Hw #8 Solution

1)(a) $\dot{m}_a = \eta_v \left(\frac{p}{RT}\right) V_D \frac{N}{2} = 0.8 \left(\frac{2.5\times10^5}{287\times393}\right) 1.29\times10^{-2} \times \frac{2000}{120} = 0.381\ kg/s = \underline{\underline{381\ g/s}}$

$\dot{m}_f = bsfc \times P = \frac{200\times294}{3600} = \underline{\underline{16.3\ g/s}}$

$A/F = \dot{m}_a/\dot{m}_f = 381/16.3 = \underline{\underline{23.3}}$

(b) $m_a = \dot{m}_a/(n\,N/2)$; where n = # of cylinders = 6

$= \frac{381}{6\times\frac{2000}{120}} = \underline{\underline{3.81\ g}}$

A large drop!

$m_f = m_a/(A/F) = \underline{\underline{0.163\ g}}$; $V_f = \frac{m_f}{\rho_f} = \left(\frac{0.163}{0.78}\right) = \underline{\underline{0.21cc}}$

(c) Denote nozzle properties by subscript z.

$V_z = \frac{V_f}{n_z A_z \tau}$ n_z = # of nozzle holes = 8, A_z = hole area, τ = inj duration:

$= \frac{0.21\times10^{-6}}{8\times\frac{\pi(0.2\times10^{-3})^2}{4}\times3.3\times10^{-3}}$ at 2000 rpm $\rightarrow$ 12°CA/ms, 40°CA $\Rightarrow \tau = 40/12 = 3.3$ ms

$= \underline{\underline{253\ m/s}}$

(d) Reynolds no $= \frac{\rho_f V_z d_z}{\mu} = \frac{253\times0.78\times10^3\times0.2\times10^{-3}}{5\times10^{-4}} = 7.9\times10^4$ turbulent.

From Moody chart, friction coeff. $f = 0.018$

$\Delta p_z = \frac{1}{2}\rho_f V_z^2 f\left(\frac{L}{D}\right)_z = \frac{1}{2}\times0.78\times10^3\times(253)^2\times0.018\times10 = \underline{\underline{45\ bar}}$

This is not significant ($\simeq$3%) compared to the pressure drop of 1350 bar across the nozzle.

(e) Discharge coeff. $C_D = \frac{V_z}{\sqrt{2\Delta p/\rho_f}} = \frac{253}{\sqrt{2(1350\times10^5)/0.78\times10^3}} = \underline{\underline{0.43}}$

(f) Droplet Diameter $d = (W_b)_{critical}\frac{\sigma}{\rho_g V_z^2} = 30\frac{0.025}{\left(\frac{50\times10^5}{287\times800}\right)(253)^2} = \underline{\underline{0.54\ \mu m}}$

(g) $N_{drop} = \left(\frac{V_f}{\pi d^3/6}\right) = \frac{0.21\times10^{-6}}{\pi(0.54\times10^{-6})^3/6} = \underline{\underline{2.5\times10^{12}\ drops}}$

Droplet spacing $= (V_c/N)^{1/3}$; $V_c = \frac{V_D}{r_c-1} = \frac{12.9\times10^{+3}}{6\,(16.5-1)} = 139cc$

Spacing $= \left(\frac{139\times10^{-6}}{2.5\times10^{12}}\right)^{1/3} = 3.8\times10^{-6}\ m = \underline{\underline{3.8\ \mu m}}$

Thus each drop has to evap. and mix with air of $\left(\frac{3.8}{0.54}\right)^3 = 348$ times its size. A difficult job

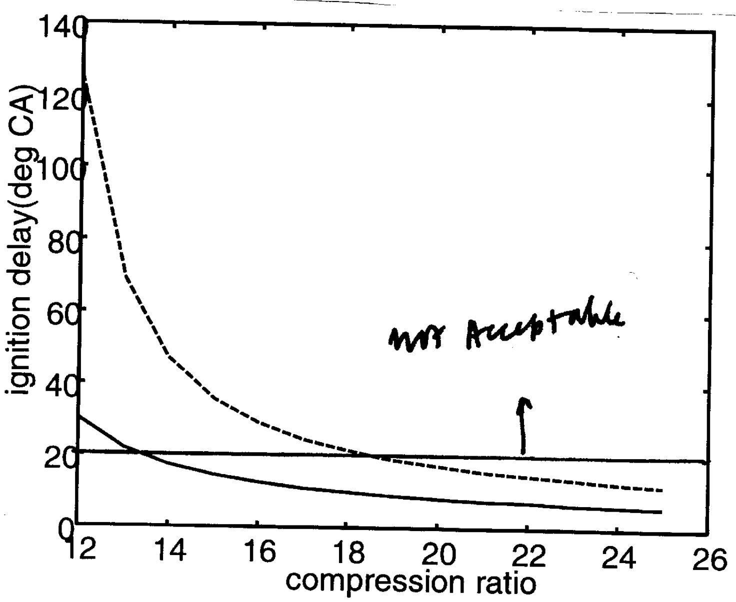
2/ Ignition delay:

$$\tau\,(CA) = \left(0.36 + 0.22\,\bar{S}_p\right) \exp\left\{ E_A \left[\left(\frac{1}{\bar{R}T}\right) - \frac{1}{17190} \right] + \left(\frac{21.2}{P-12.4}\right)^{0.63} \right\}$$

$$E_A = 618840/(CN + 25)$$

$$T = T_i\,(CR)^{n-1} \;;\quad P = P_i\,(CR)^{n}$$

Results:



CR	ign delay (a)	ign delay (b) in °CA
12.00	30.34	125.73
13.00	22.15	69.04
14.00	17.56	47.10
15.00	14.64	35.88
16.00	12.64	29.18
17.00	11.17	24.76
18.00	10.05	21.63
19.00	9.17	19.30
20.00	8.46	17.50
21.00	7.87	16.07
22.00	7.38	14.90
23.00	6.96	13.93
24.00	6.59	13.11
25.00	6.28	12.40

To have $\tau_{id} \leq 20°CA$

For the large truck engine, the minimum compression ratio is <u>14</u>.

For the small passenger car engine, the minimum is <u>19</u>.

8.3) Mass Diffusivity $\underset{(m^2/s)}{D} = 1.8\times10^{-5}\left(\frac{p_0}{p}\right)\left(\frac{T}{T_0}\right)^{1.81}$

Diffusion Time $\tau_{diff} = d^2/D$

Conditions	(a)	(b)	(c)
p (bar)	100	70	30
T (K)	2500	2000	1400
D (m^2/s)	8.36×10^{-6}	7.97×10^{-6}	9.75×10^{-6}
τ_{diff} - 10 nm	1.2×10^{-11} s	1.25×10^{-11} s	1.03×10^{-11} s
τ_{diff} - 100 nm	1.2×10^{-9} s	1.25×10^{-9}	1.03×10^{-9}
τ_{diff} - 1000 nm	1.2×10^{-7} s	1.25×10^{-7}	1.03×10^{-7}

All small values compared to oxidation time

Oxidation Time

mass conservation $\rho(4\pi r^2)\frac{dr}{dt} = -(4\pi r^2)w$

$\Rightarrow \quad \frac{dr}{dt} = -\frac{w}{\rho}$

integrating $r - r_0 = -\frac{wt}{\rho}$; thus time to oxidize particle of radius r_0

is $t_0 = \frac{r_0 w}{\rho}$ — The time is proportional to r_0.

For the Nagle-Strickland Constable formula, $p_{O_2} = p\cdot x_{O_2}$ then the numerical values are: (for 100 nm = 0.1 μm diameter particle)

oxidation time (s) for 0.1 micron soot

T(k)	p(bar)	time@xo2=0.1%	1%	10%
2.5000e+003	1.0000e+002	1.5798e-002	1.9327e-003	6.9704e-004
2.0000e+003	7.0000e+001	8.8868e-003	4.9975e-003	4.6617e-003
1.4000e+003	3.0000e+001	2.5396e-001	1.9351e-001	1.8747e-001

Note the sensitivity to temperature. Sufficient O_2 has to be mixed in early (at high T) to oxidize the soot.