

## 2.737 Mechatronics

### Laboratory Assignment 1: Servomotor Control

Assigned: 2/14/06

Pre-lab due: 2/22/06 in class

Reports due: week of 2/27/06 at your checkoff

Reading: Simulink text or online manual, Feedback system notes, Ch. 3-6 [1ex]

## 1 Lab organization

**Lab kits** The lab kits consist of a set of consumables (resistors, capacitors, IC's, wire, etc.) in a plastic box, and a set of returnables (lab kit, tools, cables, meter, and meter leads). The consumables are yours to keep. The returnables must be promptly returned to us at the end of the semester. The kits cost \$500 to replace, and you will be billed for this amount if they are not returned promptly at the end of the semester. If the kits are returned late you will be billed a \$25 late fee. If parts of the kit are missing you will be billed accordingly. You will be assigned a locker and a combination lock to store your kit in the lab. You may take the kit home if you desire. You are responsible for the kits; if they are lost/stolen you will be charged \$500 for the replacement cost.

### 1.1 General Lab Guidelines

- Keep the lab clean and in good working order. No food or drink is allowed in the vicinity of the lab benches. You may eat or drink at the conference table by the white board, but you must discard all trash, clean the table surface, etc. There are paper towels, and a brush and dustpan available over by the sink. (If the sink fills up with water, open the cabinet below the sink, and gently but confidently kick the tank, which will dislodge the float, and start the pump motor. Don't laugh, I've been trying for about 7 years to get MIT physical plant to fix this!)
- *Do not* remove or borrow any equipment from the lab. This includes manuals, scope leads, etc. You may take your lab kit home with you, however.
- Do not leave valuables in the lab or in your lockers.
- Do not let people into the lab who are not in the class. Before you leave the lab, be sure that anyone remaining is a student in the class. If you're the last to leave, please turn off the lights.
- Report any problems with lab equipment to the staff so that we can repair them.
- Be careful not to cut the cord of your lab kit with the wire strippers when you pack up your kit. If you find that your cord has been cut, bring it to the staff's attention and we will repair it. **Do not use any kit with a frayed power cord as this is a significant safety hazard!**

## 1.2 Lab report format

Your lab report must answer the questions posed in this handout and also show the depth of your understanding of the lab material. You are not required to have a formal lab report format, i.e., no list of equipment or procedures is needed. The report is written to tell the teaching staff what you've observed and what depth of understanding you've gained. Take the time to examine secondary effects and anomalous behaviors, and tell us what you think causes them. The labs are open-ended, so don't just try to give canned answers to the questions in the handout. Try to develop an in-depth understanding of what you observe. Back it up with relevant calculations. Include informative data, figures, etc. Your report can be hand-written or word processed as you prefer. The report is due at the time of your checkoff. Late reports will not be accepted except in the case of illness or other circumstances beyond your control.

## 1.3 Checkoffs

A significant portion of your lab grade depends upon an interview with a member of the teaching staff. A signup sheet will be available shortly in the lab. Please sign up for a time which you are sure you can attend; we will not generally allow students to move their checkoff once scheduled. During the checkoff we will ask you to demonstrate the operation of your system and to explain features of its behavior. We will also try to clarify any points of confusion you may have.

We expect you to be in the lab with your system running in advance of your checkoff time so that we can use the time slot most effectively. That means you should come early and be sure that everything is hooked up and running properly well ahead of the checkoff time. Checkoffs will not generally be rescheduled, so be sure to be on time.

## 1.4 Collaboration policy

We strongly encourage you to make use of the other students in the course as a resource. You should introduce yourself to others working near you in the lab. The staff is not in the lab at all hours; in many cases other students in the lab will be able to help you. Conversely, if someone asks you a question, do your best to help them figure it out; you will learn a lot in the process.

You are allowed to freely discuss/share ideas with others in the lab. Your lab grade is based upon the level of understanding you demonstrate in checkoffs and in your report. We find that much of learning comes from interacting with others who may have a different perspective on the material.

We recommend that you work individually on the lab at first and try to do it without discussing with others *a priori*. When you get stuck on a point, then use others to help you get unstuck. We *do not* recommend group efforts from a cold start on the labs, as this is more a social occasion than a learning experience. But by this point in your careers, you will know how you learn best.

This relatively free collaboration policy does not mean however that you can simply copy material

and hand it in as your own. This is clearly academic dishonesty. Also, if you get a good idea from a discussion with another student, please credit them in your lab report as a useful source. This is standard good scholarship and will be viewed as a positive in your lab report.

In summary, use others as a resource to learn effectively, but be sure that your lab report represents your individual understanding of the material. If you have any questions on this policy, feel free to ask one of the teaching staff.

## 2 Lab overview

In this laboratory, we study digital control of a D.C. motor. Each station is equipped with a D.C. motor, with an integral tachometer to measure angular velocity, a single turn potentiometer to measure angular position, and a power amplifier to drive the motor. First, we develop a model for the motor. This model is then used to design velocity and position controllers. For the purpose of this lab, controller design is done using continuous time methods, and the real-time workshop within Matlab/Simulink emulates these with discrete time algorithms.

Initial experiments focus on velocity control. A proportional velocity controller is implemented first. Resulting steady-state errors are observed and the effect of increasing gain on steady-state error is seen. Next we implement a PI velocity controller to remove steady-state errors. Problems with integral windup are observed and an anti-windup scheme is implemented. Finally, natural frequency and damping ratio specifications are met by designing a continuous-time controller.

Next, position control of a D.C. motor is studied. Lead-lag compensators are designed using Matlab, implemented on the real-time hardware, and the resulting performance is studied.

Finally, a velocity minor loop/position major loop controller is implemented. Proportional gains for both loops are selected based on frequency domain criteria. The controller thus implements state feedback control with velocity and position as state variables.

## 3 Power Amplifier Connections

Each station is equipped with a D. C. motor and a power amplifier. **It is important that you read these instructions carefully so that you do not destroy the power amplifier!** As shown in Figure 2 the power amplifier provides a voltage gain of 2 for the signal from the DAC. (Verify this.) We will study this power amplifier in more detail in the next lab.

**Note: The power amplifiers have an absolute maximum of  $\pm 25$  volts. Irreversible damage will occur above this level. The supplies in the lab can provide more than  $\pm 30$  volts and thus are capable of destroying the amplifier. Therefore, before shutting off or turning on the supply *always* set it to 0 volts on both sides.**

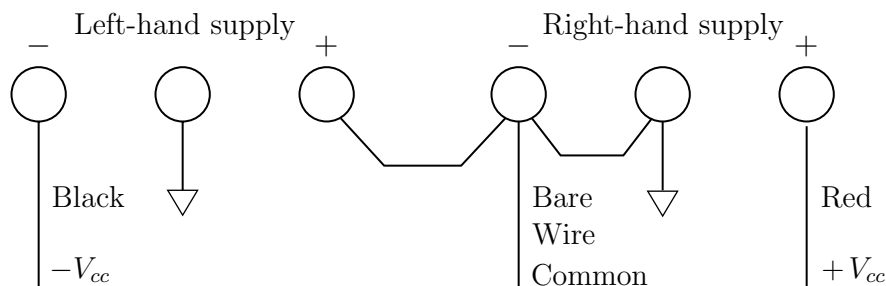


Figure 1: Amplifier/supply connections

**Supply connections:** In the following, we refer to the two 0–30 volt supplies available on the Tektronix power supply. The unit also has a 5 volt supply which we do not use in this lab. The supplies in the lab have been furnished with jumpers and connected for series operation by depressing the appropriate push button on the front panel of the supply. Please keep the supply in this configuration throughout the laboratory. In this mode, the right-hand voltage control sets the voltage output for both supplies. **Never set the power supply to more than  $\pm 17$  V.** Also carefully check for the correct polarity with a voltmeter when the amplifier is first attached to the supply. As connected, the right-hand supply provides  $+V_{cc}$  at its positive terminal, and the left-hand supply provides  $-V_{cc}$  at its negative terminal. Common is taken from the jumpered center connections (the negative terminal of the right-hand supply and/or the positive terminal of the left-hand supply).

Set the current limits on both supplies to the maximum. Whenever you are about to turn on the supply, first adjust both voltage controls to zero. Set the right-hand meter to monitor voltage. Set the left-hand meter to measure current. Then power up the supply. At this point you may increase the supply voltage up to the desired  $\pm 17$  volt level. (The reason for choosing this voltage is that it is comfortably below the amplifier's absolute maximum rating of  $\pm 25$  volts.) The amplifier should be wired to the supply as shown in Fig. 1.

The negative and positive terminals of the op amp are brought out on green and red wires respectively. The board ground plane is also brought out on a black wire for connection on the low-power side. On the high-power side, we have brought out the power supplies with  $+V_{cc}$  on a red wire,  $-V_{cc}$  on a black wire, and common on an uninsulated wire. The amplifier output is brought out on a yellow wire along with another connection to common on a black wire.

## 4 D.C. Motor model

Each station has a D.C. motor and a power amplifier. Please follow the above precautions before powering up your setup. The D.C. motor has a tachometer to measure velocity and a potentiometer to measure angular position. The motor parameters are :

$$\begin{aligned} R &= 7.5 \, \Omega \\ L &= 5.55 \, \text{mH} \end{aligned}$$

$$\begin{aligned}
\Rightarrow \tau_e &= \frac{L}{R} = 0.7 \text{ ms} \\
K_T &= 0.024 \text{ Nm/A} \\
J &= 1.5 \times 10^{-5} \text{ Kgm}^2 \text{ (all loads referred to motor shaft)} \\
K_{\text{tach}} &\approx 0.023 \text{ V/rad/sec} \\
K_{\text{pot}} &= \frac{V_{pp}}{2\pi \cdot (\text{gear ratio})} \approx 0.5 \text{ V/rad (of motor)} \\
\text{No. of teeth on pinion} &= 36 \\
\text{No. of teeth on driven gear} &= 216 \\
\text{Gearing ratio, } N &= 6 \\
\Rightarrow \tau_m &= \frac{JR}{K_T^2} \approx 200 \text{ ms} \\
\Rightarrow \frac{\omega_m(s)}{V_m(s)} &= \frac{1}{K_T(\tau_m s + 1)} \tag{1}
\end{aligned}$$

In this derivation, we are assuming that the motor inductance is low enough to be ignored, i.e.,  $L \simeq 0$ . How can this assumption be justified? Please show how the simplified transfer function above results from this assumption.

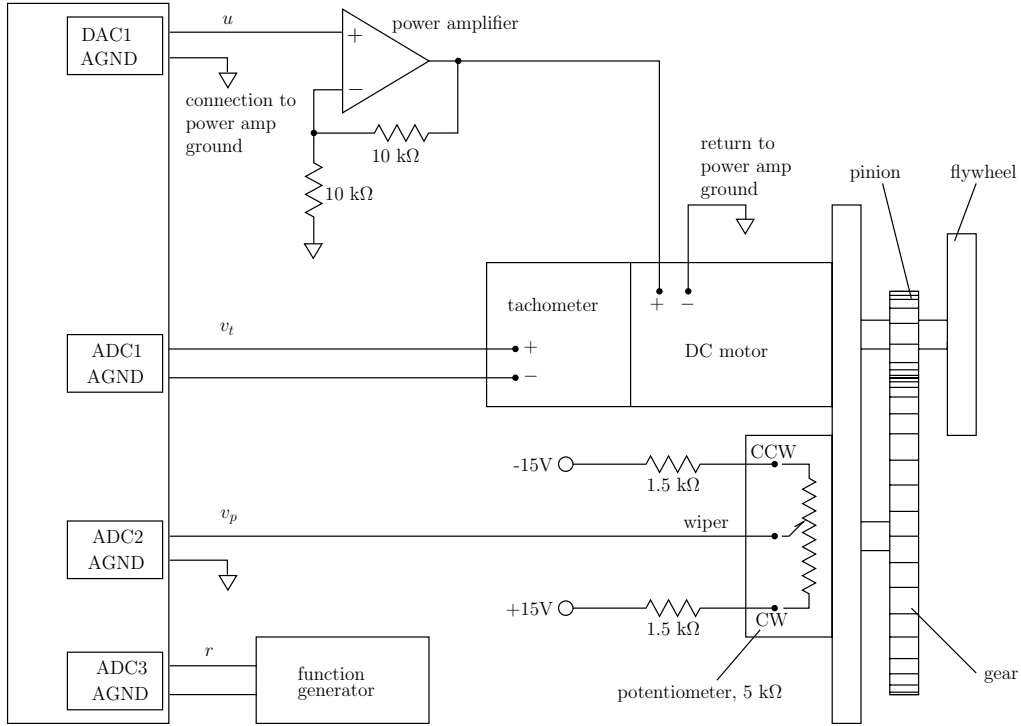


Figure 2: Connections for the DC motor setup.

The connection of the motor setup is shown in Figure 2. For the purposes of this lab, the reference signal  $r$  is generated by the function generator. Connect the function generator to AD channel 3, the tachometer signal to AD channel 1, and the potentiometer signal to AD channel 2. Also, connect the input of the power amplifier to DA channel 1.

In order to match the A/D input range, we connect the potentiometer to operate at a little less than  $\pm 10$  volts ( $\pm 9.4$  V). To do this, we use the  $\pm 15$  volt power source from the protoboard, and use a pair of resistors to reduce the voltage to the desired value. The value of the potentiometer resistance is  $5\text{ k}\Omega$ , thus the pair of voltage dropping resistors are chosen as  $1.5\text{ k}\Omega$ . These resistors are soldered in place on the motor setup, so you don't need to supply them on your protoboard.

- a) Draw a block diagram model which represents the power amplifier, motor, tachometer, and potentiometer using the parameters given above. Throughout this lab, we will drive the motor with the power amp configured for a voltage gain of 2 as shown in the figure.
- b) Derive the transfer functions of the system from input  $u$  to outputs  $v_t$  and  $v_p$ . That is, calculate the transfer functions  $v_t(s)/u(s)$  and  $v_p(s)/u(s)$ . First calculate these in terms of the variables defined above, then substitute in the numerical values. Use Matlab to plot the frequency response for these transfer functions in Bode form.
- c) Use your block diagram to develop a Simulink model for the plant with input  $u$  and outputs  $v_t$  and  $v_p$ . Your model should include the effect of power amp saturation at  $\pm 13$  V. Simulate the plant driven by a step in  $u$  and plot the outputs versus time. Verify that you are getting the proper response from the model. Can you see the effects of power amp saturation for sufficiently large signals?

## 5 Velocity control

We now use the motor model developed in the previous section for designing velocity control of the D.C. motor. The tachometer output is used as the feedback signal. In this and subsequent sections of the lab, use a sampling time of 0.1 msec for the implementation on the real-time hardware. This is fast enough that the effects of sampling are negligible.

- a) Design a proportional velocity controller of the form  $G_1(s) = K_p$ , which realizes a *closed-loop* system time constant of  $\tau = 20$  ms. Implement this controller on the real-time hardware. Collect step response data and compare with the predicted step response. What mechanical impedance (i.e., what equivalent spring/mass/dashpot) is created at the output shaft? Why? (You can sense this impedance most easily at the potentiometer shaft, since the gearing makes this the higher stiffness point in the drivetrain.)
- b) What happens as you increase the gain  $K_p$  above its nominal design value? What limits the achievable bandwidth? What physical mechanisms might explain this?  
  
(Note: In the lab you will see that the tachometer voltage has significant ripple when the motor is spinning. This is due to the brushed construction of the tachometer.)

## 5.1 PI velocity control

Next, we implement PI control which removes the DC errors that were present with a proportional controller. Use a PI controller of the form

$$G_2(s) = K_p \left( 1 + \frac{1}{T_i s} \right)$$

The proportional gain  $K_p$  and the integral or reset time  $T_i$  are chosen to meet specifications on the response.

The presence of an integrator in your control law can lead to *integrator windup*. This can be noticed if you cause the error to accumulate in spite of saturation of the actuators. The integrating action of the controller keeps accumulating the error and leads to large overshoots. You can observe this by setting a constant velocity command, and by imposing a velocity error on the flywheel by clamping it with your hand. Be careful not to get yourself hurt while doing this (*i.e.*, *Keep your fingers out of the gear mesh*). When you release the flywheel, you should be able to see a rather large overshoot. For the purpose of observing windup, use the DC offset of the signal generator to set the velocity to an essentially constant level. Note that the size of the overshoot is dependent on the time for which you hold the flywheel stationary. It is also interesting to conduct this experiment with the velocity command set to zero.

- a) Design a PI velocity controller which nominally creates closed-loop roots with  $\zeta = 0.5$  and  $\omega_n = 30$ . Explain how you calculated the necessary values of  $K_p$  and  $T_i$ . What is the resulting crossover frequency and phase margin for this design?

Implement this controller on the real-time hardware. When building the controller Simulink block diagram, use independent integrator and proportional blocks so that you can easily change the integrator saturation limits to prevent windup. Note that due to the scaling in the DAC block, a saturation limit of  $\pm 1$  is required to limit the DAC output voltage to  $\pm 10$  V.

For a non-zero DC velocity reference, and with the integrator saturation limits set to infinity, clamp the flywheel with your hand for some period of time, and notice the large overshoots when you release it. Modify the integrator saturation limits to prevent windup and re-run the program. Again, impose velocity errors by holding the flywheel with your hand. Notice the greatly improved recovery performance when the integral term is bounded. Explain the difference in the response. What is the ‘best’ value for the saturation limit? Why?

- b) What mechanical impedance (*i.e.*, equivalent spring/mass/dashpot) is created at the output shaft under PI velocity control? Why? How does this impedance change when anti-windup is implemented? Why? This impedance is best sensed with the velocity command set to zero.

## 6 Position control

Position control is in some sense more difficult than velocity control, due to the additional integrator in the transfer function. In position control, feedback is taken from the potentiometer. We use a

one-turn 5 k $\Omega$  servo potentiometer in this setup. The term servo potentiometer means that it is specified for continuous rotation with a reasonable life time. Conventional pots will have a rotation stop at the extreme of rotation, and also will not endure continuous mechanical operation. Given the selected 1.5 k series resistors, and for the  $\pm 15$  V supplies, the potentiometer voltage output varies from  $-9.4$  to  $+9.4$  volts. There is a discontinuity at the end of the range as the voltage transitions from the maximum negative to the maximum positive.

We have used the above information when developing the relationship between potentiometer output voltage and motor shaft angle. This also depends upon the gear ratio of the drive train. Due to the way the potentiometer is connected, the motor angle is 6 times the potentiometer angle, which is what you measure. In the model, we refer all variables to the motor shaft.

A lead-lag compensator with its lag pole at  $s = 0$  can be written as

$$G_3(s) = K \frac{(\tau_1 s + 1)(\alpha \tau_2 s + 1)}{\tau_1 s (\tau_2 s + 1)} \quad (2)$$

The compensator parameters are to be chosen to satisfy the system requirements.

- a) Design a lead-lag compensator as given above to implement a position controller which achieves a crossover frequency of 50 rad/s and a phase margin of 45 degrees. Implement this controller on the real-time hardware. Record the system step response and compare with that predicted in your paper analysis.
- b) With the controller running, notice the effect of increasing the gain  $K$  on the servo stiffness. Do you notice any deterioration of position stability with increasing loop bandwidth? What is this due to?
- c) With the controller running, and with a fixed reference position, try manually rotating the potentiometer shaft away from a fixed reference point. Notice the restoring torque of the servo increases with time due to the integral control term. Record the transient response of recovery from this induced position error. Can you see the rather large overshoots caused by integral windup? Implement anti-windup measures in your controller and repeat this experiment. Comment on the difference with anti-windup in place.

## 7 State feedback

As a final design approach, we combine both position and velocity feedback to implement state feedback. We consider the problem of controlling the position of the motor shaft with both position and velocity feedback as shown in Figure 3. In this case, the controller consists of two gains  $K_{pos}$  and  $K_{vel}$ , with no dynamic compensation elements.

With velocity feedback, we use the tachometer to obtain the derivative of the position, *i.e.*, velocity instead of physically differentiating the output position in a lead compensator. Note that differentiation amplifies noise effects. Velocity feedback however, leads to an increased damping ratio without affecting the natural frequency of the system and hence can lead to better control



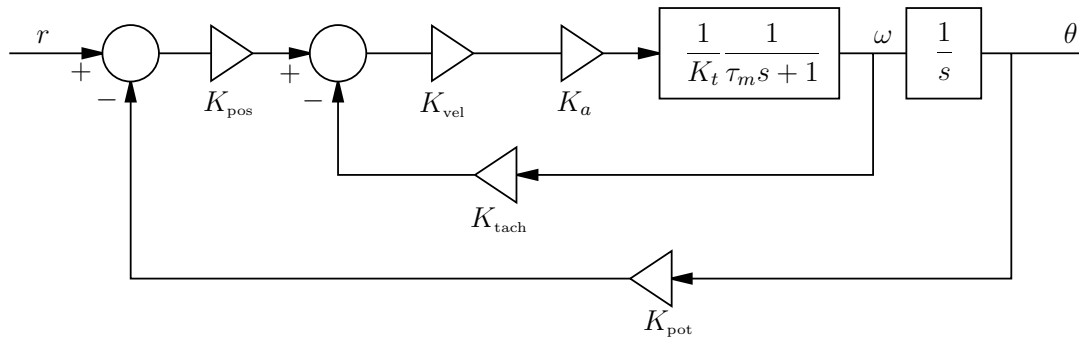


Figure 3: Velocity minor loop with position major loop.

performance. However, the tachometer ripple not affects the accuracy of trajectory following at nonzero velocities. A similar controller can also be designed using pole placement techniques (see the Matlab functions `acker` and `place`) for a state variable model, although we will not take this viewpoint here.

We implement this with the classical viewpoint of a velocity minor loop and a position major loop. The velocity minor loop is usually designed with a crossover on the order of 3 to 10 times higher in frequency than the position loop.

- a) Design a velocity minor loop with a crossover of 200 rad/s and a position major loop with a crossover of 50 rad/s. Find the gains  $K_{pos}$  and  $K_{vel}$  which implement the controller. Run this controller on the real-time hardware. Record the position step response of the controller. How is this affected by varying the gains  $K_{pos}$  and  $K_{vel}$ ? Why?

## 8 Saving scope traces to disk and plotting them in Matlab

To save scope data onto a floppy disk, you need to do the following :

Insert a formatted (3.5 inches) floppy (DOS Format) into the floppy disk drive of the Oscilloscope. Have the waveform that you need to record on the display of the scope and press the **Save/Recall Waveform** button on the scope. This brings up the **Waveform save** menu on the scope display. Press the button corresponding to **Save Format** on the main menu first. Choose **MathCad** from the submenu to save data in the MathCad or ASCII format. Then, press the **Save Wfm Ch1/2** button on the main menu to save the file to disk.

To read the data into Matlab, first copy the data file from the floppy disk to your working directory. Assuming that your data was saved in the file `lab1.dat` in your current working directory, at the Matlab prompt type:

```
>> load lab1.dat
```

```
>> plot(lab1(5:504))  
>> title ('Put in the title for plot here')
```

Note that the first four pieces of data give details of the record length, sampling time etc. Actual data starts only from the fifth entry of the data vector. You will need to generate a time scale for the X axis using regular Matlab commands. The sampling time for the data is included in the first two data entries. You can use this time to generate a time vector.

In the above we have assumed that the scope is set to acquire record lengths of 500 points. You can change the record length to up to 30,000 points using menus on the scope. However since longer records tend to slow down the scope update rate, we suggest leaving the record length set to 500.

It is also possible to save data in *tif* format. This allows easy inclusion in a Word document for adding data to your lab reports. The tif format is selected in the scope's save format menu.