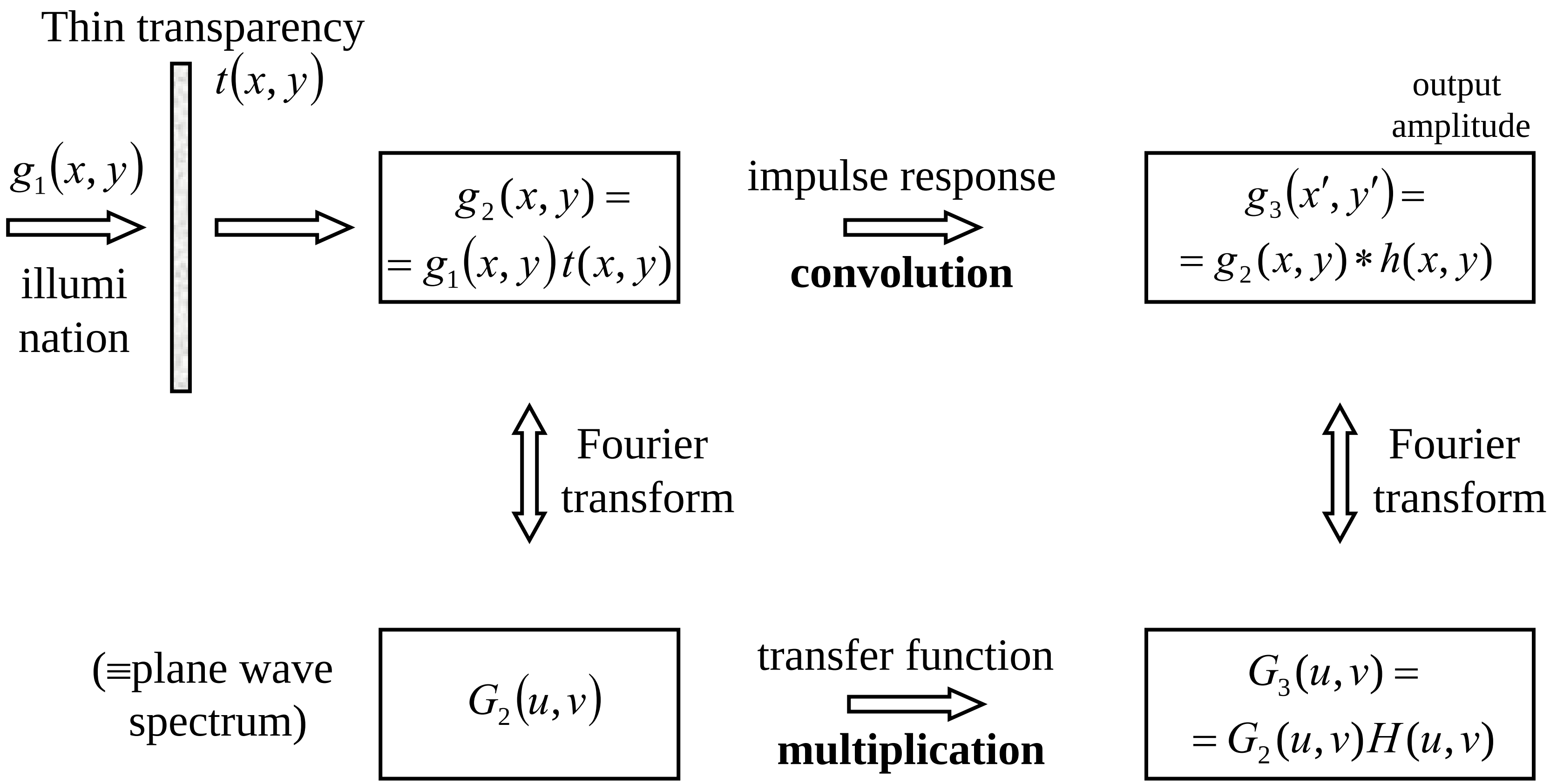
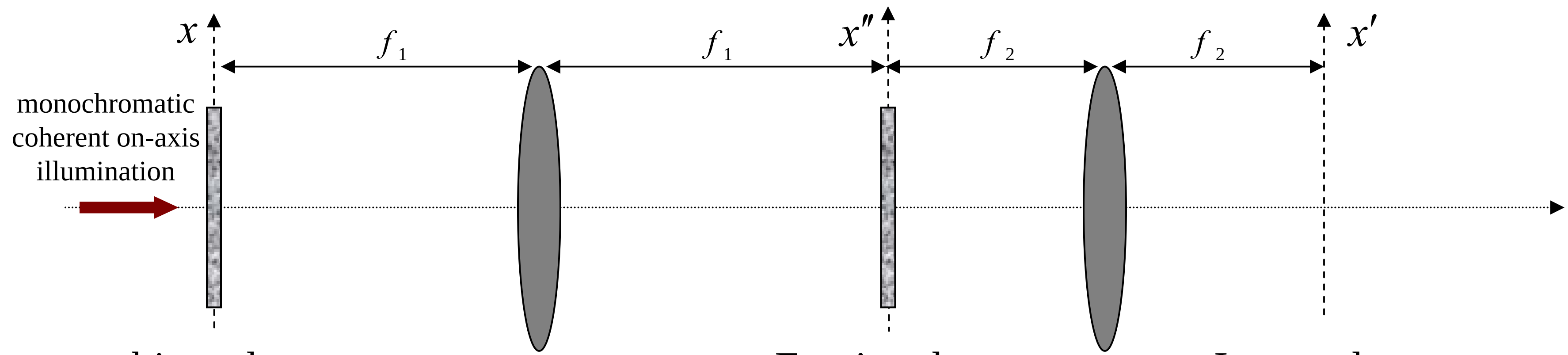
Coherent imaging
as a linear, shift-invariant system



transfer function $H(s_x, s_y)$: aka pupil function

Spatial filtering with the 4F system



object plane
transparency
$g_{\text{in}}(x, y)$

$\mathfrak{F}$ Fourier transform

field arriving
at Fourier plane
$G_{\text{in}}\left(\frac{x''}{\lambda f_1}, \frac{y''}{\lambda f_1}\right)$

Fourier plane
transparency
$H(x'', y'')$

field departing
from Fourier plane
$G_{\text{in}}\left(\frac{x''}{\lambda f_1}, \frac{y''}{\lambda f_1}\right) \times H(x'', y'')$

$\mathfrak{F}$ Fourier transform

Image plane
observed field
$(g_{\text{in}} * h)\left(-\frac{f_1}{f_2}x', -\frac{f_1}{f_2}y'\right)$

Single-lens imaging system

Impulse response (PSF)

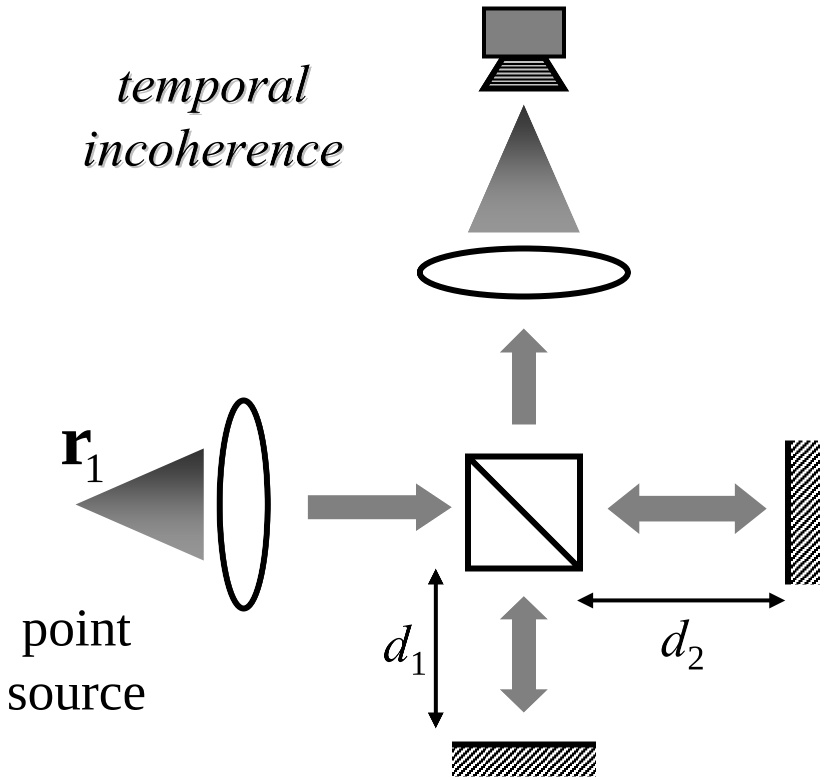
Ideal PSF: $$h(x,y;x',y') = \delta(x'-mx)\,\delta(y'-my)$$

Diffraction-limited PSF: $$h(x,y;x',y') = \operatorname{jinc}\left(\frac{R}{\lambda}\sqrt{\left(\frac{x'}{s'}-\frac{x}{s}\right)^2+\left(\frac{y'}{s'}-\frac{y}{s}\right)^2}\right)$$

Imaging with incoherent light

Two types of incoherence

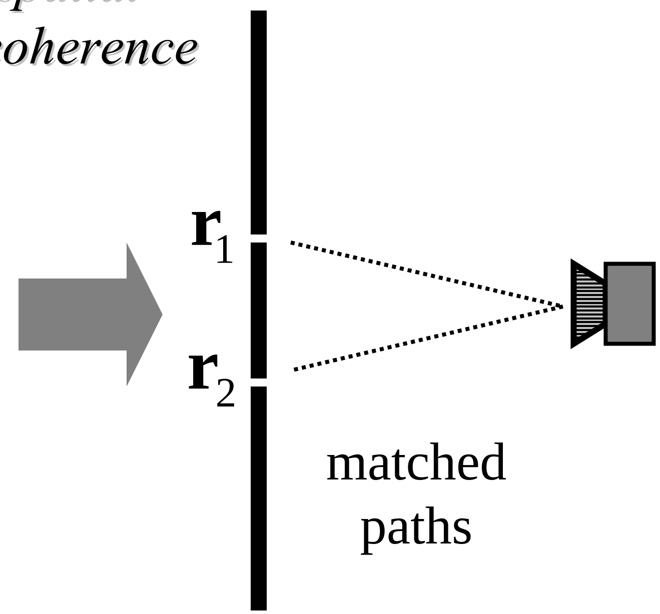
temporal incoherence



Michelson interferometer

poly-chromatic light
(=multi-color, broadband)

spatial incoherence

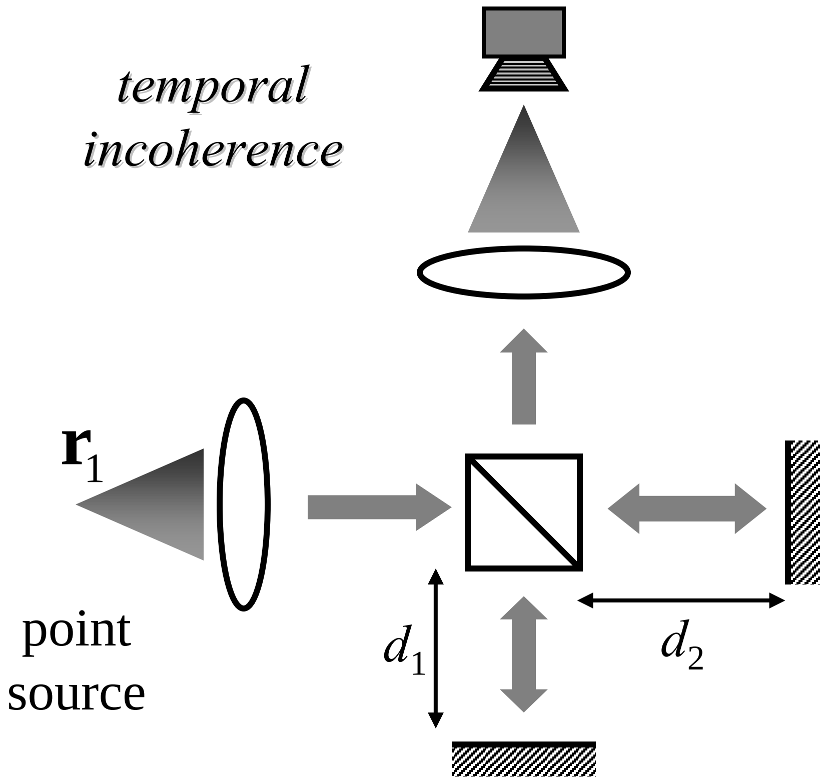


Young interferometer

mono-chromatic light
(= single color, narrowband)

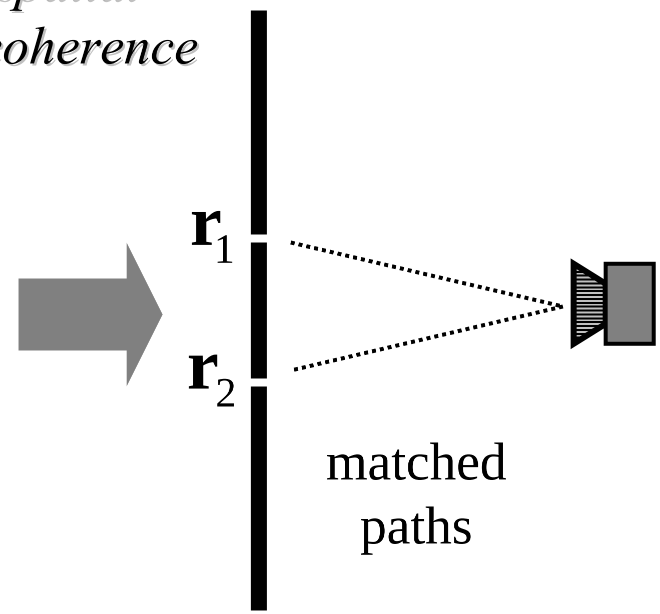
Two types of incoherence

temporal incoherence



waves from unequal paths
do not interfere

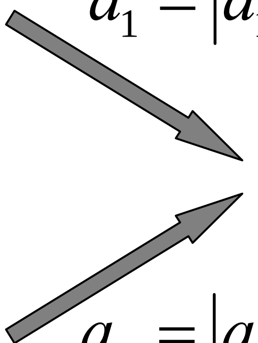
spatial incoherence



waves with equal paths
but from different points
on the wavefront
do not interfere

Coherent vs incoherent beams

$$a_1 = |a_1| e^{i\phi_1}$$



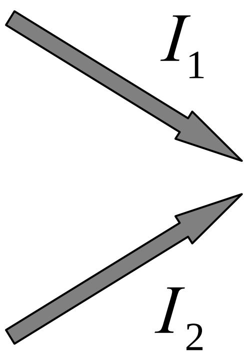
$$a_2 = |a_2| e^{i\phi_2}$$

<u>Mutually coherent</u>: superposition field *amplitude* is described by *sum of complex amplitudes*

$$a = a_1 + a_2 = |a_1| e^{i\phi_1} + |a_2| e^{i\phi_2}$$

$$I = |a|^2 = |a_1 + a_2|^2$$

$$I_1$$



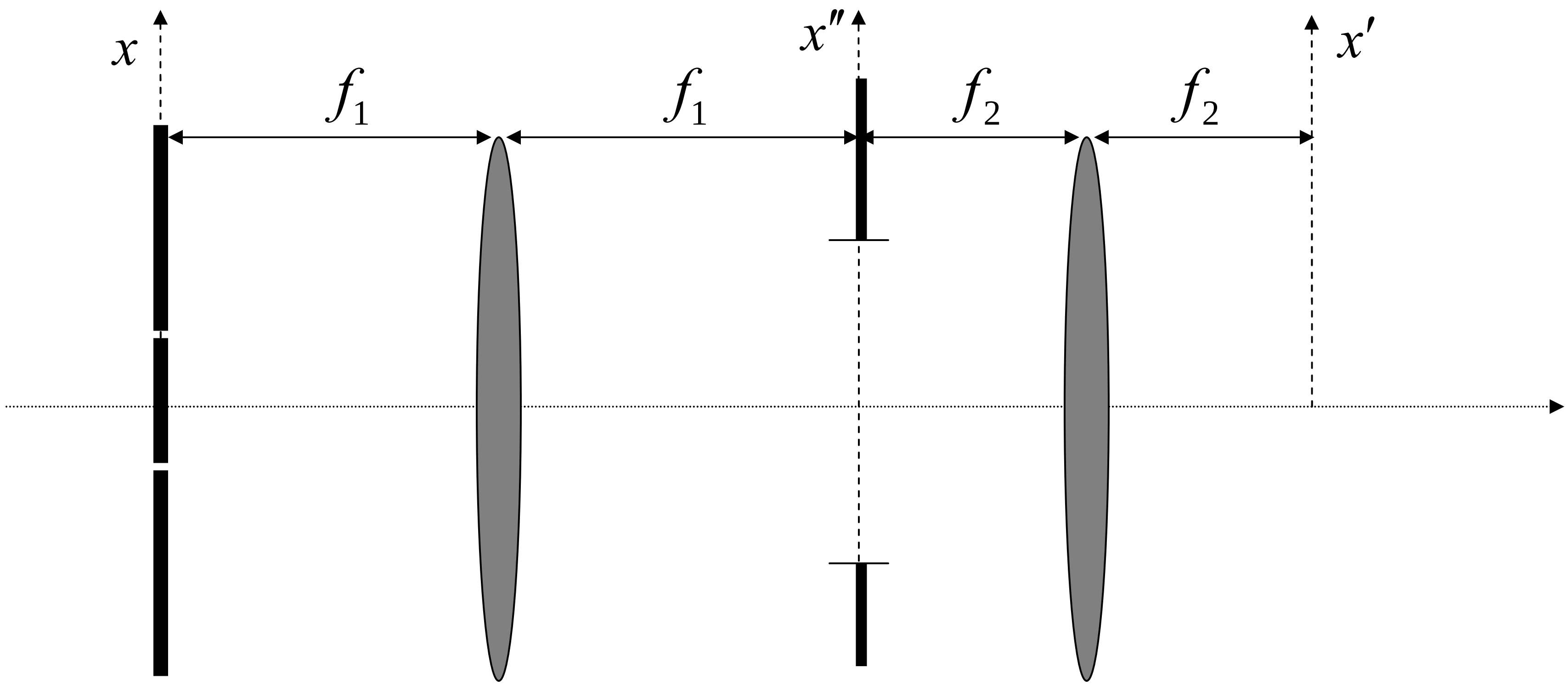
$$I_2$$

<u>Mutually incoherent</u>: superposition field *intensity* is described by *sum of intensities*

$$I = I_1 + I_2$$

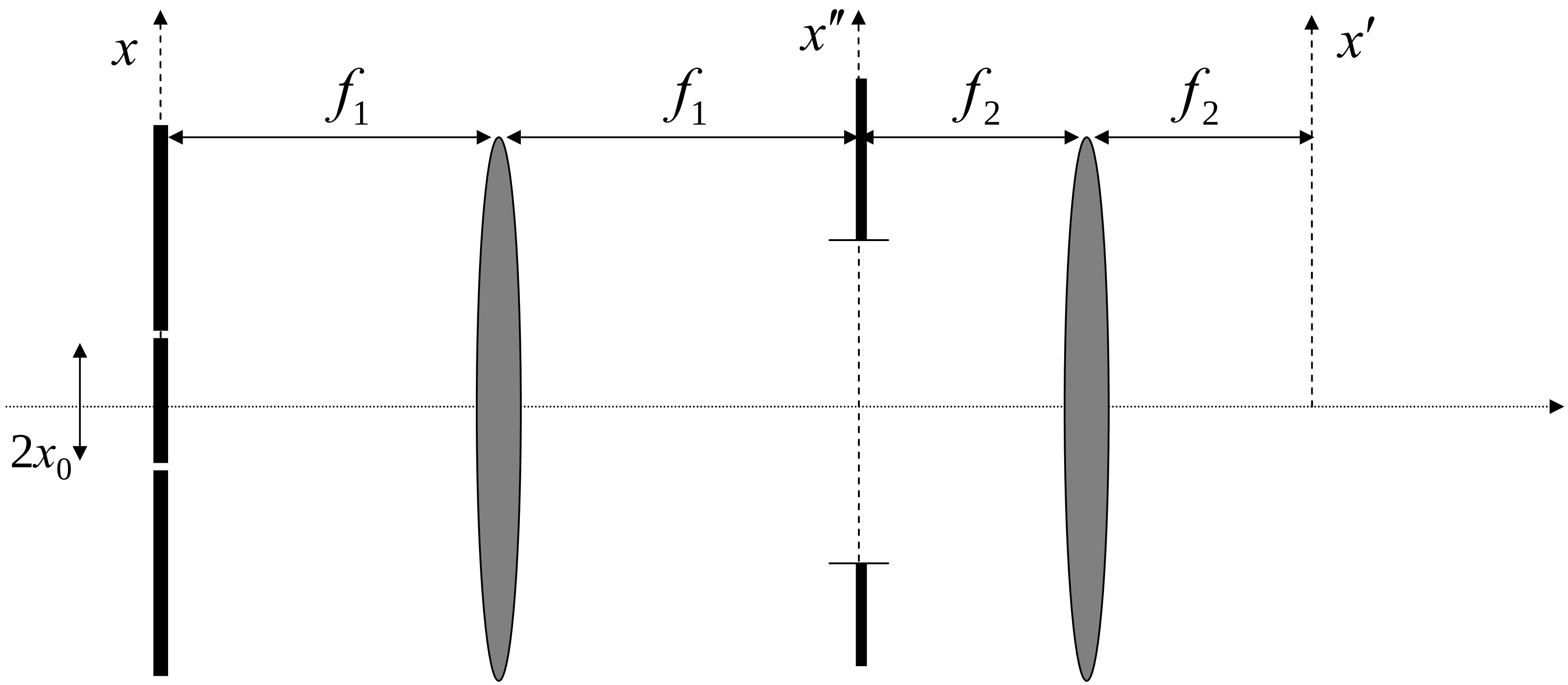
(the phases of the individual beams vary randomly with respect to each other; hence, we would need statistical formulation to describe them properly — <u>*statistical optics*</u>)

Imaging with spatially incoherent light



simple object: two point sources
narrowband, mutually incoherent
(input field is *spatially incoherent*)

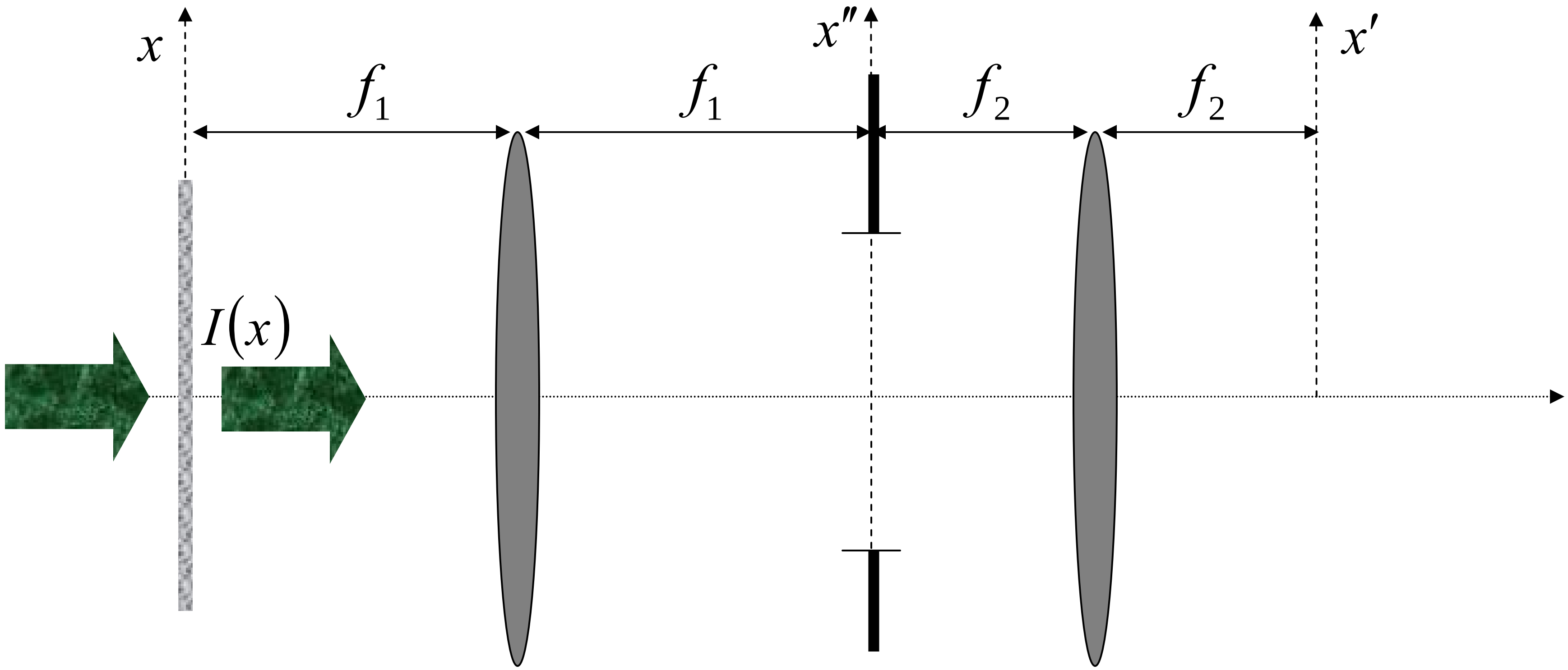
Imaging with spatially incoherent light



incoherent: adding in intensity ⇒

$$I(x') = |h(x' - x_0)|^2 + |h(x' + x_0)|^2$$

Imaging with spatially incoherent light

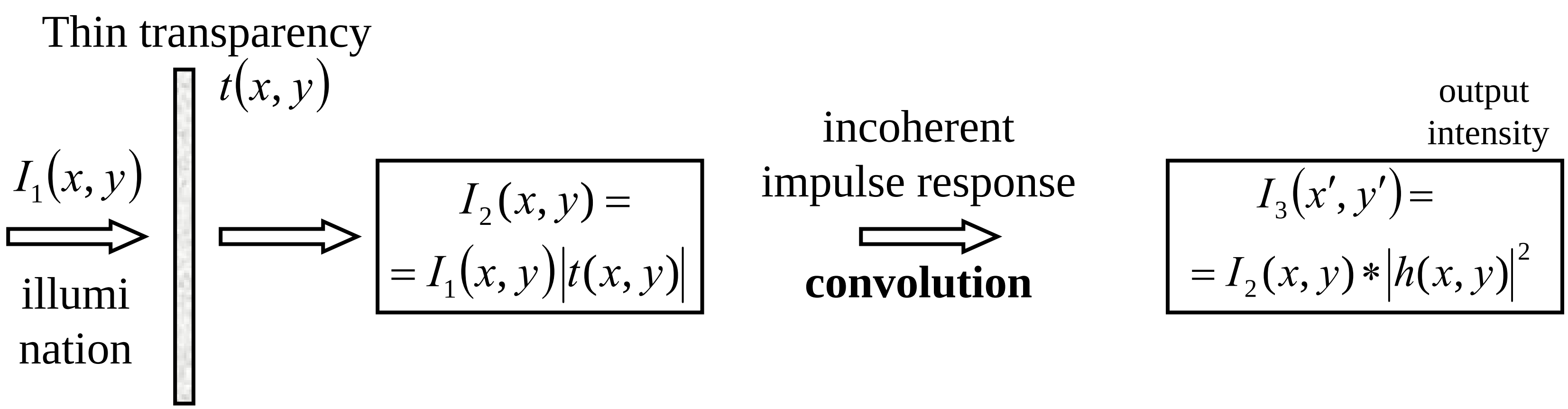


<u>Generalizing:</u>
thin transparency with
sp. incoherent illumination

$$I(x') = \int I(x) |h(x'-x)|^2 \, \mathrm{d}x$$

intensity at the output
of the imaging system

Incoherent imaging
as a linear, shift-invariant system

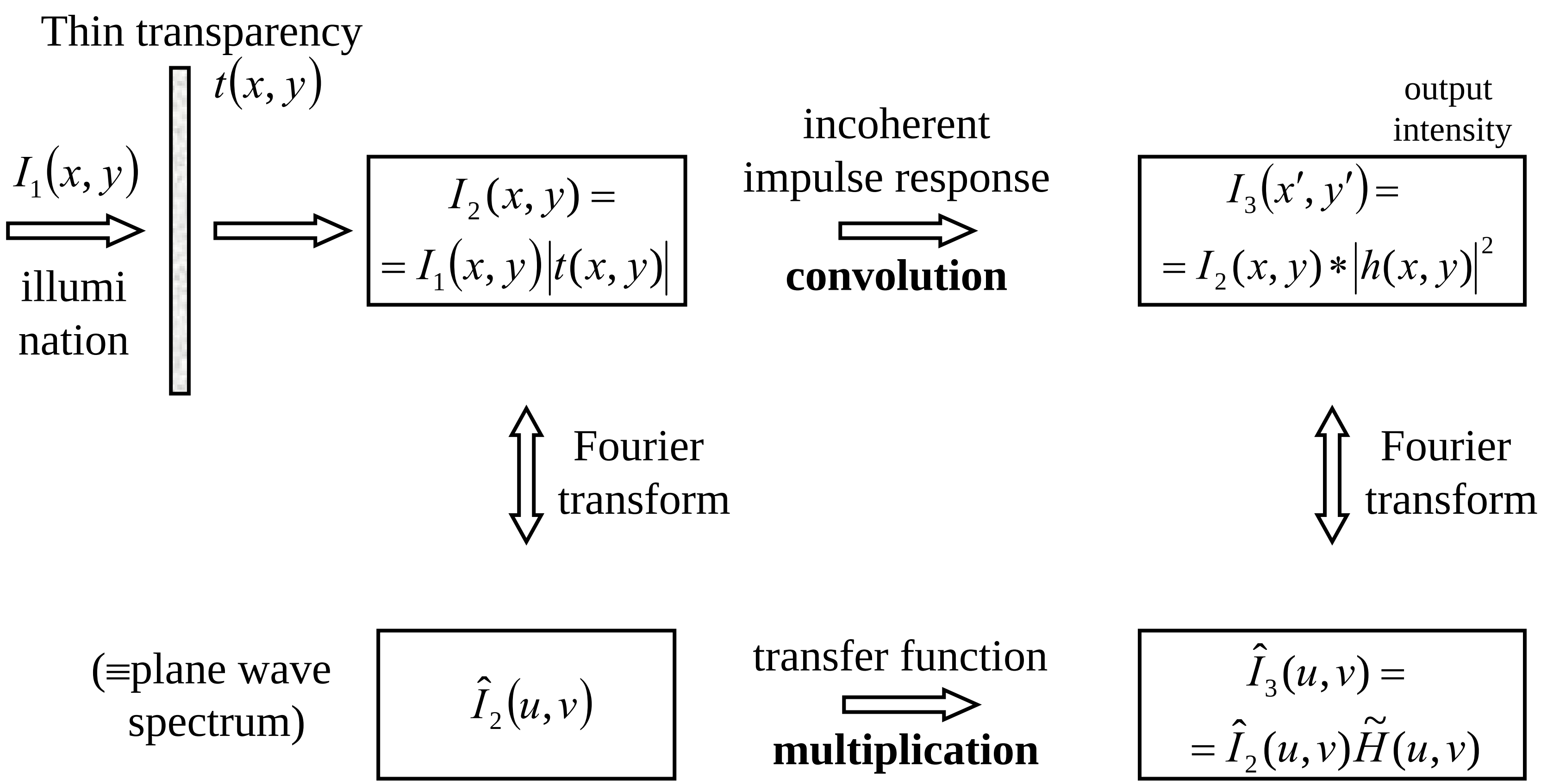


Incoherent imaging is ***linear in intensity***
with <u>incoherent</u> impulse response (iPSF)

$$\tilde{h}(x,y) = |h(x,y)|^2$$

where $h(x,y)$ is the <u>coherent</u> impulse response (cPSF)

Incoherent imaging
as a linear, shift-invariant system

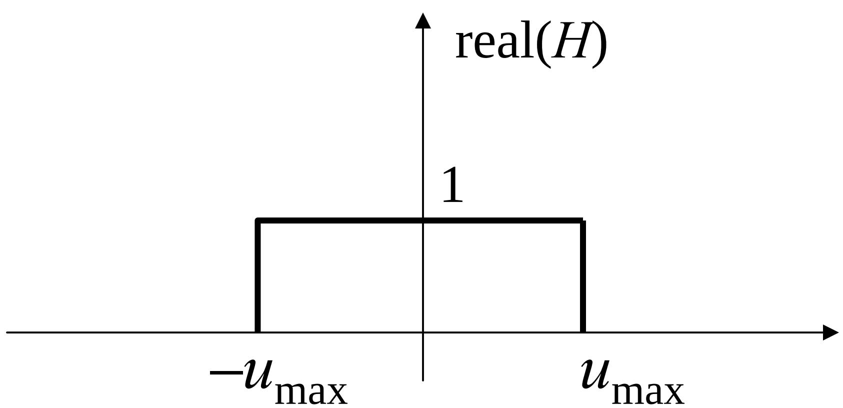


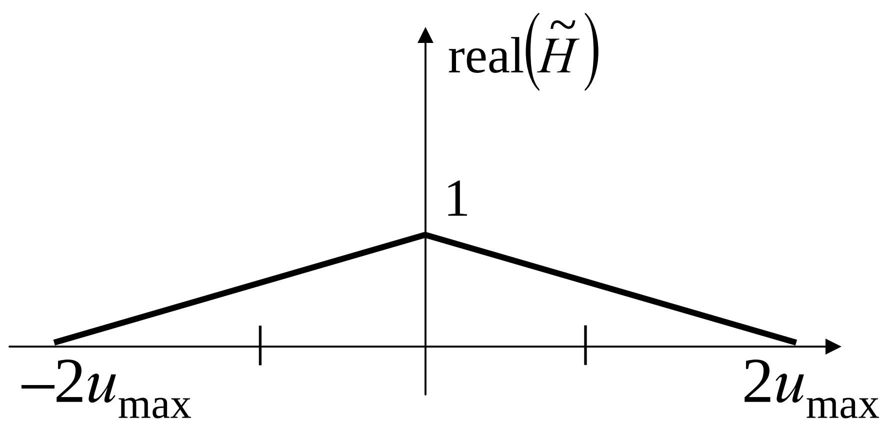
transfer function of incoherent system: $\tilde{H}(s_x, s_y)$ **optical transfer function (OTF)**

The Optical Transfer Function

$$\tilde{H}(u,v) \equiv \Im\left\{ \left| h(x,y) \right|^2 \right\} \quad \text{normalized to 1}$$

$$= \frac{\iint H(u',v') H^*(u'-u, v'-v)\, \mathrm{d}u'\mathrm{d}v'}{\iint \left| H(u',v') \right|^2 \mathrm{d}u'\mathrm{d}v'}$$

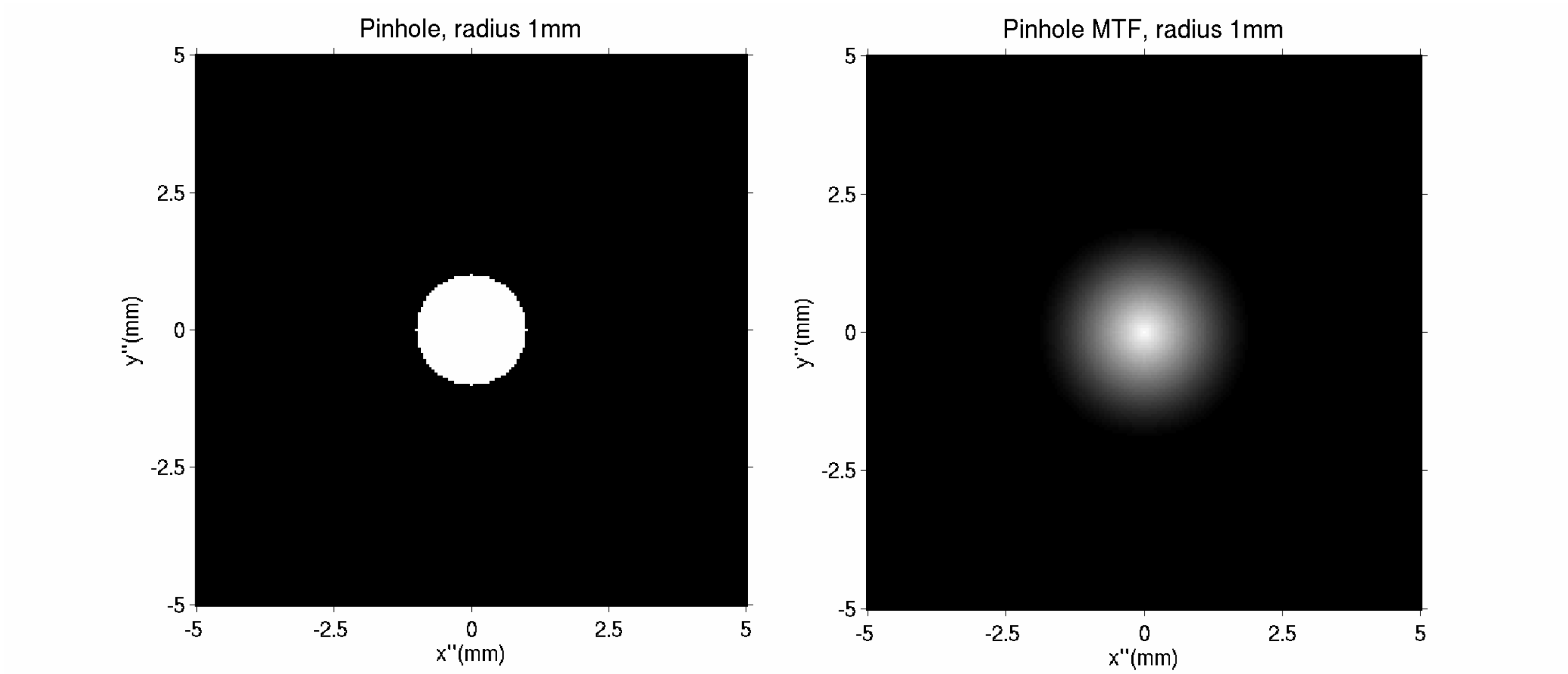




some terminology ...

$H(u,v)$	Amplitude transfer function (coherent)
$\tilde{H}(u,v)$	Optical Transfer Function (OTF) (incoherent)
$\left\lvert \tilde{H}(u,v) \right\rvert$	Modulation Transfer Function (MTF)

MTF of circular aperture

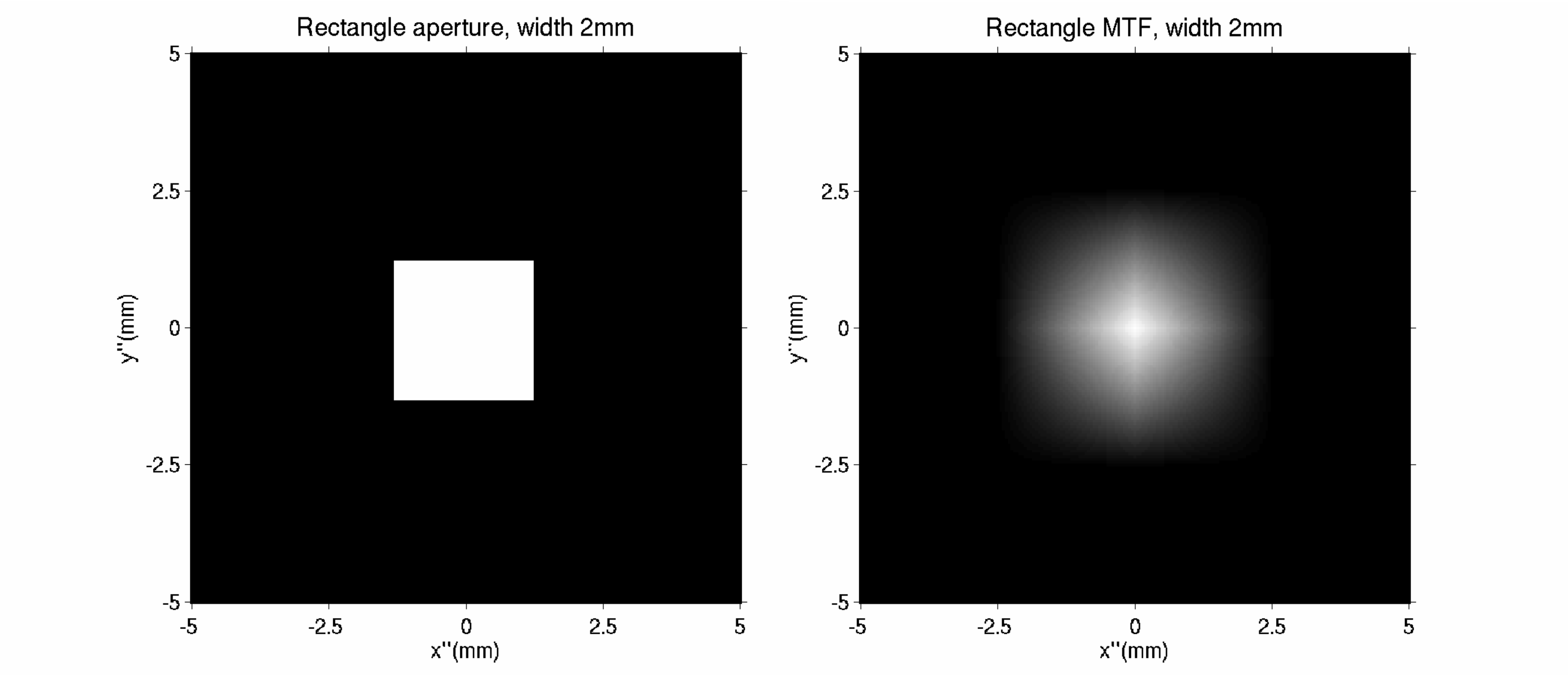


f_1=20cm λ=0.5µm

physical aperture

filter shape (MTF)

MTF of rectangular aperture

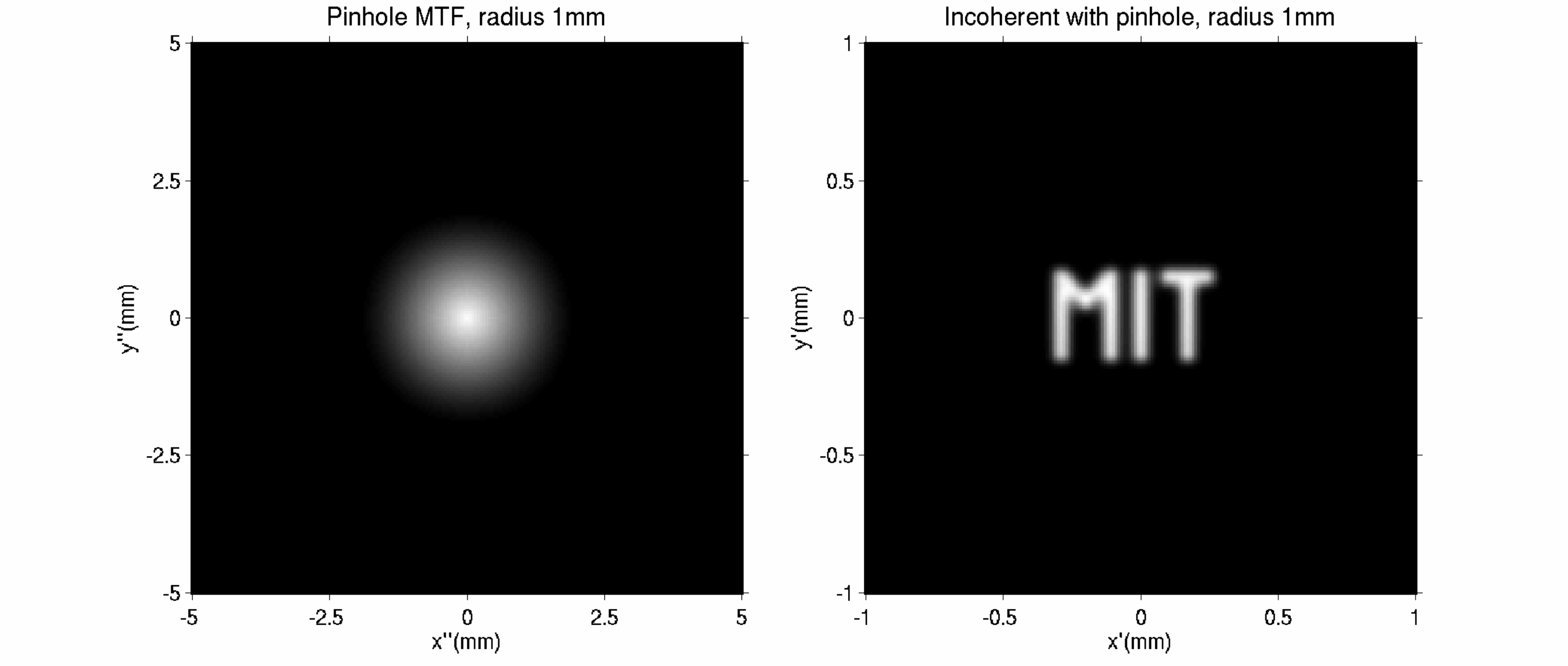


f_1=20cm
λ=0.5μm

physical aperture

filter shape (MTF)

Incoherent low–pass filtering

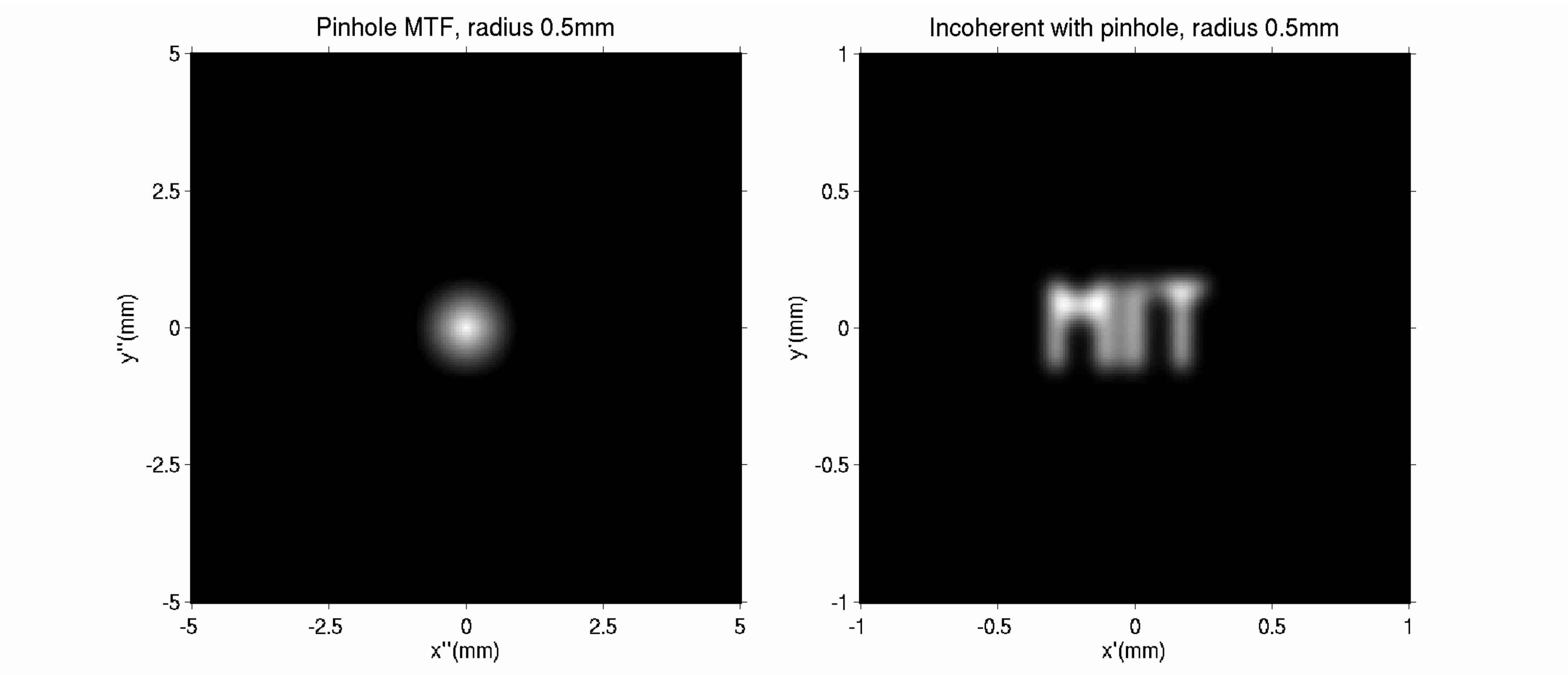


f_1=20cm
λ=0.5μm

MTF

Intensity @ image plane

Incoherent low–pass filtering

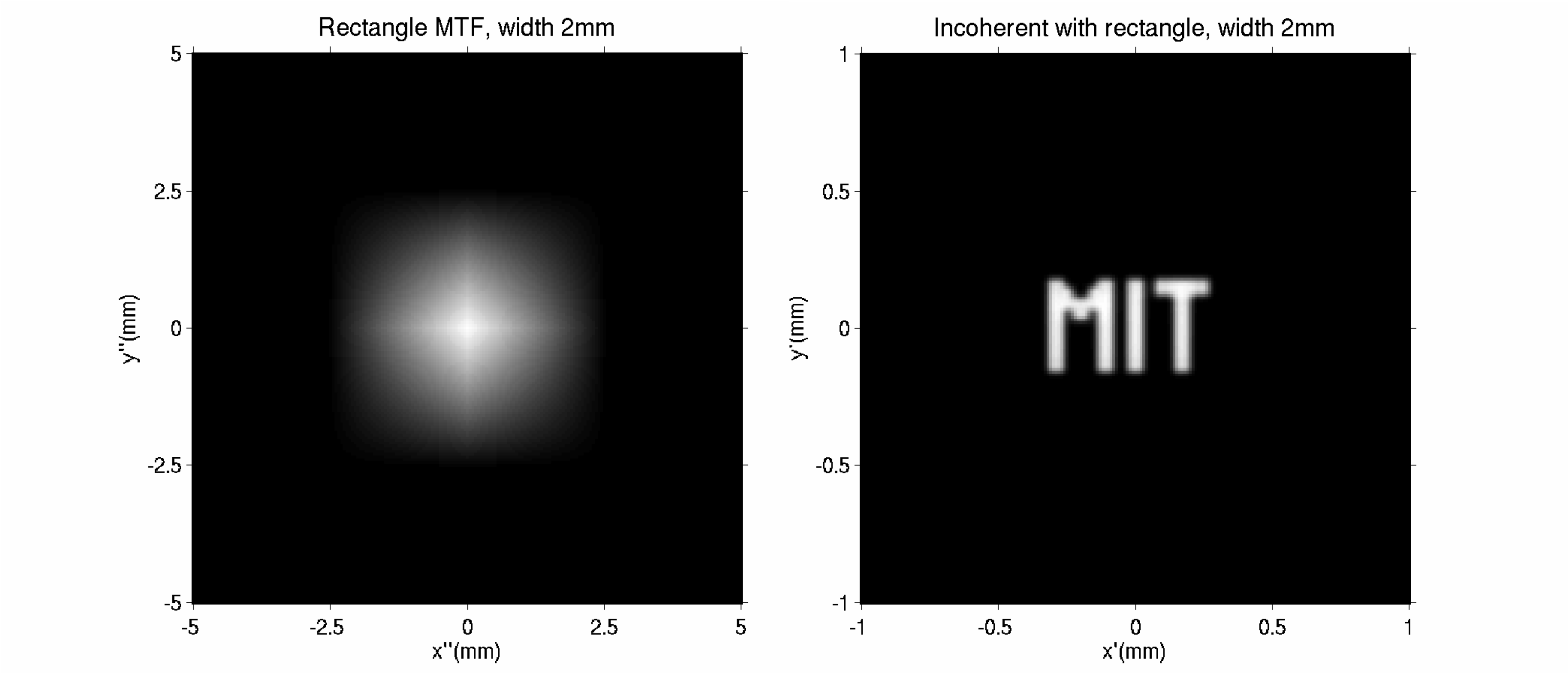


f_1=20cm
λ=0.5μm

MTF

Intensity @ image plane

Incoherent low–pass filtering

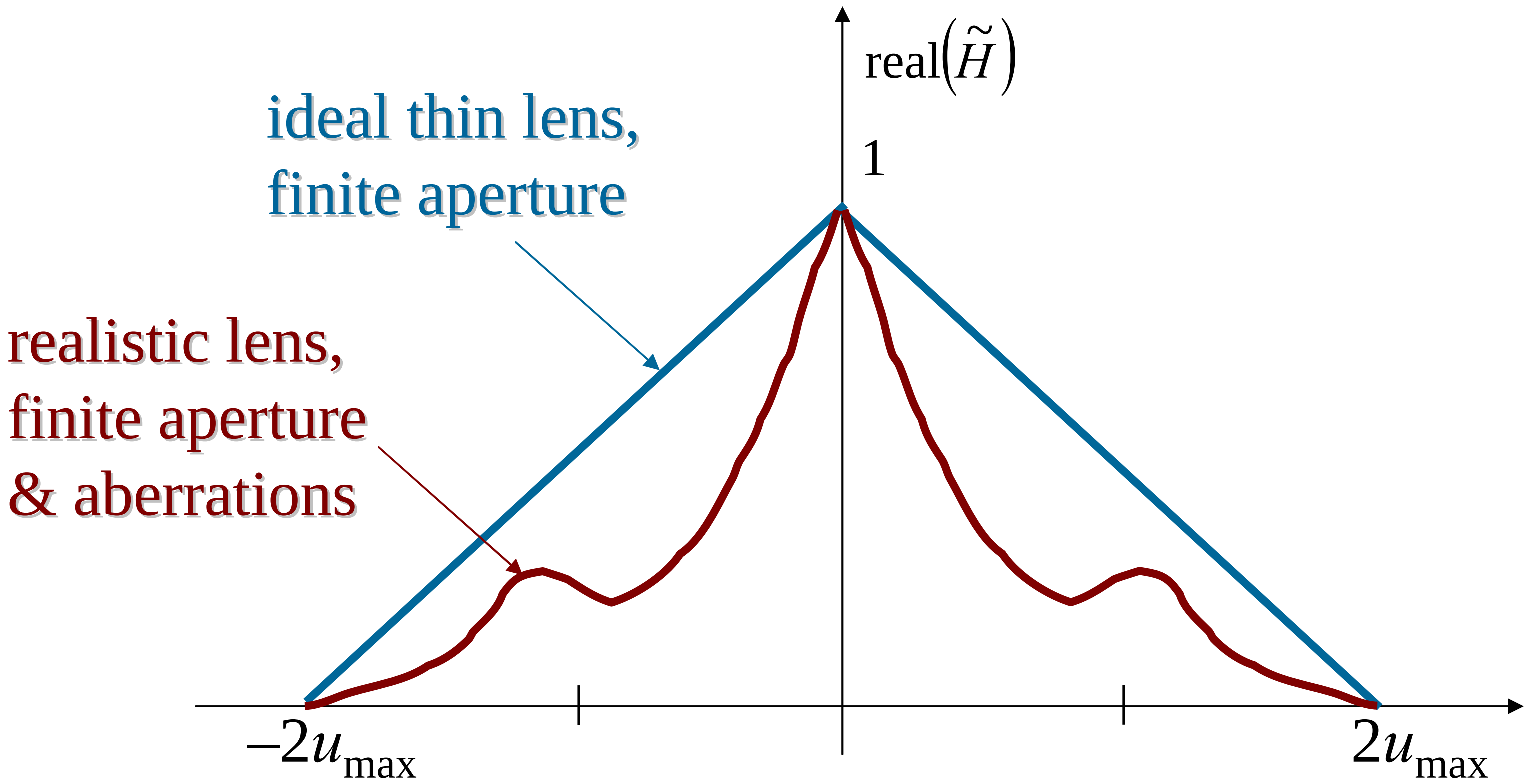


f_1=20cm
λ=0.5μm

MTF

Intensity @ image plane

Diffraction-limited *vs* aberrated MTF



Imaging with polychromatic light

Monochromatic, spatially incoherent response at wavelength λ_0:

$$I(x', y'; \lambda_0) = \iint I(x, y; \lambda_0) \left| h(x' - x, y' - y; \lambda_0) \right|^2 \mathrm{d}x \mathrm{d}y$$

Polychromatic (temporally <u>and</u> spatially incoherent) response:

$$I(x', y') = \int I(x', y'; \lambda_0) \, \mathrm{d}\lambda_0$$
$$= \int \iint I(x, y; \lambda_0) \left| h(x' - x, y' - y; \lambda_0) \right|^2 \mathrm{d}x \mathrm{d}y \, \mathrm{d}\lambda_0$$

Comments on coherent *vs* incoherent

- Incoherent generally gives better image quality:
  - no ringing artifacts
  - no speckle
  - higher bandwidth (even though higher frequencies are attenuated because of the MTF roll-off)
- However, incoherent imaging is insensitive to phase objects
- Polychromatic imaging introduces further blurring due to chromatic aberration (dependence of the MTF on wavelength)