Derivation of the Rassweiler & Withrow method

During combustion $\quad V = V_u + V_b \quad — (1)$

Assumed that the unburned volume V_u may be "back tracked" to condition o (i.e. before combustion) by a polytropic relationship:

$$V_{u,0} = V_u \left(\frac{p}{p_0}\right)^{1/n} \quad — \quad (2)$$

Similarly, the burned gas volume V_b may be "forwarded" to the final end of combustion volume by polytropic condition

$$V_{b,f} = V_b \left(\frac{p}{p_f}\right)^{1/n'} \quad — \quad (3)$$ (n' may not be equal to n)

Since $\quad x_b = 1 - \dfrac{V_{u,0}}{V_0} \quad — \quad (4)$

and $\quad x_b = \dfrac{V_{b,f}}{V_f} \quad — \quad (5)$

Substituting (2) & (3) into (1)

$$V = V_{u,0}\left(\frac{p_0}{p}\right)^{1/n} + V_{b,f}\left(\frac{p_f}{p}\right)^{1/n'}$$

and replacing $V_{u,0}$ & $V_{b,f}$ by x_b using (4) & (5)

$$V = (1-x_b)\, V_0\left(\frac{p_0}{p}\right)^{1/n} + x_b V_f \left(\frac{p_f}{p}\right)^{1/n'}$$

Solving for x_b: $\quad x_b = \dfrac{V - V_0\left(\frac{p_0}{p}\right)^{1/n}}{-V_0\left(\frac{p_0}{p}\right)^{1/n} + V_f\left(\frac{p_f}{p}\right)^{1/n'}} = \dfrac{p^{1/n}V - p_0^{1/n}V_0}{\left(\frac{p^{1/n}}{p^{1/n'}}\right)p_f^{1/n'}V_f - p_0^{1/n}V_0}$

Usually assume $n' = n$.