2.61 Homework #6 solution

1) The energy released in a flame ball of radius R is

$$E = \left(\frac{4}{3}\pi R^3 \rho_b\right)\frac{Q_{LHV}}{(1+A/F)}(1-x_r)$$

Thus the radius at which the energy corresponds to 30 mJ is

$$R = \left\{\frac{E}{\frac{4}{3}\pi \rho_b\left(\frac{Q_{LHV}}{1+A/F}\right)(1-x_r)}\right\}^{1/3}$$

At ignition under light load condition, using the data from the previous HW: $p \sim 3.3$ bar; $T_b \simeq 2200$K; $\rho_b = \frac{p}{RT/W} = \frac{3.3\times10^5}{8314\times2200/29}$

$= 0.52$ kg/m^3

$$R = \left\{\frac{30\times10^{-3}}{\frac{4}{3}\pi\times0.52\times\left(\frac{4.4\times10^6}{1+14.6}\right)(1-0.2)}\right\}^{1/3} = \underline{\underline{4\,\text{mm}}} \xrightarrow{\text{size}} \bigcirc$$

The value will be even smaller at higher load.

2) The change in charge temperature may be estimated by

$$m_a c_p \Delta T_1 = \Delta m_f h_{fg}$$

(This assumes adiabatic evaporation. In practice, much of the energy comes from the port wall. Thus the ΔT will be smaller)

$$\Delta T_1 = \frac{\Delta m_f h_{fg}}{m_a c_p} = \left(\frac{\Delta m_f}{m_f}\right)\left(\frac{m_f}{m_a}\right) h_{fg}$$

$h_{fg} \sim 305\times10^3$ J/kg $\Rightarrow$ $\quad = 0.2\,\frac{1}{14.6}\,\frac{305\times10^3}{1000} = 4\,^\circ$K

$c_p = 10^3$ J/kg

The compression temperature change is $\Delta T_2 = (C_R)^{\gamma-1}\Delta T_1$

$= (9^{1.33-1})4 = 8\,^\circ$K

The change of compression temp. due to change in γ is

$$\Delta T_2 = T_1\left(\frac{\partial (C_R)^{\gamma-1}}{\partial\gamma}\right)d\gamma = T_1 (C_R)^{\gamma-1}(\log_e C_R)\,d\gamma$$

Assume $T_1 = 300$K ; $\Delta T_2 = 300(9^{0.33})(\log_e 9)\,0.03 = 41\,^\circ$K

together about $\underline{\underline{50^\circ\text{K drop}}}$

3/ Constant volume combustion: 1^{st} law of thermodynamics $\Rightarrow \frac{d}{dt}(m c_v T) = \dot{Q}$

For ideal gas with constant properties, $c_v = \frac{R}{\gamma-1}$; Thus $\frac{d}{dt}\left(\frac{pV}{\gamma-1}\right) = \dot{Q}$

or $\dot{p} = (\gamma-1)\dot{q}$

where $\dot{q}$ is the volumetric heat release rate (W/m^3)

Integrating over the combustion period $\Delta p = (\gamma-1)\, q$

where q is the energy release per unit volume.

$$q = \left(\frac{m_f\, LHV}{V}\right) = (LHV)\left(\frac{m_f}{m_A}\right)\frac{(1-x_r)}{(1+F/A)}\cdot \rho$$

where the local charge density ρ, at TDC, may be related to the trapped charge density ρ_0 at IVO by the effective CR_e

$$\rho = \rho_0\, CR_e$$

Thus

$$\Delta p = (\gamma-1)(LHV)(F/A)\frac{(1-x_r)}{(1+F/A)}\,\rho_0\, CR$$

Note the scaling on CR & ρ_0 (compression ratio and boosting effects.)

The temperature may be obtained via the ideal gas law (assuming burned gas and unburned gas have same molecular wt W)

$$T = \frac{(p+\Delta p)}{\left(\rho_0 CR_e \frac{R}{W}\right)}$$; where p is the pre-knock pressure

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Numerical values

$$\Delta p = (1.33-1)(4.4\times10^7)\left(\frac{1}{14.6}\right)\frac{(1-0.1)}{\left(1+\frac{1}{14.6}\right)}\times 1\times 9$$

$$= \underline{75.4 \text{ bar}} \rightarrow p + \Delta p = \underline{95.4 \text{ bar}}$$

$$T = \frac{(20+75.4)\times10^5}{1\times 9\times \frac{8314}{29}} = \underline{3700\,K}$$

(The actual temperature is lower because of dissociation)