

Think of problem 2 as consisting of the spring, mass and dampers moving for a long time ($-\infty < t < 0$) and then we stop the mass M as $t=0+$ (this is clear from the given velocity source pulling M .)

For $t < 0$ the velocity source can be achieved by, say, a force F acting on Mass M pulling it with a constant velocity $v_1(t < 0) = 1$. Given this force on M , the spring attains an equilibrium extension, after which its ends move at the same velocity. If not, i.e. if $v_1 > v_2$ or $v_2 > v_1$ we will have the spring getting indefinitely extended or compressed respectively.

The extension of the spring for equilibrium at $t=0-$ is $x(0-) = b_2 v_2(0-)/k$ where $v_2(0-) = v_1(0-)$

We obtain the above from equilibrium of the massless damper. You can't do something like $x(0-) = b_1 v_1(0-)/k$, since if you consider the equilibrium of the mass M , you get $F = kx(0-) + b_1 v_1$.

You will see that F has to be a constant 10N force so that the velocity source $v_1(t < 0) = 1$ is achieved.

$x(0+)$ must be the same as $x(0-)$ for continuity. Note however that $\dot{x}(0+)$ need not be the same as $\dot{x}(0-)$

Solution to Part C is clear since you know that for $t=0+$, Mass M is at rest. The free body diagram of the massless damper should give you the answer. In Part(d) you use the initial conditions in Part (b) to get the solution to the differential equation.