Homework #5 solution

6.1 (a) Mixture of 1 mole H_2 to 1 mole iC_8H_{18}

$$H_2 + C_8H_{18} + 13(O_2 + 3.773N_2) \rightarrow 10H_2O + 8CO_2 + 49.05N_2$$

Stoich. air fuel ratio $= \frac{13 \times 4.773 \times 28.96}{2+114} = \underline{15.5}$ (compared with 15.13 with neat iC_8)

(b) Heating value of fuel $Q_{LHV} = \frac{2 \times 120 + 114 \times 44.4}{2+114} = \underline{\underline{45.7\ MJ/kg}}$

(c) Relate BMEP to known quantities

$$BMEP = \eta_{f,ig}\,\eta_m\,\eta_v\left(\frac{\rho_{a0}V_D}{A/F}\right)\frac{LHV}{V_D} \quad ; \quad \eta_m = \frac{BMEP}{BMEP + PMEP + FMEP}$$

The PMEP may be estimated by $p_E - p_i$

The volumetric efficiency difference is due only to quasi-static effects

$$\eta_v = \underbrace{\left(\frac{W}{W_A}\right)\left(\frac{1}{1+F/A}\right)}\left(\frac{p_i}{p_{a0}}\right)\underbrace{\left(\frac{T_{a0}}{T_i}\right)\left(\frac{T_i}{T_1}\right)(1-x_r)\left(\frac{r_c}{r_c-1}\right)}_{\text{assume same for both cases}}$$

$$\frac{W}{W_A}\frac{A}{A+F} = \underbrace{\frac{W}{W_A}\frac{A}{A+F+R}}_{=\tilde{x}_a}\;\underbrace{\frac{A+F+R}{A+F}}_{=\underbrace{\frac{1}{1-x_r}}_{\text{same for both cases}}}$$

$\tilde{x}_a$: mole fraction of air in inducted fresh charge

x_r = residual mass fraction

Case (1) C_8H_{18} at $\phi = 0.8$

$$C_8H_{18} + \frac{12.5}{0.8}(O_2 + 3.773N_2) \rightarrow 8CO_2 + 9H_2O + 58.95N_2 + 3.13O_2 \;;\; \tilde{x}_a = 0.987$$

Case (2) $C_8H_{18} + H_2$ at $\phi = 0.5$

$$C_8H_{18} + H_2 + \frac{13}{0.5}(O_2 + 3.773N_2) \rightarrow 8CO_2 + 10H_2O + 98.1N_2 + 6.5O_2 \;;\; \tilde{x}_a = 6.984$$

← same

The $\tilde{x}_a$ values are about the same because at fuel lean, it is mostly air

Thus $\frac{\eta_{v_2}}{\eta_{v_1}} = \frac{x_{a_2}}{x_{a_1}}\frac{p_{i2}}{p_{i1}} \simeq \frac{p_{i2}}{p_{i1}}$

Comparing at the same BMEP = 2.75 bar

$$1 = \frac{BMEP_2}{BMEP_1} = \frac{\eta_{f,ig_2}}{\eta_{f,ig_1}}\frac{(BMEP + FMEP + PMEP)_1}{(BMEP + FMEP + p_E - p_{i_2})_2}\frac{p_{i2}}{p_{i1}}\frac{(A/F)_1}{(A/F)_2}\frac{LHV_2}{LHV_1}$$

Put in numerical values: $1 = \frac{0.4}{0.35}\left(\frac{2.75 + 1.38 + 0.55}{2.75 + 1.38 + 1 - p_{i_2}}\right)\left(\frac{p_{i2}}{1}\right)\frac{15.13\left(\frac{1}{.8}\right)}{15.5\left(\frac{1}{.5}\right)}\frac{45.7}{44.3}$

solving: $p_{i2} = \underline{\underline{0.62\ bar}}$ $\quad PMEP_2 = 1 - 0.62 = \underline{\underline{0.38\ bar}}$

Problem 2) The unburned and burned gas temperatures are tabulated. (parts (i) - (ii))

(iii) The laminar flame speed is

$$S_L = S_{L0}\left(\frac{T_u}{T_0}\right)^{\alpha}\left(\frac{p}{p_0}\right)^{\beta} f \quad \text{where } f = 0.4 \text{ for a residual mole fraction of } 20\%$$

$$\alpha = 2.4 - 0.271\,\phi^{3.51}$$

$$\beta = -0.357 + 0.14\,\phi^{2.77}$$

$$S_{L0} = 30.5 + (-54.9)(\phi - 1.21)^2$$

(cm/s)

(iv) Expansion velocity $S_E = S_L\left(\frac{\rho_u}{\rho_b}\right) \overset{\text{uniform pressure}}{=} S_L\left(\frac{T_b}{T_u}\right)$

(v) $\frac{d}{d\theta}(x_b)$ can be found by finite difference

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(vi) Unburned gas vol. $V_u = m_u R_u T_u$

burned gas vol. $V_b = m_b R_b T_b$

$$\Rightarrow \left(\frac{1 - \frac{V_b}{V}}{V_b/V}\right) = \frac{(1-x_b)}{x_b}\,\frac{W_b}{W_u}\,\frac{T_u}{T_b}$$

where W_u and W_b are the molecular weights. Thus

$$\left(\frac{V_b}{V}\right) = 1 \Big/ \left[1 + \left(\frac{1}{x_b} - 1\right)\frac{W_b}{W_u}\,\frac{T_u}{T_b}\right]$$

For stoichiometric gasoline combustion: $\begin{cases} W_b = 28.98 \simeq 29 \\ W_{u,f} = 30 \text{ for fresh mixture} \end{cases}$

For gasoline, $(\underbrace{C_{7.9}H_{1.87\times 7.9}}_{MW=110,\ H/C=1.87})$ at $\lambda = 1$

$$C_{7.9}H_{14.8} + 11.6(O_2 + 3.773N_2) \rightarrow 7.9CO_2 + 7.4H_2O + 43.8N_2$$

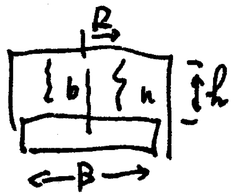
$$W_u = W_{u_f}\tilde{x}_r + (1 - \tilde{x}_r) + W_b(\tilde{x}_r)$$

$$= 30 \times (1 - 0.2) + 29(0.2)$$

$$= 29.80$$

(v) The burned gas volume $V_b = (\pi R^2)h$

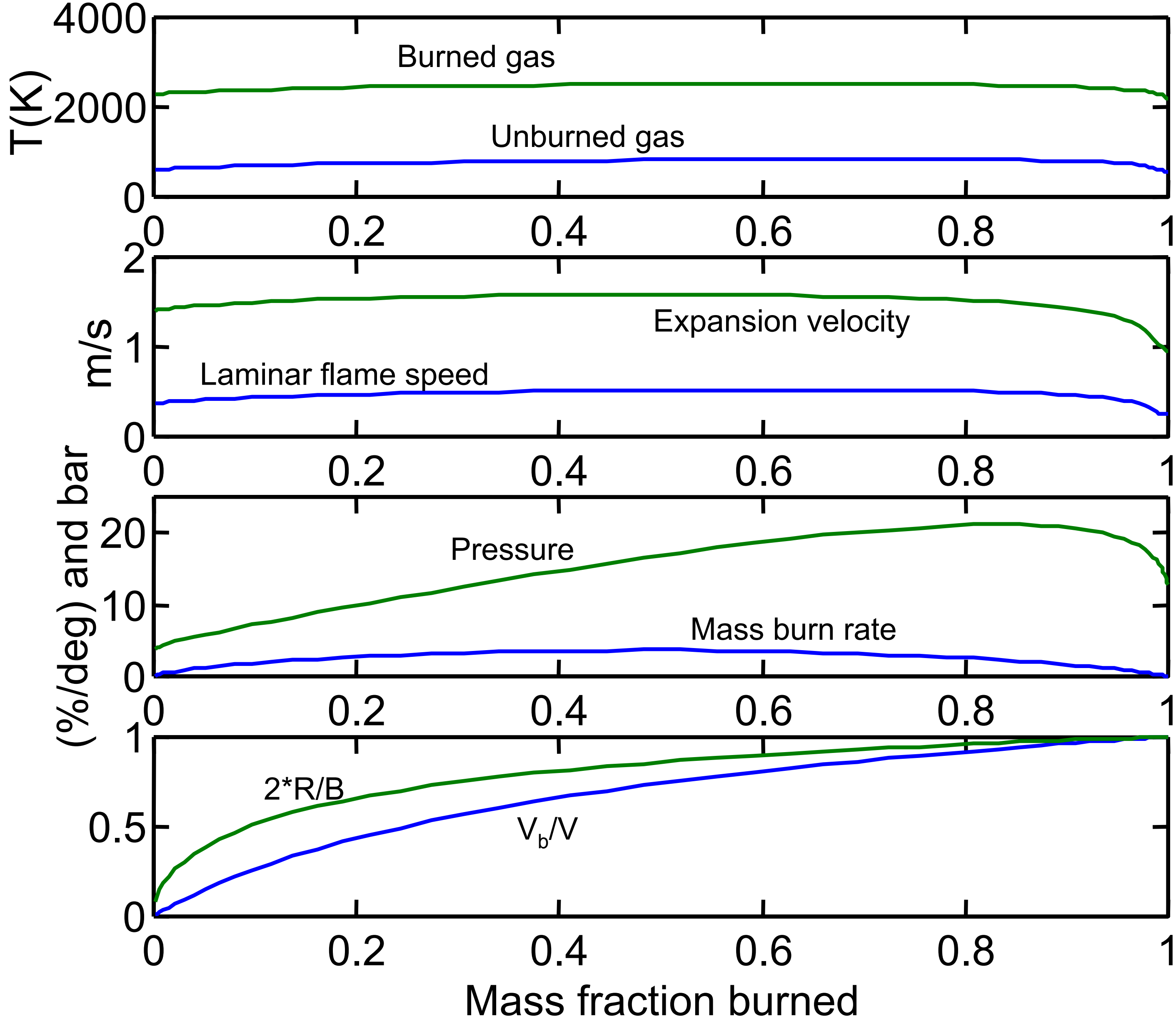
The chamber volume $V = \left(\frac{\pi B^2}{4}\right)h$



Thus $\left(\frac{V_b}{V}\right) = \left(\frac{2R}{B}\right)^2$

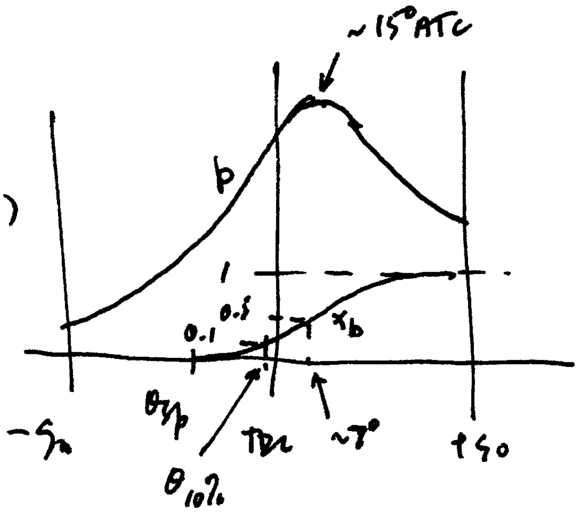
$$\boxed{\frac{R}{B} = \frac{1}{2}\sqrt{\left(\frac{V_b}{V}\right)}}$$

The various quantities are plotted on the next page.



T(K)
4000
2000
0
Burned gas
Unburned gas
0
0.2
0.4
0.6
0.8
1
m/s
2
1
0
Expansion velocity
Laminar flame speed
(%/deg) and bar
20
10
0
Pressure
Mass burn rate
1
0.5
0
2*R/B
V_b/V
Mass fraction burned

Problem 3/ (a)



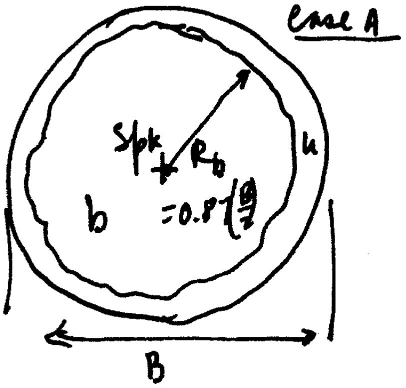
(b) $V_u = m_u R_u T_u$, $V_b = m_b R_b T_b$ $\Big\}$ $\left(\frac{1-\frac{V_b}{V}}{V_b/V}\right) = \frac{m_u}{m_b}\frac{R_u}{R_b}\frac{T_u}{T_b}$

Since burned & unburned gas have approximately the same M.W., $R_u \simeq R_b$

Thus $\left(\frac{V_b}{V}\right) \simeq \left[\left(\frac{1-x_b}{x_b}\right)\frac{T_u}{T_b}+1\right]^{-1}$

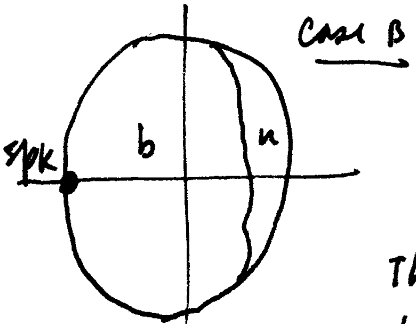
$\frac{T_u}{T_b} \simeq 1/3$ at $x_b = 0.5$; $\left(\frac{V_b}{V}\right) = \frac{1}{\left(\frac{1-0.5}{0.5}\right)\frac{1}{3}+1} = \underline{\underline{0.75}}$

(c) (i) At the 50% burn point $\frac{V_b}{V} = 0.75$. For centered flame $\left(\frac{R_b}{B/2}\right) = \sqrt{\frac{V_b}{V}} = 0.87$

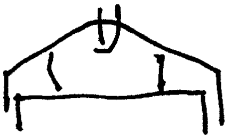


(ii) In case B, the geometry is such that much of the flame front is blocked by the combustion chamber wall. Reduction of A_f ⇒ slower flame propagation.

Also note that in case B, the flame has to travel longer for burning up the whole charge. Thus slower overall burn time.



(iii) For case C



- The combustion is faster at first compared to A because of the larger A_f.
- Towards the end of combustion, the reverse is true.
- For MBT timing, 50% burn point is fixed (~7° ATC). Therefore the faster initial burn (case C) will have more retard timing than case (A).