
2.003 Spring 2003
Quiz 2 - Sample problem Set 2 Solutions

Problem A - RLC circuit analysis

1.

$$\frac{V_o}{V_i} = \frac{1}{LCs^2 + RCs + 1}$$

2.

$$\omega_n = 2 * \pi * 5000 = 31,400 \text{ r/s} = \frac{1}{\sqrt{LC}}$$

$$L = \frac{1}{\omega_n^2 C} = 0.001 \text{ H} = 1 \text{ mH}$$

$$\frac{R}{L} = 2\zeta\omega_n = 2 * 0.707 * 31,400$$

$$R = 44.4 \Omega$$

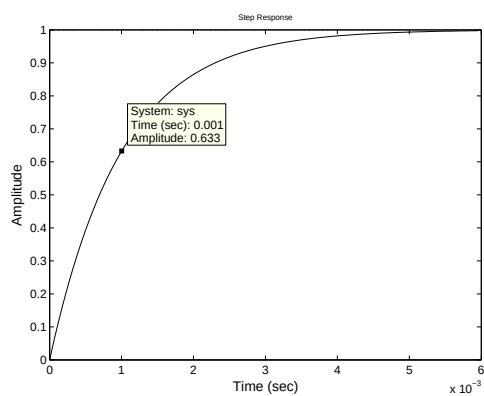
3. There are no zeros, poles at roots of

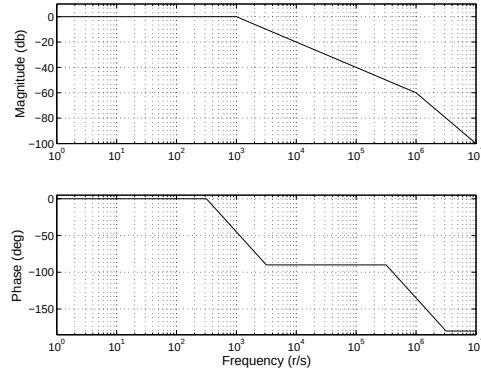
$$s^2 + \frac{1000}{0.001}s + \frac{1}{1e-6 * 1e-3} = 0$$

$$s_1 \approx -1e6$$

$$s_2 \approx -1e3 \text{ dominant pole}$$

$$x_{ss} = 1$$





4.

5. $s = -1e3 = s_2$

6.

$$\text{For circuit with R\&C in } \parallel \quad \frac{v_o}{v_i} = \frac{R_2}{R_2 LC s^2 + (R_1 R_2 C + L)s + R_1 + R_2}$$

$$\text{For circuit with R\&C in series} \quad \frac{v_o}{v_i} = \frac{R_2 C s + 1}{LC s^2 + (R_1 + R_2) C s + 1}$$

Problem B

1.

$$\begin{aligned} T(s) &= \frac{K}{s^2 + 20s + K} \\ \omega_n &= \sqrt{K} \\ 2\zeta\omega_n &= 2\sqrt{K} = 20 \\ K &= 100 \end{aligned}$$

2.

$$\begin{aligned} \frac{V_{out}}{V_1} &= \frac{(s+3)(6s+1)}{(s+3)(6s+1) + (8s+7)} \\ \frac{V_{out}}{V_2} &= \frac{(6s+1)}{(s+3)(6s+1) + (8s+7)} \end{aligned}$$

3. Solve using superposition

$$\begin{aligned} V_{out}(6s^2 + 27s + 10) &= V_1(6s^2 + 19s + 3) + V_2(6s + 1) \\ 6\ddot{V}_{out} + 27\dot{V}_{out} + 10V_{out} &= 6\ddot{V}_1 + 19\dot{V}_1 + 3V_1 + 6\dot{V}_2 + V_2 \end{aligned}$$

Problem C

The transfer function for this system is

$$\begin{aligned}\frac{x(s)}{f(s)} &= \frac{1}{m_{eq}s^2 + cs + k} \\ \omega_n &= \sqrt{\frac{k}{m_{eq}}} \\ 2\zeta\omega_n &= \frac{c}{m_{eq}}\end{aligned}$$

1. From graph, we measure the following

$$\begin{aligned}T &\approx 1.0s \Rightarrow \omega_d = \frac{2\pi}{T} = 6.28r/s \\ \omega_d &= \omega_n\sqrt{1-\zeta^2} \\ M_p &\approx 100\frac{0.75-0.5}{0.5} = 50 \\ \zeta &= \frac{A}{\sqrt{\pi^2 + A^2}} = 0.215 \\ A &= \ln\frac{100}{M_p} = 0.693 \\ \omega_n &= 6.43r/s \\ m_{eq} &= \frac{\omega_n^2}{k} = 4.8 \approx 5kg \\ c &= 2\zeta\omega_n m_{eq} = 13.8Ns/m \approx 14Ns/m\end{aligned}$$

Alternately, you could determine ζ using the log decrement method.

- 2.

$$\begin{aligned}m_{eq} &= m + \frac{I}{r^2} \\ I &= 0.5 kg m^2\end{aligned}$$

Problem D

The transfer function for this system is

$$\begin{aligned}\frac{\omega(s)}{\phi(s)} &= \frac{k}{Js^2 + cs + k} \\ \text{Thus } \omega_n &= \sqrt{\frac{k}{J}} \\ 2\zeta\omega_n &= \frac{c}{J}\end{aligned}$$

1. There are a couple of ways to solve this part of the problem. First, you can read $\omega_r = 9 r/s$ and $M_p = 5 dB$ from the bode plot and use the following

relationships

$$\begin{aligned} M_p &= \frac{1}{2\zeta\sqrt{1-\zeta^2}} \\ \omega_r &= \omega_n\sqrt{1-2\zeta^2} \end{aligned}$$

to find $\zeta \approx 0.3$ and $\omega_n \approx 10 \text{ r/s}$. Or you can read $\omega_n = 10 \text{ r/s}$ directly from the phase plot ($\theta = -90^\circ$)

2. $k = 1500 \text{ Nm/r}$, $c = 90 \text{ Nms/r}$
- 3.

$$\begin{aligned} \omega = 1.1 \text{ r/s } \theta(t) &\approx \sin(1.1t + 0) \\ \omega = 10 \text{ r/s } \theta(t) &\approx 1.58 \sin(10t - \pi/2) \\ \omega = 20 \text{ r/s } \theta(t) &\approx 0.3 \sin(20t - 2.75) \end{aligned}$$