2.615 HW#7 Soln

1) (Prob. 11.3 of Text)

Exhaust mass flow $\dot{m}_e = \dot{m}_f (1 + A/F)$

To change mass fraction to mole fraction $\tilde{x}_i = x_i \frac{\bar{W}}{W_i}$

where $\bar{W}$ is the av. exhaust M.W.; W_i is the M.W. of species i

For gasoline stoichiometric combustion: $CH_{1.87} + 1.47(O_2 + 3.773 N_2)$

$$\rightarrow CO_2 + 0.935 H_2O + 5.54 N_2$$

$$\text{Av. exh. M.W.} = \frac{44 + 0.935 \times 18 + 5.54 \times 28.12}{1 + 0.935 + 5.54} = 28.96 \simeq 29$$

Exhaust species mole fractions: $$\tilde{x}_{NO_2} = \frac{\dot{m}_{NO_2}}{\dot{m}_f (1 + A/F)} \frac{\bar{W}}{W_{NO_2}} = \frac{1.5}{120 \times (1 + 14.6)} \frac{29}{46}$$

$$= 5.05 \times 10^{-4} = \underline{\underline{505 \text{ ppm}}}$$

$$\tilde{x}_{HC} = \frac{2}{120 \times (1 + 14.6)} \frac{29}{13.85} = 2.24 \times 10^{-3} = \underline{\underline{2240 \text{ ppm}}}$$

$$\tilde{x}_{CO} = \frac{28}{120 \times (1 + 14.6)} \frac{29}{28} = 1.11 \times 10^{-2} = \underline{\underline{11000 \text{ ppm}}} \text{ or } \underline{\underline{1.1\%}}$$

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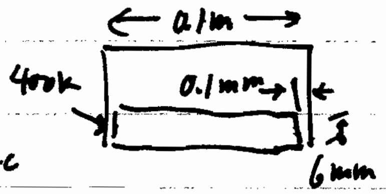
2) (11-9 of Text) (a) Mass of charge $= m_a + m_f + m_r = \frac{m_a (1 + F/A)}{(1 - x_r)}$; assume $x_r = 10\%$

$$m_a = \rho_{a0} V_D \eta_v = \left(\frac{10^5}{287 \times 303}\right)\left(\frac{\pi \times 0.1^3}{4}\right) \times 0.8 \text{ kg} = 0.72 \text{ g}$$

$$m_{charge} = \frac{0.72 \,(1 + 1/14.6)}{1 - 0.1} \text{ g} = 0.85 \text{ g}$$



$$V_{crev.} = \pi D \delta L = (\pi \times 0.1 \times 100 \times 10^{-6} \times 6 \times 10^{-3}) \text{ m}^3 = 0.19 \text{ cc}$$

Molecular wt of both burned and unburned gas are about the same; $\bar{W} \simeq 29$

$$\text{Peak mass in crevice} = \frac{P_{peak} V_{crev}}{(\tilde{R}/\bar{W}) T_{crev}} = \frac{3 \times 10^6 \times 0.19 \times 10^{-6}}{8314/29 \times 400} \text{ kg} = 4.9 \text{ mg}$$

$$\frac{m_{crev}}{m_{total}} = \frac{4.9 \times 10^{-3}}{0.85} = \underline{\underline{0.6\%}}$$

(b) Unburned gas in exhaust

$$m_u = \underbrace{m_{cr}\,\tfrac{2}{3}}_{\text{unburned fraction in crevice}} \cdot \underbrace{\left(1-\tfrac{1}{2}\right)\left(1-\tfrac{1}{3}\right)}_{\text{oxidation}} = 4.9 \times \tfrac{2}{3} \times \tfrac{1}{2} \times \tfrac{2}{3} = 1.09\,\text{mg}$$

unburned fuel (HC) in exhaust

$$m_{HC} = m_u\left(\frac{1}{1+A/F}\right) = 1.09 \times \frac{1}{15.6} = 0.070\,\text{mg}$$

unburned mole fraction of HC, $\tilde{x}_{HC} = \left(\dfrac{m_{HC}}{m_a(1+F/A)}\right)\dfrac{\bar{W}}{W_{HC}}$

for $W_{HC} = 13.85$ ($H/C = 1.85$)

$$\tilde{x}_{HC} = \frac{0.070\times10^{-3}}{0.72\left(1+\frac{1}{14.6}\right)}\,\frac{29}{13.85} = 1.9\times10^{-4} = \underline{\underline{190\,\text{ppm}}}$$

(c) Brake specific HC emission

$$BSHC = \frac{m_{HC}}{\underset{m_a\left(\frac{F}{A}\right)}{m_f}}\;\underbrace{\frac{m_f}{W_{brake}}}_{bsfc} = \left(\frac{0.07\times10^{-3}}{0.72\times\frac{1}{14.6}}\right)300 = \underline{\underline{0.43\,\text{g/kW-hr}}}$$

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3) (11-10 of Text)

$$\left.\frac{d[NO]}{dt}\right|_0 = 2k_1^+\,[N_2]_e\,[O]_e \qquad \text{for [ ] in mol/cc}$$

Since $[N_2] = n\,\tilde{x}_{N_2}$ where n is the total mole/cc

and $n = \left(\dfrac{p}{\tilde{R}T}\right)$ kmol/m^3 $= \left(\dfrac{p}{\tilde{R}T}\right)\dfrac{\times10^3}{10^6}$ mol/cc $= \left(\dfrac{p}{\tilde{R}T}\right)\times10^{-3}$ mol/cc

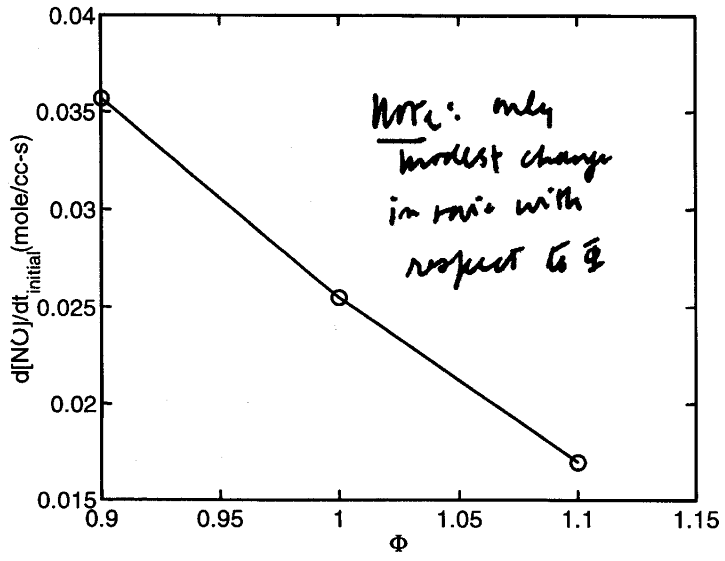
$$\left.\frac{d[NO]}{dt}\right|_0 = 2k_1^+\left(\frac{p}{\tilde{R}T}\times10^{-3}\right)^2 \tilde{x}_{N_2,eq}\,\tilde{x}_{O,eq} \;;\quad \tau_{NO,eq} = \frac{\left(\frac{p}{\tilde{R}T}\times10^{-3}\right)x_{NO,eq}}{[\dot{NO}]_0}$$

(mol/cc-s)

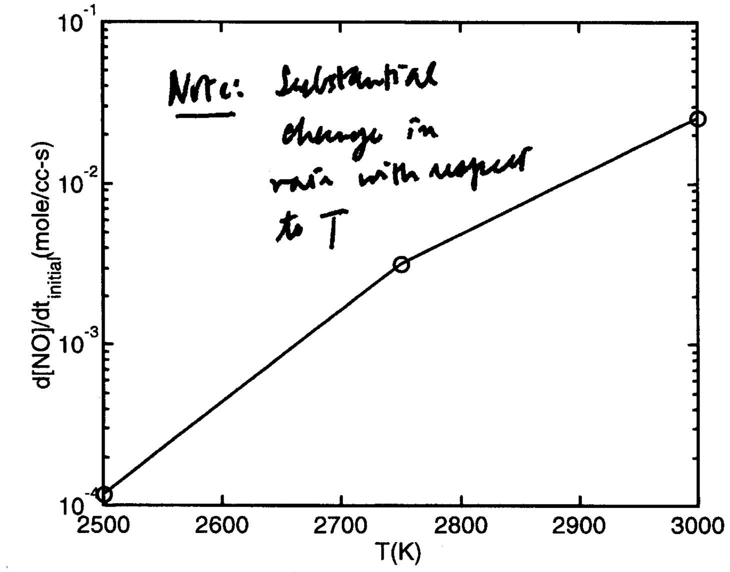
$k_1^+ = 7.6\times10^{+13}\exp\left(\dfrac{-38000}{T}\right)$ where T is in K

ϕ	T	$\tilde{x}_O$	$\tilde{x}_{N_2}$	$[\dot{NO}]_0$ mol/cc-s
0.9	↑	2.1×10^{-3}	↑	3.57×10^{-2}
1.0	3000	1.5×10^{-3}	0.73	2.55×10^{-2}
1.1	↓	1×10^{-3}	↓	1.7×10^{-2}

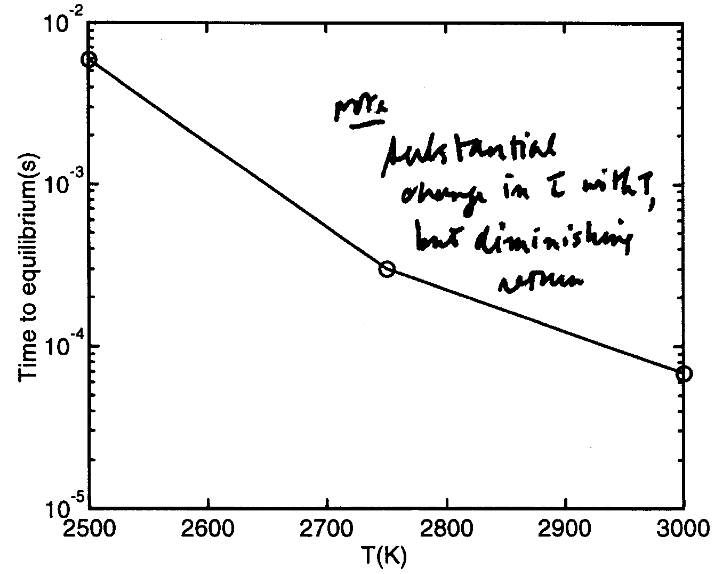
T	$\tilde{x}_O$	$\tilde{x}_{N_2}$	$\tilde{x}_{NO,eq}$	$[\dot{NO}]_0$ mol/cc-s	τ (s)
2500	6×10^{-5}	↑	2.6×10^{-3}	1.17×10^{-4}	5.9×10^{-3}
2750	5×10^{-4}	0.73	4×10^{-3}	3.2×10^{-3}	3×10^{-4}
3000	1.5×10^{-3}	↓	8×10^{-3}	2.6×10^{-2}	6.8×10^{-5}



Note: only modest change in rate with respect to Φ



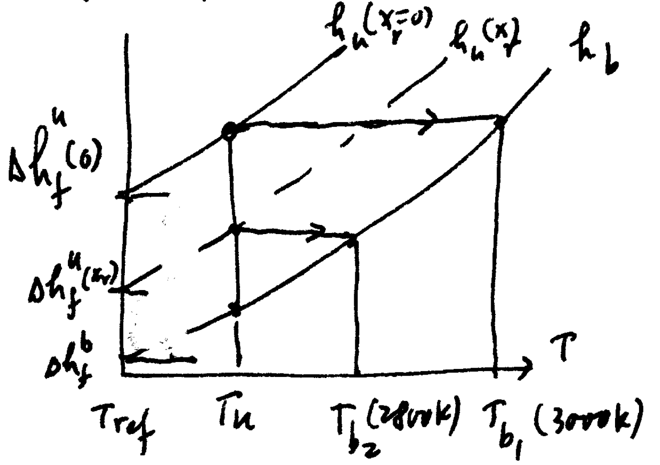
Note: Substantial change in rate with respect to T



Note
substantial change in τ with T, but diminishing return

Factor of 4 reduction in $[\dot{NO}] \Rightarrow [\dot{NO}]$ reduces from 2.55×10^{-2} mol/cc-s to 0.64×10^{-2} mol/cc-s. From the graph, Temperature has to drop from 3000K to <u>2800K</u>

Graphically, the x_r needed is illustrated as



combustion at prevailing pressure

$h_b = h_u$, Thus

sensible part ↘

$$h_s^b(T_{b_1}) + \Delta h_f^b = h_s^u(T_u) + \Delta h_f^u(x_r=0)$$

$$-)\quad h_s^b(T_{b_2}) + \Delta h_f^b = h_s^u(T_u) + \Delta h_f^u(x_r)$$

$$h_s^b(T_{b_1}) - h_s^b(T_{b_2}) = \Delta h_f^u(x_r=0) - \Delta h_f^u(x_r)$$

Using Eq 4.32, at φ=1

$$\Delta h_f^u(x_r=0) - \Delta h_f(x_r) = x_r \cdot 2951 \text{ kJ/mole air}$$

$$= x_r\, 2951 \times \frac{15.6}{14.6} \text{ kJ/mole fuel/air mixture}$$

From Fig 4-17, $c_{p,b} \simeq 2000$ J/kg K

Thus

$$x_r = \frac{c_{p,b}(T_{b_1} - T_{b_2})}{2951 \times 10^3 \times \frac{15.6}{14.6}} = \frac{2000\,(200)}{2951 \times 10^3 \times \frac{15.6}{14.6}}$$

$$= \underline{\underline{0.13}}$$