2.737 Mechatronics
Pre-Lab 0: Controls Review

Assigned: 2/10/06
Due: 2/15/06 in class

Welcome to 2.737! The purpose of this pre-lab homework assignment is to get you thinking about and reviewing controls. This material should be familiar to you from 2.14 or some other equivalent controls course. If you get stuck, you can get help from your fellow students, but make sure that what you turn in is your own work. You can also get help from the staff during office hours which are held in the lab, 1-004.

Lab Kits: We will be handing out lab kits next Tuesday, February 14th from 2:30-5 pm. Please come to the lab during this time to pick up your kit as you will need it for Lab 1 which starts next week.

Problems

1. **First-Order Response.** For the system

$$H(s) = \frac{10}{0.05s + 1}$$

   (a) Sketch the response to a unit step input.
   (b) Sketch the system Bode plot.
   (c) Verify the step response and Bode plot in Matlab.

2. **Second-Order Response.** For the system

$$H(s) = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1}$$

   with $\omega_n = 100$ and $\zeta = 0.1$

   (a) Sketch the step response to a unit step input.
   (b) Sketch the system Bode plot.
   (c) Verify the step response and Bode plot in Matlab.

3. **Doublet Response.** For a system

$$H(s) = \frac{\alpha\tau s + 1}{\tau s + 1}$$

   (a) Sketch the Bode plot and show that for $\alpha > 1$ this is a *lead* network and for $0 < \alpha < 1$ this is a *lag* network.
   (b) Find an expression for the system response to a unit step input. Sketch representative responses for the cases $\alpha > 1$ (*lead*) and $0 < \alpha < 1$ (*lag*).

4. **Block Diagram.** (This problem is taken from Roberge, J.K., *Operational Amplifiers: Theory and Practice*, Wiley, 1976.) Figure 2.20 shows a block diagram for a linear feedback system. Write a complete, independent set of equations for the relationships implied by this diagram. Solve your set of equations to determine the input-to-output gain of the system.

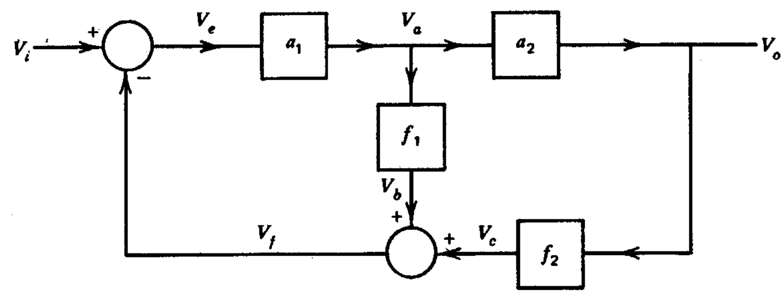

Figure 2.20 Two-loop feedback system.

5. **Nonlinear Feedback.** (This problem is taken from Roberge, J.K., *Operational Amplifiers: Theory and Practice*, Wiley, 1976.) Plot the closed-loop transfer characteristics for the nonlinear system shown in Fig. 2.22.

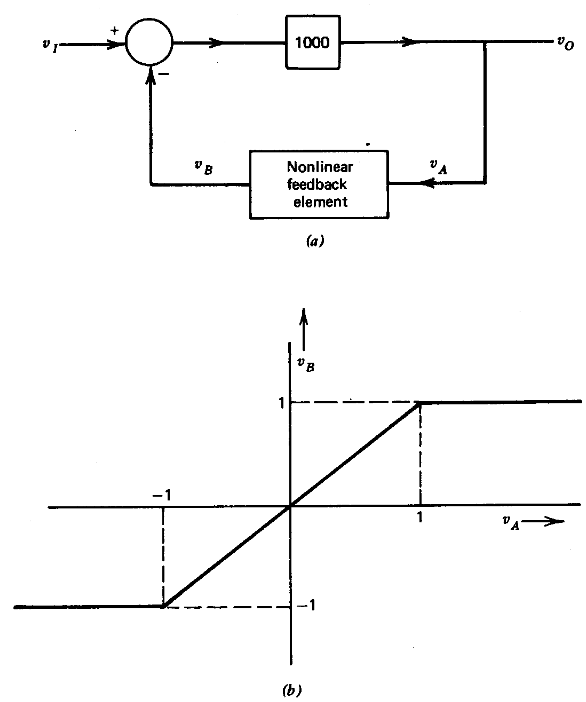

Figure 2.22 Nonlinear feedback system. (*a*) System. (*b*) Transfer characteristics for nonlinear element.

6. Servomechanism with Disturbance. For the servomechanism of Fig. 1.18 let the parameters be

$$\begin{aligned} J_T &= 10^{-5} \text{ kg} \cdot \text{m}^2 \\ B &= 10^{-3} \text{ N} \cdot \text{m} \cdot \text{s/rad} \\ K_m &= 0.1 \text{ N} \cdot \text{m/A} \\ K_t &= 2 \text{ V/rad} \end{aligned}$$

Note that the amplifier with gain K_a is a current-drive which determines the motor current I_m directly. Thus in this model, the motor back emf does not affect the dynamics. The motor torque is given by $T_m = K_m I_m$.

(a) Choose gain K_a to yield a system with a step-response peak overshoot of 20%. What is the resulting damping ratio ζ and natural frequency, ω_n?

(b) Calculate the disturbance transfer function $\frac{\Theta(s)}{T_d(s)}$. How is the disturbance rejection affected by varying K_a?

(c) Sketch a root locus for this system as K_a varies between 0 and ∞. How is stability affected with increasing K_a?

(d) Verify your root locus in Matlab.

(e) For the value of K_a found in part (a), build a Simulink model of the form of Figure 1.19 and simulate responses to steps in V_i and T_d. (Note: There should not be gaps near the summing junction and output in Figure 1.19.)

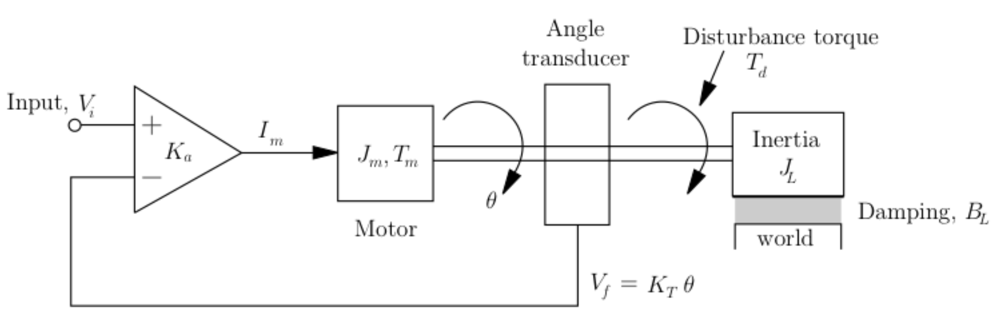

Figure 1.18: Servomechanism with an added disturbance torque.

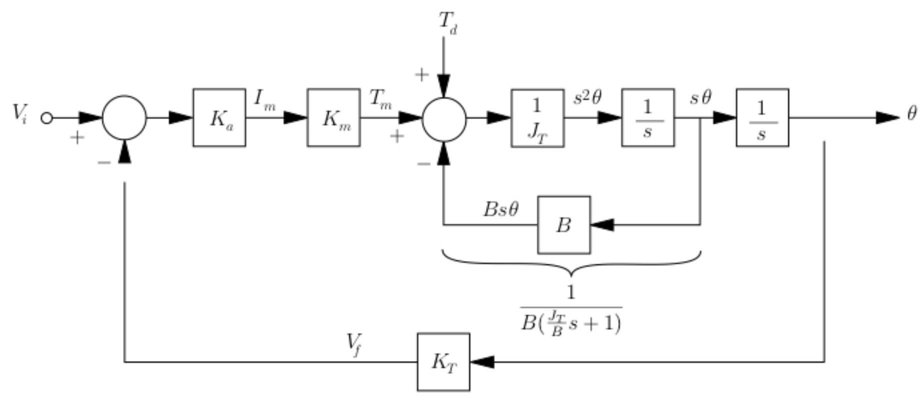

Figure 1.19: Block diagram representation of servomechanism with external disturbance.