Homework #4 solution

1) WOT fuel required per cycle $m_f = \eta_{v,i} V_D \left(\frac{p_i}{RT_i}\right)\left(\frac{F}{A}\right)$

where $\eta_{v,i}$ is the volumetric efficiency based on the manifold condition.
At WOT, $\eta_{v,i} \simeq 0.8$ (because of the valve timing etc.)

$$m_f = 0.8 \times (500 \times 10^{-6}) \left(\frac{10^5}{287 \times 300}\right) \frac{1}{14.6} \text{ kg} = \underline{\underline{31 \text{ mg}}}$$

The max. time available is to have the injector on continuously.
Thus τ = cycle time $= \frac{60}{6500 \times 1/2}$ for 4-stroke engine $= 18.5$ ms

The injector flow rate is then $\dot{m}_f = \frac{31 \text{ mg}}{18.5 \text{ ms}} = \underline{\underline{1.68 \text{ g/s}}}$

At idle, intake pressure is about 0.3 bar, fuel required = 0.3 × 31 mg = 9.3 mg
Injection duration = $\left(\frac{9.3 \times 10^{-3}}{1.68}\right)$ s $= \underline{\underline{5.54 \text{ ms}}}$ (In practice, because the residual fraction at idle is much higher, the fuel required is about half of that.)

---

2) $x - \tau$ model: discrete case

$$M_i - M_{i-1} = x f_i - k M_{i-1}$$
$$m_i = (1-x) f_i + k M_{i-1}$$

(a) At equilibrium with fuel per cycle set at f_0, puddle mass

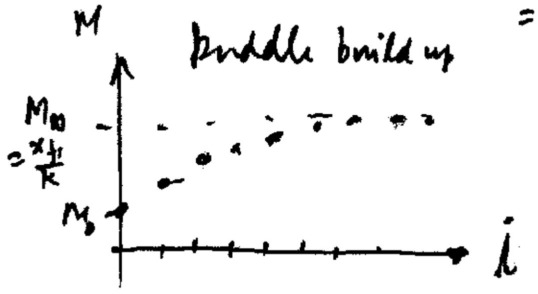
$$0 = x f_0 - k M_0 \Rightarrow M_0 = \underline{\underline{\frac{x f_0}{k}}}$$

fuel delivered
$$m_0 = (1-x) f_0 + k M_0 = \underline{f_0}$$

(b) The sequence of port puddle accumulation is

$$M_1 = x f_1 + (1-k) M_0$$
$$M_2 = x f_1 + (1-k)[x f_1 + (1-k) M_0] = x f_1 + (1-k) x f_1 + (1-k)^2 M_0$$
$$\vdots$$
$$M_i = \underbrace{x f_1 + (1-k) x f_1 + (1-k)^2 x f_1 + \cdots}_{i \text{ terms: geometric series}} + (1-k)^i M_0$$

$$= \underline{\underline{x f_1 \left[\frac{1-(1-k)^i}{k}\right] + (1-k)^i M_0}} \rightarrow \text{for } i \to \infty \quad M_\infty = \frac{x f_1}{k}$$



puddle build up

Fuel delivered

$$m_i = (1-x) f_1 + k M_{i-1} = f_1 + (k M_{i-1} - x f_1)$$

putting in above expression for M_{i-1}

$$m_i = f_1 + k\left\{x f_1 \left[\frac{1-(1-k)^{i-1}}{k}\right] + (1-k)^{i-1} M_0\right\} - x f_1$$
$$= \underline{\underline{f_1 + (1-k)^{i-1}(k M_0 - x f_1)}} \rightarrow \text{for } i \to \infty \quad m_\infty = f_1$$