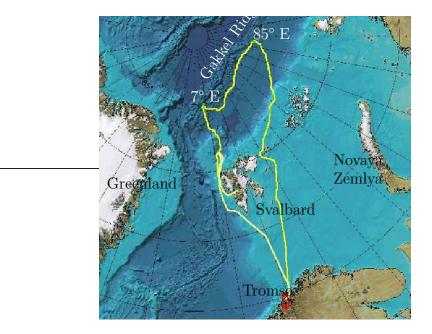


Figure 1: AGAVE cruise track showing study sites on the Gakkel Ridge at 85° N, 7° E and at 85° N, 85° E. Long baseline transponders were deployed at both survey sites, two at 7° E and four at 85° E.

survey with track lines spaced hundreds of meters apart to revisiting specific locations tens of meters in size for detailed investigation. We therefore endeavored to attain geo-referenced navigation commensurate with typical O(m) open ocean accuracies, except under ice. We report the successful deployment and under-ice operation of conventional long baseline (LBL) acoustic navigation along with several adaptations required by various operational scenarios imposed by working with AUVs in the ice-covered Arctic.

Weather conditions affect the ice thickness, availability of open leads, ice drift speed, and ambient acoustic noise levels, which together dictated the nature of our operations. Weather during the expedition was relatively mild with temperatures hovering near 0 °C and consis-



Figure 2: The Jaguar AUV being brought back on deck after a mission.

tently low winds.¹ Ice cover at our two study sites was close or very close packed ice (8/10 to 9/10 mean areal coverage) with a thickness of 2 m to 5 m. Ice drift speeds typically varied between 0.05 m/s and 0.1 m/s and occasionally up to 0.2 m/s. These speeds were sufficient to require several repositionings of the ship per day in support of operations over fixed sites on the seafloor and precluded the establishment of a semi-permanent ice camp on the ice itself. While dense ice cover complicated nearly every aspect of AUV operations relative to open ocean deployments, we used available open leads extensively to avoid breaking ice with the ship as much as possible and never required the expedition's ice cutter. We report our methods for beacon deployment and survey, tracking during routine vehicle recovery, and impaired-vehicle localization in Sec. 3.

2 Background

2.1 Previous Under-ice AUV operations

A few authors have discussed the design and performance of AUV navigation systems for polar latitudes. (Vestgård, 1985) suggests an under-ice LBL positioning system based on a combination of ice-moored and seafloor beacons. (von der Heydt et al., 1985) describe a self-calibrating array of acoustic hydrophones and pingers affixed to moving ice-floes that could be adapted for AUV tracking, although autonomous navigation would require telemetering the array geometry to the vehicle. (Deffenbaugh et al., 1993) describe an algorithm for very long baseline acoustic navigation utilizing multipath sound propagation for near ice navigation. (McEwen et al., 2005) describe under-ice AUV navigation experiments from 2001 with dead-reckoned navigation from a commercial coupled Doppler Velocity Log (DVL) and Inertial Navigation System (INS).

AUV operations in ice-covered seas are also not without precedent. Pioneering work by (Francois and Nodland, 1972) in the 1970s resulted in the development of the UARS AUV along with a near-surface tracking system based on ice-moored acoustic beacons. These were used track UARS during missions designed to produce high resolution profiles of the underside of the ice (Francois, 1977). More recently, (Bellingham et al., 1994) reports under-ice work with Odyssey class AUVs in the Arctic. In 1996 the Theseus AUV layed cable for an under-ice acoustic array over a distance of several hundred kilometers and then returned to its launch location (Thorleifson et al., 1997). Theseus navigated using a medium-grade inertial navigation unit and bottom-tracking DVL augmented by a series of widely-spaced acoustic beacons placed at critical locations along the vehicle's intended flight path. More recently, the Autosub AUV flew a number of under-ice missions in both polar oceans,² including a 50 km mission beneath Antarctica's Fimbul Ice Shelf (Nicholls et al., 2006). Other recent projects include drift-ice studies (Wadhams et al., 2003) and AUV deployments in ice-covered lakes (Forrest et al., 2007).

These expeditions differed from the AGAVE mission in their objectives, navigation method-

¹During AUV operations median temperature was 0.0 °C with lower and upper quartiles of -0.6 °C and 0.5 °C, respectively. Median wind speed during the same period was 4.5 m/s with lower and upper quartiles of 1.5 m/s and 6.2 m/s, respectively.

²Autosub under ice website: http://www.noc.soton.ac.uk/aui/.

ologies, and deployment/recovery strategies. Most sought characterizations of ice cover or ice-related processes and thus required near-surface operation in proximity to the ice; most relied on dead-reckoning for navigation; and all deployed and recovered their vehicles in relatively open water at the ice edge or through a maintained hole at an ice camp. In contrast the AGAVE objectives called for deep (4000 m) near-bottom survey work in a relatively small operating area of a few square kilometers with deployment and recovery through permanent but moving ice cover from a mobile ship.

2.2 Beacon-based Long-Baseline Navigation

Long Baseline (LBL) acoustic positioning uses travel times converted to ranges from two or more widely spaced (long baseline) stationary beacons (a net) to trilaterate the location of a moving receiver (vehicle) in two or three dimensions (Hunt et al., 1974; ?). The beacons are typically stationary (fixed baseline) and moored to the seafloor with tethers. The beacons can also be held in fixed relative locations on a moving platform such as a ship (moving baseline). Their signals must be uniquely identifiable which is typically accomplished through the assignment of unique coded pulses or frequencies to each beacon. The beacon locations must be determined during an initial off-line survey. During operation the receiver either actively interrogates (pings) the beacons acoustically and measures the round-trip travel-time to each beacon or else passively listens to the net being interrogated remotely or triggered on a synchronized time base. Travel times are converted to slant ranges which yield spherical constraints on vehicle position (active interrogation) or hyperbolic constraints (passive listening). Additional information, such as an independent estimate of the receiver's depth, can also constrain the position fix. When the number of constraints matches the number of unknown positional degrees of freedom (exactly determined) a position fix can be attained geometrically. The over-determined case is typically solved using least-squares methods.

While the basic premise of spherical and hyperbolic LBL navigation is straightforward, its implementation for deep ocean navigation requires addressing beacon survey, sound velocity profile compensation, and various systematic and random noise sources. Appendices A through C summarize these issues as they apply to typical open-water deployment of LBL nets in order to provide essential background for our discussion of LBL beneath the Arctic ice for readers unfamiliar with its more typical use.

3 Under-Ice Experiences

Fig. 3 schematically depicts the basic operational setup of the transponders at each of the study sites. The accompanying table indicates which of the positioning techniques and acoustic paths were used for various operations during the expedition. The following sections describe in detail each of these navigation methodologies as they apply to navigation and localization beneath permanent moving ice cover.

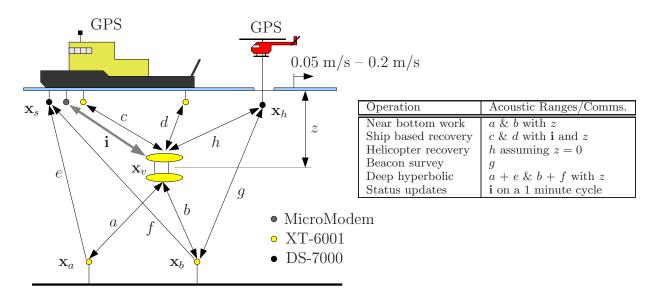


Figure 3: Schematic of the operational setup showing the acoustic ranges employed by the various navigation methodologies reported here (not all concurrently). The vehicle interrogated up to four XT-6001 transponders, two deployed from the ship and two on the bottom. Modem communication (i) was also established between the ship and vehicle. The modem telemetered travel-times and state information recorded on the vehicle to the ship and guidance information from the ship back to the vehicle. The ship's helicopter was used to range to the bottom transponders to survey their initial locations and also on two occasions to localize immobilized vehicles directly beneath the ice. The table summarizes the acoustic paths used during the different positioning operations. The speed range indicates the over-bottom ice drift speeds observed during AGAVE.

3.1 Beacon deployment, survey, and recovery

We deployed Benthos XT-6001 acoustic beacons with integrated releases at both study sites: two at 7° E and four at 85° E. Nominal beacon locations were selected based on pairwise coverage over features of interest on the seafloor. The last two beacons at 85° E were deployed several days apart and after the initial pair because the criteria specifying features of interest necessarily evolved over the course of exploratory work at the site. Beacons were deployed either over the side of the ship or from the ice with 35 kg descent weights on steel tethers between 100 m and 200 m in length. The tether lengths were chosen based on available bathymetric information to provide the vehicles with line of sight to the beacons from within the intended survey area. Beacons descended at approximately 35 m/min with minimal horizontal drift over the course of nominal 4000 m descents. Descending beacons typically drifted less than 100 m horizontally between their drop and surveyed locations indicating weak depth-averaged currents on the order of 1.5 cm/s.

To survey the beacons and obtain their geo-referenced locations using a typical ship-based survey method (App. A) would have been at best extremely time consuming and at worst impossible due to the difficulties with ship noise and keeping a hydrophone over the side and in the water while breaking ice. Instead a helicopter-based survey was used. Operators aboard the ship's helicopter used a portable Benthos DS-7000 acoustic ranging unit to acquire travel-times to each beacon at five to six locations arranged in a one-water-depth (4000 m typical) radius circle around the beacon's nominal (drop) location. At each location the helicopter pilot hovered at a fixed altitude and lowered the transducer below (Fig. 4) such that a mark on the transducer wire 10 m above the transducer head was held at the sea surface. In order to keep the transducer in open water, the pilot was forced to maintain the helicopter's position relative to the drifting ice. The helicopter was therefore in motion relative to the earth.

Fig. 5 (top) shows the sequence of slant ranges collected while surveying the 10.5 kHz beacon at the 85° E site. These slant ranges were computed from raw round-trip travel-times using a sound speed profile derived from lowered CTD data and the ray-tracing algorithm described in (Hunt et al., 1974, pg. 20–22). The beacon location was then determined in a geographic (latitude/longitude) coordinate frame by a least squares fit to the slant ranges given the known ranging locations.

The slant range residuals depicted in Fig. 5 (bottom) all show a location-dependent systematic drift that dominates any random noise except for a single obvious outlier (ranging 26). We attribute these systematic errors to ice drift exacerbated by a deficiency in our survey methodology. Each set of rangings took approximately one minute (10 s per ranging) to acquire. Because the helicopter pilot was forced to maintain position relative to the moving ice, the each ranging was collected in a slightly different location; however, only a single GPS location was manually recorded for each set of rangings.

If in fact the helicopter moved with the ice while ranging, the ranges should predictably grow or shrink according to the dot product between the ice drift velocity and the vector connecting the ranging location with the actual transponder location. Ranges collected downstream will grow while ranges collected on radii perpendicular to the ice drift velocity



Figure 4: Hydrophone being lowered into an open lead during a helicopter survey of a long baseline transponder's location on the seafloor approximately 4000 m below. Acoustic ranges were opportunistically collected at 5–6 open leads per transponder. A GPS receiver on board the helicopter enabled geo-referencing of the survey positions and ultimately of the sub-sea transponder itself.

will stay approximately constant. The sets of ranges and residuals in Fig. 5 show systematic drifts that are consistent with a single bulk ice movement across the survey area.

Fig. 6 shows an error surface calculated as the RMS difference in range drift speed between the observed and predicted values for a range of possible ice drift velocities. The contours provide a sensitivity measure for how well the range data constrain the ice drift direction and indicate the data are consistent with a north-northeast drift direction. Independent ice drift data collected from a buoy fixed to the ice and approximately 15 km to the northeast of the survey site showed a north-by-northwest drift at between 0.11 m/s and 0.14 m/s during the day of the survey in rough but not unreasonable agreement with our best-fit ice drift velocity of 0.14 m/s to the north-northeast.

To reduce the influence of this drift on our transponder survey results we used only the first three rangings at each location. (These were recorded temporally closest to the recorded GPS location.) Using the first three rangings as shown resulted in an RMS slant range residual for this transponder survey of 1.2 m. This figure compares well with typical deep sea LBL positioning accuracy (e.g. 3.97 m as reported in (Bingham and Seering, 2006)). Nevertheless, considering the evidently dominant role played by ice drift in the slant range residuals of Fig. 5, in retrospect the quality of our survey data could have been further improved even without automated data collection by simply having noted the GPS location of the helicopter before and after collecting a set of rangings.

It is worth considering our final survey methodology relative to the other options available to us in the field: (1) using the ship to drive a traditional circular path around the beacons, (2) our initial strategy of landing the helicopter on the ice to collect rangings. Using the ship directly was not attempted due to slow progress while breaking ice (often < 2 km/h when constrained to a specific course), ship noise, and the risk associated with keeping a hydrophone over the side while under way. Use of the ship's helicopter offered the advantage

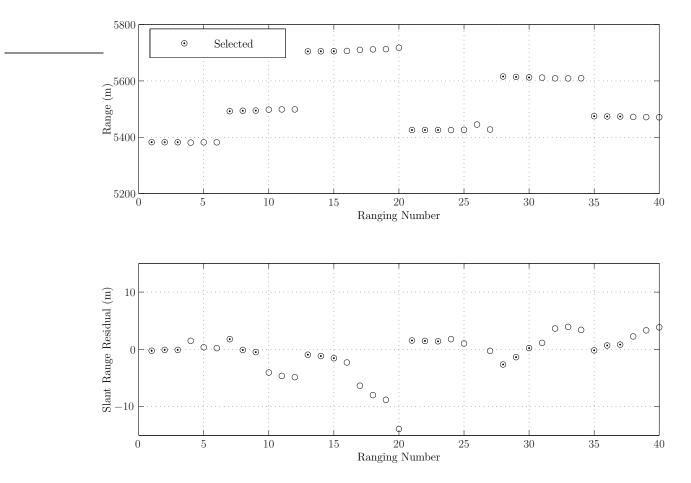


Figure 5: Helicopter-based beacon survey results for a 10.5 kHz beacon deployed at the 85° E site: (top) slant ranges compensated for sound-velocity profile showing the rangings selected for beacon localization; (bottom) residual slant range error relative to the least-squares solution to beacon location in geographic coordinates.

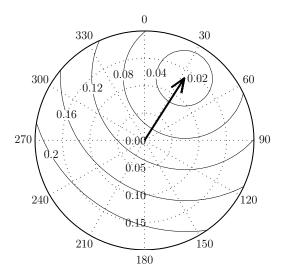


Figure 6: Error surface contours showing the RMS difference between observed and predicted range drift speeds as related to possible ice-drift velocities for the transponder survey data shown in Fig. 5. The arrow shows the best fit ice drift velocity, 0.14 m/s at a heading of 33° true.

that surveys with good geometry could be accomplished rapidly (< 1 h) so long as open leads were sufficiently plentiful. Landing the helicopter on the ice at each survey point was tried initially but provided no significant advantage over hovering surveys, required additional time, and exposed the helicopter crew to the potential danger associated with landing on the ice. The number of rangings collected by our helicopter-based surveys was far less than can typically be collected by a ship executing a traditional survey (thousands); however, acquiring a large number of rangings is only important if random noise dominates the slant range residuals. In our case, we attained RMS slant range residuals commensurate with typical 10 kHz LBL system accuracy and our dominant error source remained a systematic one, namely ice-induced motion of the helicopter.

The residual range errors for all beacon surveys were similar to that depicted in Fig. 5. It would nevertheless have been useful to independently quantify the accuracy of our helicopterbased transponder surveys by collecting range data from three or more transponders simultaneously and at depth where the effects of a variable sound speed profile are minimal. Unfortunately, our AUVs were limited to listening to only two transponders at a time and thus the travel time data from AUV deployments cannot provide independent confirmation of a consistent net geometry. A single data set was acquired using a towed asset at 85° E that included reception of three transponders, although only along a nearly vertical trajectory in a single horizontal location. This data suggested one of the three transponders recorded (the 10.0 kHz shown in Fig. 8) may have been incorrectly surveyed; however, no reasonable subset of the original survey data admitted a transponder location that was also consistent with the three transponder data. A persistent bottom-bounce would explain the apparently anomalous ranges in the three transponder data taken at this single location.

Our survey procedures during the actual expedition were simplified relative to the techniques

used to arrive at the results described above. In particular, we employed a depth-averaged sound speed to compute slant ranges from travel-times and solved for beacon location in a local Cartesian coordinate frame based on a conversion between degrees latitude/longitude and meters north/east derived in Bowditch for the Clarke 1866 Spheroid (Bowditch, 1966, pgs. 1186–1187). The average difference for the transponder survey shown between slant ranges computed using a mean sound speed and using ray-tracing was less than a meter and its effect on surveyed transponder location further minimized by the radial symmetry of the survey plan. In contrast, solving for this beacon's location in a geographic coordinate system reduced the RMS slant range residuals by a factor of four relative to the realtime estimate, and moved the solution by several meters. This was not unexpected since at polar latitudes the distortion created by projecting lines of longitude into parallel vertical lines can become significant on the scale of AUV operations.³ In terms of achieving our goals during the expedition, these simplifying assumptions (also employed by the vehicles in real time) proved insignificant because our surveys were designed to cover somewhat enlarged areas thereby accounting for small errors in absolute positioning. The difference between real-time and post-processed vehicle position was on the order of 10 m and an order of magnitude smaller than the smallest AUV survey extent executed. On the other hand, the influence of ice-induced helicopter motion during transponder surveys was obscured until post-processing, which might otherwise have precipitated a change in our survey methodology.

We were able to recover two of the six deployed transponders by acoustically tracking the ascending beacons until they reached the underside of the ice, and then using the ship to break up the ice above them until they became visible in the crush. The buoyancy and toughness of the Benthos 6000 series beacons made these recoveries possible. Of the four unrecovered transponders, one failed to release and three others were tracked to the underside of the ice but time constraints prohibited any attempt to break ice above them. Based on the ease with which the two successful recoveries were accomplished, we are confident that these remaining three transponders might also have been recovered had time permitted. Deciding whether it is in fact worth recovering a transponder requires balancing the cost of the ship time required to transit to the transponder's location (dependent on ice conditions and competing scientific objectives) and the cost of the transponder itself.

3.2 Deep LBL

Our AUVs use the WHOI MicroModem for both acoustic ranging and modem communications (Singh et al., 2006; Freitag et al., 2005). The modem software is configured to record travel-times at four unique frequencies in the 9 kHz - 12 kHz range. Because two channels were reserved for the ship-LBL transponders on every AUV dive, only exactlydetermined two-beacon fixes, incorporating the measured depth of the vehicle, were computed autonomously on the vehicle. These were computed using a preselected pair from among the four deep transponders. The modems were configured to run on two minute cycles that interlaced three 10 s ranging pings to the beacons with one uplink and one downlink for acoustic communications between the vehicle and ship. Acoustic communications consisted of summarized vehicle state (uplink) and control commands (downlink).

 $^{^3\}mathrm{At}\;85^\circ$ N, true north changes by 0.1° for each kilometer of easting.

Fig. 7 shows an example of travel-times recorded by one of our AUVs during a high-altitude hydrothermal plume survey mission. The record is typical of dives at both sites, showing reception of both deep and ship-mounted beacons at the surface prior to descent, excellent reception of the surface beacons until they were hauled back aboard at a vehicle depth of 500 m (approximately GMT 14:00:00) and then reliable reception of both deep beacons during descent beyond about 2000 m (approximately GMT 16:00:00). The travel-time records show almost no evidence of consistent multipath returns, though shadowing by surrounding bathymetric features did occur on low altitude dives. Instead, bad travel-times typically exhibited a uniformly distributed randomness and were automatically rejected by a ring buffer median filter.

Excepting the repeatable deep beacon outage at the surface, beacon reception was remarkably consistent. This may in part reflect relatively low levels of ambient acoustic noise. In ice-free seas and in the absence of proximal shipping, the dominant sources of noise in the 1 kHz to 10 kHz range are wind-driven free surface agitation and precipitation (Wenz, 1962). During AGAVE, nearly total ice cover combined with low winds and only light snowfall largely eliminated these noise sources. However, the ice pack itself generates noise through various mechanisms of stress relief and this spectrum extends into the kHz range (Dyer, 1984). The upper end of this spectrum is dominated by a wide band of noise associated with thermal cracking which is most intense during periods of cooling (Milne, 1972). Given that the expedition took place during the 24-hour daylight of the Arctic summer and that only relatively small variations in atmospheric temperature were observed, noise from thermal cracking may well have been minimal; however, we lack direct ambient noise measurements to support this conjecture.

The repeatable deep beacon outage observed during ascent and descent may have been the result of acoustic shadowing of the vehicle's transducer by its foam filled hulls in combination with self-generated noise created by the vehicle's vertical thruster during powered ascent and descent. The LBL transducer on our vehicles is located between the hulls and has a hemispherical beam pattern oriented upwards. This arrangement creates shadows both directly above and directly below the vehicle. These shadows are inconsequential in the approximately planar LBL net geometry encountered during deep operation and when tracking near the surface, but they come into play when substantial vertical offsets exist between the beacons and the vehicle. The upward-refracting sound velocity profile of the Arctic Ocean (App. B) tends to trap acoustic waves generated near the surface in a shallow waveguide (Kutschale, 1969); however, it is unlikely to have been a factor here since the acoustic channel was nearly vertical, and the vehicle generally did hear the deep beacons while on the surface and not actively ascending or descending. LBL reception while on the surface is apparent at the beginning and end of both travel-time records in Fig. 7.

Fig. 8 shows the real-time fixes computed from the travel-time data in Fig. 7 in comparison to the AUV's mission way-points. High altitude operation facilitates acoustic LBL navigation by reducing bathymetric obstructions to line-of-sight, nevertheless it is noteworthy that the vehicle did not compute a single bad fix on this dive despite using only a median filter for rudimentary outlier rejection. Lower-altitude dives suffered some multipath and occasional outages. For these dives, LBL navigation was employed in real time only to guide the vehicle during its descent to a pre-determined landing site. Once the vehicle was sufficiently close

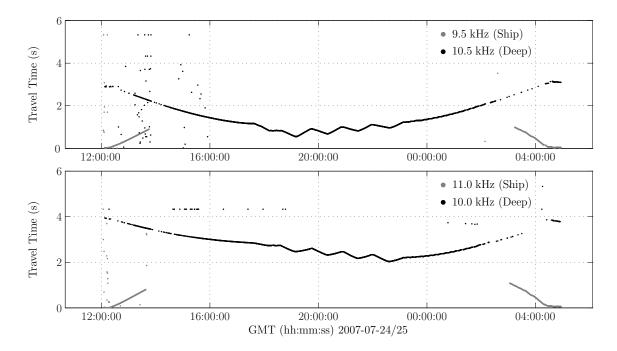


Figure 7: LBL one-way travel-times: (top) one ship and one deep beacon; (bottom) the remaining ship and deep beacon. A constant-depth high-altitude (nominally 300 m) AUV dive generated this record. It is remarkable for its almost complete lack of consistent multipath returns or other noise, cf. (Yoerger et al., 2007; Olson et al., 2006).

to the seafloor to attain bottom-lock with its on-board DVL (30 m) it switched to deadreckoning with its DVL and north-seeking fiber-optic gyrocompass. In these cases, we fused LBL data with the dead-reckoned track in post-processing. In keeping with standard practice, during real-time operation the vehicle computed navigation fixes in a local Cartesian frame and used a mean sound velocity to convert travel times to ranges. For the dive shown, post-processed re-navigation completed after the expedition resulted in an average position difference of 17.4 m consisting almost entirely of a uniform offset. Most of this change was due to refined beacon locations and the recomputation of navigation fixes in a geographic coordinate frame. This difference between real-time and post-processed navigation was typical and an order of magnitude smaller than the extent of our smallest near-bottom surveys. The impact of this discrepancy on data acquisition was therefore minimal.

3.3 Ship-LBL

Recovery required navigating the vehicle into a pool of open water near the ship typically only a few tens of meters across. Two acoustic transponders suspended from the ship between 7 m and 11 m below the surface forward and aft on the port side provided moving-baseline spherical tracking of the vehicle in support of these recoveries. The baseline between the transponders was 47 meters, half the ship's total length, and as large as possible with respect to constraints imposed by the layout of the deck and the need for quick deployment and recovery in response to encroaching ice. Assuming isotropic range noise, the highest navigational accuracy for a two-beacon LBL system will be achieved when the target is $\frac{1}{\sqrt{2}}$ times

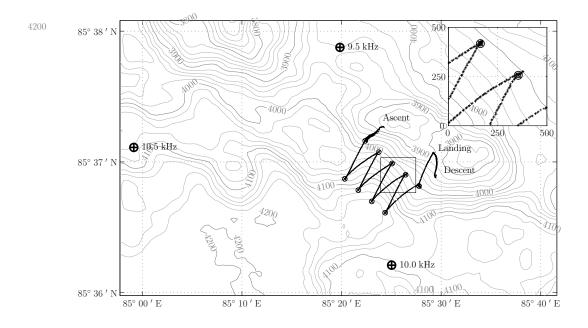


Figure 8: Real-time LBL position fixes overlayed on the survey plan for a constant-depth AUV dive (300 m nominal altitude) at the 85° E study site. The inset shows a closeup of a 500 m by 500 m area. The survey plan consisted of a series of nine way-points. Each is indicated in the plot by a circled 'x'. Beacon locations are shown along with their associated frequencies. For this dive, the vehicle used the 10.0 kHz and 10.5 kHz beacons. To execute the survey, the vehicle controlled its heading to point at each successive way point. Under the influence of a weak north-west current this control strategy produced the slight bowing visible in the vehicle's path between way points. Fig. 7 shows the raw travel-time data for this same survey.

the baseline length away from either transponder (i.e. when the vectors to each transponder are orthogonal). This system provided nearly optimal geometry during the final stages of a recovery. In most cases we were able to guide a vehicle into an opening while cruising at 5 m depth so that it could be visually sighted by observers once clear of overlying ice, after which we commanded it to surface. In fact, knowledge of the boundaries of open water, for which we had only visual estimates, proved to be the limiting factor during recovery rather than tracking accuracy.

Ranges from the ship-mounted beacons provided the ranges c and d in Fig. 3. Position fixes from this "ship-LBL" system were computed on-board the ship using travel-time information acoustically telemetered from the vehicle, transformed into world coordinates, and plotted along with ship position in order to provide the visual feedback necessary to manually steer the vehicle via telemetered commands for recovery (Fig. 9). An ultra-short baseline (USBL) system represents a viable alternative to our system;⁴ however, the extra weight of a USBL array or responder would have reduced valuable scientific payload capacity on our AUVs. Short-baseline systems employ transducers arranged similarly to our ship-LBL system; however, the SBL navigation equations are inaccurate at the short ranges (relative to baseline length) required for recovery.⁵

Tracking information was acquired at horizontal ranges up to 2 km from the ship. Larger ranges would likely have been possible but were never required because we were able to reposition the ship within this range during recoveries. However, the relatively short baseline imposed by the ship's length resulted in poor angular resolution when ranges exceeded a few baseline lengths. This behavior is evident in Fig. 9 which shows steadily decreasing range throughout the recovery with large initial jumps in bearing until the vehicle closes to within about six baseline lengths. During recoveries, the ship-LBL system was deployed only after repositioning the ship near open pools suitable for recovery. Repositioning was timed to coincide with the vehicle having reached a depth of between 1000 and 500 meters from the surface so that its final ascent could be tracked. The ship-LBL beacons could not be deployed over the side while underway because of the danger to equipment posed by breaking ice, consequently operators were blind to the vehicle's position while repositioning the ship (this same caveat would have applied to a USBL system).

A few operational subtleties with respect to operations using the setup in figure 3 and the ship based LBL bear mention:

1. Acoustic communication with the vehicle was necessary not only to send the commands necessary to drive the vehicle to the ship but also to telemeter travel-time and vehicle depth to the surface. In principle, ship position could have been telemetered to the vehicle (Eustice et al., 2006) and a position computed by the vehicle subsea. We opted against this approach to avoid relying on the vehicle's outlier rejection algorithms and to allow for occasional changes in beacon locations on the ship to improve baseline geometry or line-of-sight to the vehicle. Since the vehicle consequently did

⁴The Autosub AUV employs a USBL system for homing in support of through-ice recovery (http://www.srcf.ucam.org/polarauvguide/tools/nav.php, retrieved May 15, 2008).

⁵The SBL navigation equations are derived assuming incoming acoustic signals impinge on the array as a plane wave and compute range and bearing to the target based on the time delay between detection at either beacon.

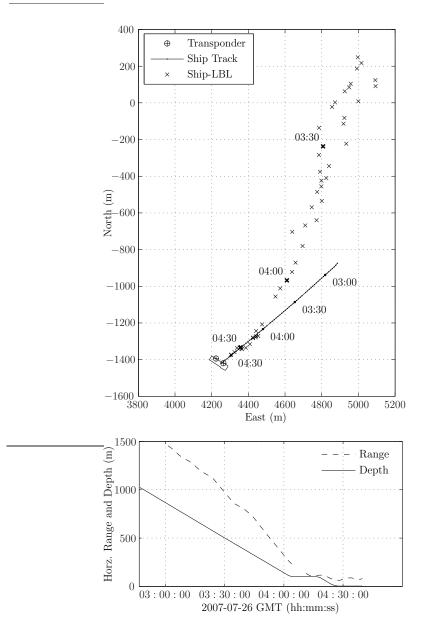


Figure 9: Guided vehicle recovery utilizing ship-relative LBL navigation: (top) ship track and vehicle position fixes plotted in an earth-fixed coordinate frame with GMT markings for the half hourly positions of each.; (bottom) converging vehicle depth and horizontal range from the ship vs. GMT. The ship's outline is plotted to scale, at recovery time, showing the positions of the transponders on the port side. The vehicle was commanded acoustically to surface at an approximate horizontal range of 80 m from the ship (judged to be within an open pool on the port side) and then recovered with the ship's small boat.

not know its own position, we steered it on courses defined by speed and heading rather than by requesting it to home to a position.

- 2. With only two transponders a baseline ambiguity exists. We typically resolved the ambiguity by arbitrarily choosing one side of the baseline and then commanding the vehicle on a heading perpendicular to the baseline. If the vehicle advanced toward the baseline, our choice was correct, if it receded, the other solution was correct. This worked as long as vehicle speed was substantially greater than ice drift velocity. During periods of relatively fast ice drift the procedure had to be repeated on a reciprocal heading to ensure that the relative motion observed was not obscured by the drift of the ship/baseline itself.
- 3. Inconsistent geometry was also an issue, particularly as the vehicle neared the baseline, until we developed methods to improve the determination of baseline geometry. We chose transponder locations on the ship with clear line of sight between them in order to permit the use of a tape measure to determine the length of the resulting baseline with sub-meter accuracy. We attained transponder depths to within a fraction of a meter by aligning marks on the transponder tethers with the sea surface. Finally, we applied an offset to telemetered vehicle depth to account for a non-zero indicated depth at atmospheric pressure. Wave-induced ship motions that would have distorted baseline geometry in open seas were negligible while lodged in the ice.

3.4 Ice-Drift Compensated Localization

On two occasions we adapted our helicopter-based transponder survey method to a moving coordinate frame for localizing immobilized AUVs pinned underneath the ice. In both cases the vehicle was impaired, being either unable to maintain depth or else offline with its batteries drained completely. Ship-LBL was unusable in these cases because in the former case the vehicle had lost line-of-sight with the ship-LBL transponders, and in the latter because acoustic communications with the vehicle had been lost. In order to recover the vehicle, its position had to be determined with sufficient accuracy for the ship to be able to break ice near enough to the vehicle to reveal it without also damaging it. Acoustic travel times to the vehicle were acquired from several helicopter survey locations using either the MicroModem or an independently powered backup relay transponder on board the vehicle.⁶ Acoustic methods were sufficient to localize the vehicle within tens of meters, after which meter-scale RF localization with a short range avalanche beacon became possible.

To account for ice drift the vehicle was localized in a translating coordinate frame fixed to the ship that was assumed to be stationary relative to the ice and vehicle. Our procedure was as follows:

- 1. Lower an acoustic transducer from the helicopter into an open lead and acquire traveltimes to the vehicle.
- 2. Relay these travel-times and the helicopter's GPS location back to the ship.

 $^{^{6}}$ The backup transponder was not a component of the original vehicle design but was added during the expedition as insurance against primary navigation system failure.

- 3. Ship-side, transform the helicopter's geographic position into a local north-aligned Cartesian frame with its origin at the ship's location at the time of the ranging.
- 4. Plot a range circle around the helicopter position.
- 5. Repeat until confident of the vehicle's position relative to the ship (3 ranges minimum to produce an initial position fix with no side-of-baseline ambiguity, plus additional close ranges based on the tentative location to refine the fix).

Fig. 10 shows the result of the second localization, in this case using the vehicle's backup relay transponder to acquire ranges. The variation in ranges for each ranging location is small (< 10 m) in comparison to the inconsistency in ranges between different ranging locations. Taken as a whole the data set produces an range residuals with an RMS value of 35 m; however, the shorter ranges taken toward the end of the procedure are more consistent with the vehicle's observed position than the longer ranges and also more self consistent (data from the three closest ranging locations yielded an RMS residual of 10 m). That observation is consistent with increased range error due to relative motion between the ranging location, ship and vehicle, or rigid body rotation of the system as a whole. Our method takes only translation into account, but could be augmented to account for rotation by localizing the vehicle in a coordinate frame also aligned with the ship's heading. This proved unnecessary; however, removing the translational component compensated for a significant several hundred meters of motion over the course of the approximately hour long localization due to an ice drift speed of 0.1 m/s. Using the vehicle's MicroModem for ranges during the other similar recovery resulted in significantly more consistent ranges (all ranges agreed to within 15 m). That improvement in consistency may simply reflect the reduced distance between the ship and vehicle in that instance (500 m as opposed to nearly a kilometer).

3.5 Hyperbolic Localization

Though not a component of regular operations, on one occasion we employed the LBL net for hyperbolic localization of an impaired AUV. The AUV had aborted its mission because it exceeded its programmed maximum depth; however, a flaw in our abort logic prevented the vehicle from returning to the surface. Instead the vehicle remained submerged and holding a constant altitude of 25 m above the seafloor. Multiple attempts to acoustically trigger correct abort behavior failed because the vehicle was never able to successfully receive a message, although a small percentage of uplinks were successfully decoded topside.⁷ To our alarm, the vehicle's reported depth exceeded charted depth in the vehicle's reported location by several hundred meters suggesting the vehicle's navigation was in error. Without the ability to acoustically trigger abort behavior, we were forced to wait for the vehicle to drain its batteries, shut down, and return to the surface under the influence of its own positive buoyancy. Knowledge of the vehicle's correct location therefore became critical for facilitating under-ice search and recovery.

An independent estimate of vehicle location was computed by listening in on the activity of the LBL net with a Benthos DS-7000 deck box.⁸ Because the time of the outgoing

⁷Acoustic reflections from the nearby seafloor likely corrupted the lengthy message packets.

⁸The Benthos DS-7000 initiates its listening cycle on an outgoing ping. To avoid triggering the LBL net, we set

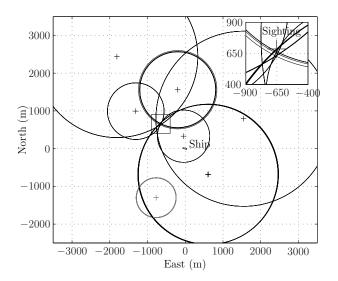


Figure 10: Ship-relative radial constraints on vehicle location during recovery of JAGUAR-02. Ranges at each survey location were acquired from helicopter to an independent relay transponder attached to the vehicle. Both the ship and vehicle were assumed to be lodged in the drifting ice and stationary relative to one another. The one set of inconsistent ranges shown in gray resulted from an erroneous ship's position and required more rangings to discover it than would have otherwise been required to localize the vehicle. The figure indicates a vehicle position of approximately 940 m from the ship's GPS antenna at a heading of 316° true. The vehicle was visually sighted from the helicopter approximately 1 m below the surface in an open lead at the location indicated in the inset on the way to a final ranging location.

interrogation ping from the vehicle was unknown, only the pairwise differences in total vehicle-transponder-ship ranges were available. In terms of the beacon pair shown in Fig. 3,

$$\delta_{a+e,b+f} = (a+e) - (b+f) .$$
 (1)

Ranges between each beacon and ship, e and f in (1), were known from geo-referenced beacon location and ship's GPS. Subtracting these off from the measured differences yields the conventional hyperbolic constraint $\delta_{ab} = a - b = \|\mathbf{x}_{\mathbf{a}} - \mathbf{x}_{\mathbf{v}}\| - \|\mathbf{x}_{\mathbf{b}} - \mathbf{x}_{\mathbf{v}}\|$.

For the purposes of confirming vehicle location we assumed a planar geometry and uniform sound velocity. Both assumptions are justified by that fact that the maximum difference between vehicle depth and beacon depths was less than 200 m and thus an order of magnitude smaller than the horizontal ranges involved. Three beacons were heard reliably from the surface resulting in the position fix shown in Fig. 11. The result confirmed that actual vehicle location coincided with the vehicle's own perception of its location. This fact ultimately resulted in the discovery, following a successful recovery, that depth was being reported as meters fresh water by the vehicle's incorrectly configured depth sensor resulting in overestimation of true depth by approximately 3%. Unfortunately, this bug also triggered our flawed abort logic by erroneously reporting the vehicle's depth as greater than its programmed abort depth.

4 Conclusion

This paper has reported underwater acoustic positioning results from the recent AGAVE expedition to the ice covered Arctic. The expedition employed two AUVs for the collection of scientific information in water more than 4000 meters deep. The expedition presented the unique challenge of producing repeatable geo-referenced deep water navigation for AUVs deployed and recovered by a vessel in ice-covered seas. That challenge was met by deploying and recovering acoustic transponders through the ice to create an otherwise standard deep sea LBL net. Beacons deployed from the ship to the seafloor were successfully geo-referenced using rapid helicopter-based surveys. LBL acoustic navigation techniques also proved adaptable to several operational scenarios beyond their typical application in open seas. A ship-based moving-baseline LBL system was set up to navigate the AUVs into open leads between the ice for recovery. LBL hardware and methods were also adapted to localize impaired vehicles under the ice and to hyperbolically locate a vehicle in deep water.

Existing leads and openings proved invaluable for nearly every aspect of AUV operations. These openings provided access to the underlying ocean for the deployment and survey of LBL beacons as well as for the deployment and recovery of the AUVs. Existing openings were especially critical for vehicle recovery as it proved difficult to maintain open pools of sufficient size without continuous use of the ship's thrusters, the wake from which would have easily overcome the AUVs' own thrusters. While wide leads simplified operations, we regularly employed leads a few meters across for transponder survey and AUV deployment. Our dependence on existing leads meant that on occasion the positions and movements of large (km²) contiguous floes dictated operations.

the outgoing frequency to one ignored by our net.

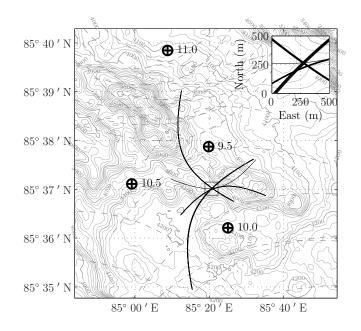


Figure 11: Two-dimensional hyperbolic constraints on vehicle location. Consistent constraints are indicated by black hyperbolae. Inconsistent constraints are indicated by dashed gray hyperbolae. Transponder locations are labeled with the associated frequency (kHz). All possible pairings of the 9.5 kHz, 10.0 kHz, and 10.5 kHz beacons provided consistent constraints. No consistent constraints were derived from pairings with the 11.0 kHz transponder presumably because of its greater distance from the vehicle. The inset shows a 500 m by 500 m closeup of the solution, the approximate location of which coincided with the vehicle's reported location.

AUVs were used as a component of the AGAVE mission primarily because, relative to the tethered assets, their motion was unconstrained by the drifting ice. Nevertheless, ice drift influenced AUV operations in several ways. The drifting ice required accurate ship-relative navigation especially when in close proximity to the ship in order to guide the vehicle into open pools before attempting to surface. Two recoveries of impaired vehicles pinned against the underside of the ice were facilitated by acoustically trilaterating their position in a moving coordinate frame affixed to ice. Finally, future attempts at helicopter-based beacon survey should ensure every individual ranging is coregistered temporally with a GPS fix to avoid limiting the attainable accuracy of beacon surveys.

Geo-referenced navigation was critical to the scientific goals of the AGAVE expedition. Georeferencing our AUV surveys allowed plume survey data to be coregistered with CTD data (navigated using topside GPS) which together guided our exploratory work on the Gakkel Ridge. The navigation requirements for plume surveys were not particularly stringent (these surveys spanned kilometers with tracklines spaced hundreds of meters apart); however, bathymetric surveys were targeted on relatively small features based on shipboard multibeam derived bathymetry or to follow up on promising near-bottom CTD data indicative of proximal hydrothermal venting. In these cases, our surveys spanned on the order of 100 m on a side, and LBL provided a reliable mechanism to guide the AUVs to the survey sites before near-bottom acquisition of bottom-lock for DVL-based dead-reckoning. Had evidence of high-temperature venting been discovered with the AUVs as hoped, we were equipped to navigate our towed assets for sample acquisition using relay transponders within the same LBL net. In this case, the O(1 m) LBL positioning precision attained would have been necessary to navigate samples and photographic data from the towed assets relative to AUV-derived maps.

Acknowledgment

Major funding for AGAVE was provided by the National Science Foundation, the National Aeronautics and Space Administration, and Woods Hole Oceanographic Institution. The expedition represented an international collaboration between scientists in the United States, Sweden, Japan, and Germany, and was part of an International Polar Year project entitled, "Biogeography and Geological Diversity of Hydrothermal Venting on the Ultra-Slow Spreading Arctic Mid-Ocean Ridge." We express our gratitude to the captain, ship's crew, and helicopter pilots of the I/B Oden for their exceptional contributions toward achieving the expedition's science and engineering objectives, in particular their delicate and competent handling of difficult vehicle recovery operations.

A Beacon survey

The absolute location of each LBL beacon in a geographic coordinate system must be obtained in order for LBL navigation to produce geo-referenced positions. This is usually done prior to operations using a survey procedure. Since the widespread availability of GPS, in open seas a support vessel will typically circle the drop location of a beacon at a constant radius (up to a water depth) to obtain a series of coregistered round trip travel times, GPS vessel positions, and vessel attitudes. Given a set of slant ranges r_i to a beacon from known ranging locations \mathbf{x}_i , the beacon's location \mathbf{x}_b can then be computed via least-squares according to

$$\hat{\mathbf{x}}_{b} = \underset{\mathbf{x}_{b}}{\operatorname{argmin}} \sum_{i} \left(\|\mathbf{x}_{b} - \mathbf{x}_{i}\| - r_{i} \right)^{2} .$$
⁽²⁾

Typical complications are removing outliers and compensating for the conversion between acoustic travel times and slant ranges for the variable sound speed of the water column. Obtaining accurate survey locations and inter-beacon (baseline) distances is important for accurate navigation because baseline error induces navigation error on the same order.

B Ranging and the sound velocity profile

The speed of sound varies with depth in the ocean first with temperature and then with pressure. The sound velocity profile (SVP) is typically determined using an expendable bathythermograph (XBT) or lowered CTD. Fig. 12 shows the CTD-derived SVP generated upon arrival at the 85° E site. This variable sound speed needs to be accounted for when computing slant ranges from travel-times along acoustic paths, particularly between points widely separated in depth, such as when surveying deep moored beacons from the surface. The distortion imposed by a gradient in sound velocity includes both the variable speed at which an acoustic pressure wave will travel along its path and the curvature of that path induced by ray-bending. The more vertical the path, the less the rays bend. Radially-symmetric survey patterns eliminate bias in horizontal beacon position introduced by, for instance, assuming a constant effective sound speed as we did for our preliminary beacon surveys. The estimated depth of the beacon will remain biased in this case; however, if the receiver/beacon geometry is nearly planar and receiver depth known, the distortion in horizontal positioning of the receiver is small.

C Noise sources

The theoretical performance bounds described in (Bingham, 2003) apply to range data corrupted only by zero-mean, uncorrelated noise. Although travel times can be measured very accurately, slant ranges computed from raw travel-times are subject to a number of potentially large systematic error sources in addition to the sound-speed related errors discussed above in App. B. The range errors associated with signal detection time jitter are typically on the order of 10 cm (Singh et al., 2006; Freitag et al., 2005) and well-characterized. Very large systematic range errors typically indicate travel times associated with acoustic multi-path resulting from bottom and surface bounces. Sporadic erroneous range returns can also be associated with environmental noise, often caused by equipment on the support vessel. Finally, the vehicle's location within the beacon net can cause location-dependent errors such as short multipath reflections from nearby bathymetry or complete loss of direct path to a beacon by terrain occlusion. When acoustic beacons are deployed their tether lengths are chosen to create clear lines-of-sight throughout the work area; however the initial bathymetry of the work area is often poorly resolved. Long tethers that improve line-of-sight

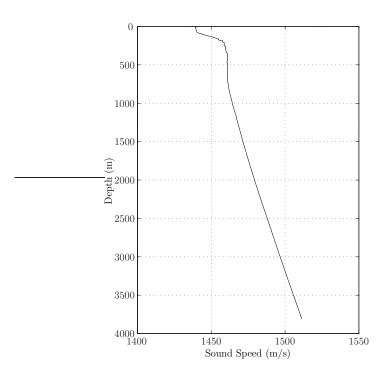


Figure 12: The CTD-derived sound velocity profile generated upon arrival at the 85° E site and used for SVP compensation. This profile shows sound velocity increasing with depth. This characteristic results in upward-refracting acoustic propagation paths peculiar to the ice-covered Arctic Ocean (Kutschale, 1969).

also increase the degree to which beacons sway in currents resulting in both bulk motion of the net and potentially also distorting its configuration. Bulk motion is unobservable from LBL data alone, whereas in principle distortion could be observed in a net with three or more beacons. The majority of large systematic and random errors can be treated by median filtering and outlier rejection in real time (Yoerger et al., 2007). More sophisticated algorithms have attempted to identify and classify outliers in real-time or in post processing (Bingham et al., 2003; Bingham, 2003).

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