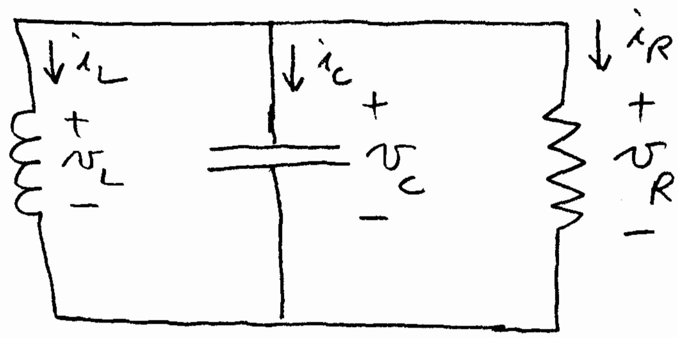
The electrical circuit shown below can be described by a 2nd-order homogeneous differential equation. We will see that it exhibits responses which are analagous to the 2nd order mechanical system studied earlier



First, recall the constitutive relationships

$$v_R = i_R R$$

$$v_L = L \frac{di_L}{dt}$$

$$i_C = C \frac{dv_C}{dt}$$

The three elements are in parallel, and thus have equal voltages

$$v_L = v_C = v_R \stackrel{\Delta}{=} v$$

Also, currents entering (leaving) a node must sum to zero. Counting entering as positive, the currents at the upper node sum as

$$-i_L - i_C - i_R = 0$$

Now we have

$$i_R = \frac{v}{R}$$

$$i_C = C\frac{dv}{dt}$$

$$i_L = \frac{1}{L}\int v\,dt$$

Substituting into the previous result gives

$$\frac{1}{L}\int v\,dt + C\frac{dv}{dt} + \frac{1}{R}v = 0$$

Taking the derivative of each term yields

$$LC\frac{d^2v}{dt^2} + \frac{L}{R}\frac{dv}{dt} + v = 0$$

Compare with the canonical 2^{nd}-order form to give

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$\zeta = \frac{1}{2R}\sqrt{\frac{L}{C}}$$

The associated initial conditions are

$$v(0) = v_c(0)$$

and

$$\dot{v}(0) = \frac{dv_c}{dt}(0) = \frac{i_c(0)}{C} = \frac{1}{C}\left(-i_L(0) - i_R(0)\right)$$

$$= \frac{1}{C}\left(-i_L(0) - \frac{v_c(0)}{R}\right)$$