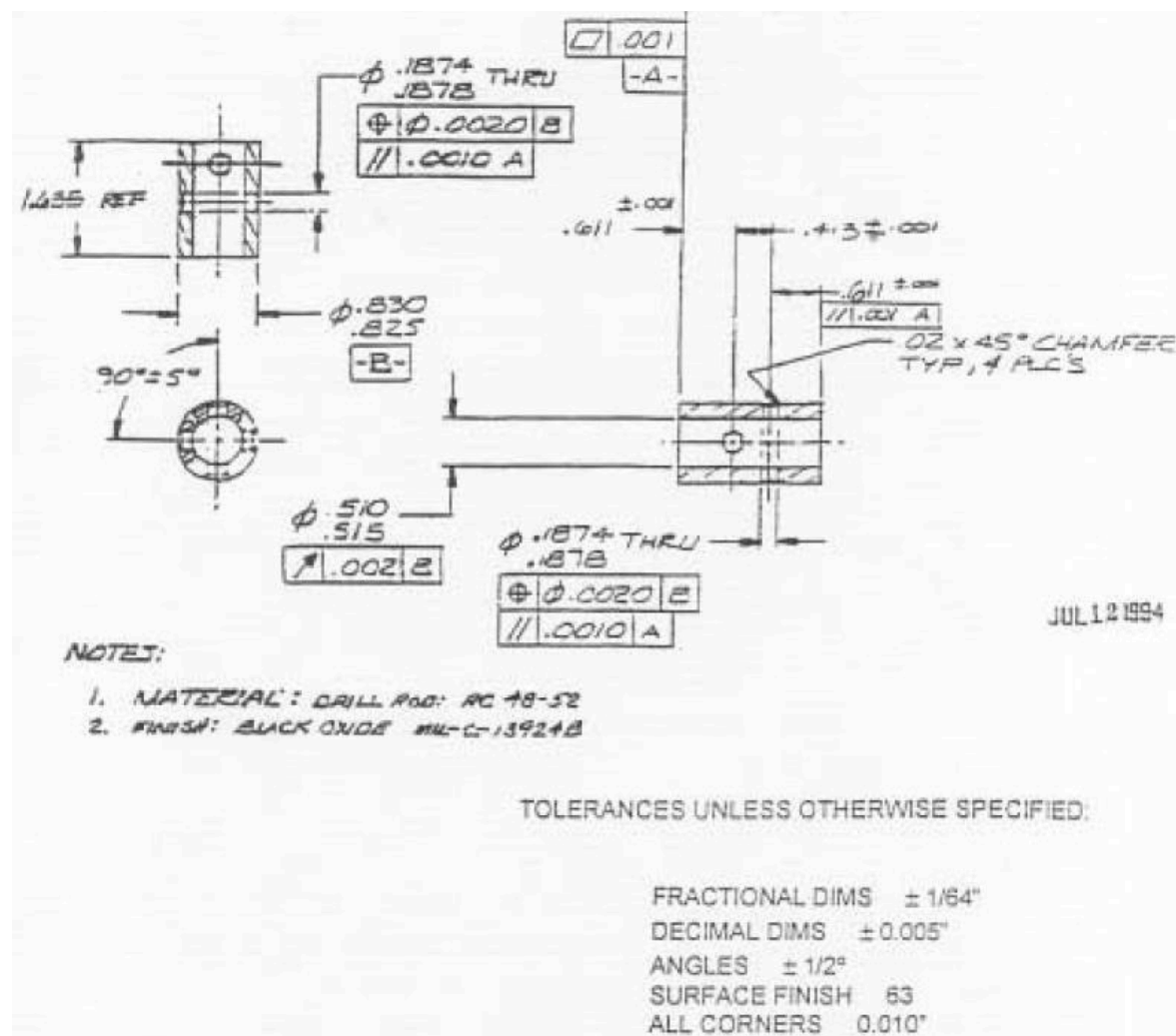


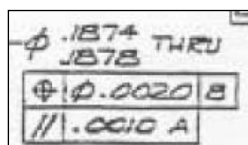
MIT 2.810 Manufacturing Processes and Systems
Homework 1 Solutions
Manufacturing Basics

September 16, 2015
(Problem 2(e) revised October 18, 2015)

Problem 1. Understanding Engineering Drawings

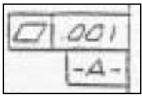


a)

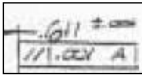


 This feature control frame defines the tolerance on the position of the axis of the hole. The datum B is the axis of the rod. The nominal location of the hole is defined in the right hand side view (0.611 + 0.43 inches from one end). The first row of the feature control frame defines the tolerance on the position. It means that the axis should lie in a cylindrical zone of diameter less than

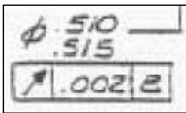
0.002 inches. In addition to the tolerance on the position, the second row defines the tolerance on the orientation of the axis. The second row states that the axis should be parallel to the datum plane A within 0.001 inches.



This feature control frame defines the acceptable flatness for the flat surface of the rod. The surface has to lie between two parallel planes 0.001 inches apart. The datum plane A is also defined here.



This feature frame demands the axis of the hole lie between parallel planes 0.001 inches apart, and themselves parallel to datum plane A.



This feature control frame defines a circular runout tolerance with respect to datum axis B. It means that all points of a circular element of the inner diameter should fall within 0.002 inches when the part is rotated 360° about the datum axis.

b) Isometric sketch:

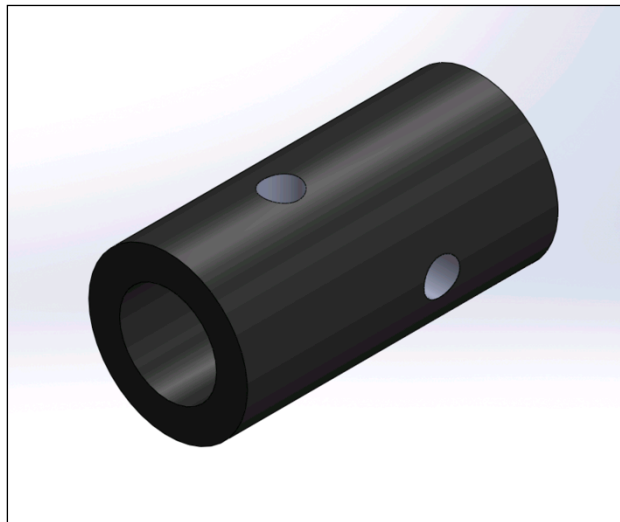


Figure 1: Isometric sketch of the connecting link

c) Heat treatment to obtain the hardness requirement, and possibly grinding and honing of the center hole or possibly reaming of that hole while the part is solid to meet the tolerances required.

d) The raw stock is a steel rod (high speed steel). After machining, the part needs to be heat treated to obtain Rockwell hardness 48-52C. (Read Section 4.8 in Kalpakjian and Schmid.)

e) The basic steps would be:

1. Preparing the materials
2. Fixturing or locating the bread
3. Applying the peanut butter and the jelly

4. Closing the sandwich

If you have to do a lot of sandwiches there are lots of options. For example:

1. The simplest and most costly is to simply add more capacity without changing the process.
2. One could automate the application of PB&J using a tube, roller application, doctor blade etc. perhaps applying both, PB&J one after the other on the same size of the bread. This will simplify handling.
3. The application could also be simplified by mixing the PB & J, but some people may not like the taste. Also, it may be less flexible if you plan to use different jelly flavors.
4. Redesign the bread to be long slices, cover the whole thing and then cutting to size.

Problem 2. Relating Processes Behavior to Engineering Fundamentals

a) Solids

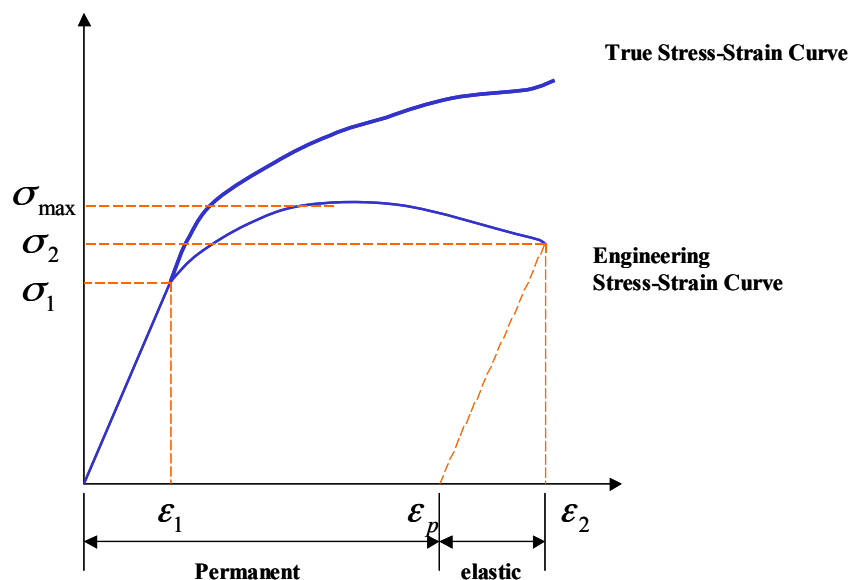


Figure 2: Stress-strain diagram

1. Stiffness/Young's Modulus: $E = \sigma_1/\epsilon_1$.
2. Yield Strength: σ_1 . Not clear from the diagram; by convention it is usually obtained by construction of a line parallel to the linear elastic region but off-set, starting at $\sigma = 0$, $\epsilon_1 = 0.002$.
3. Ultimate Strength: $\sigma_{\max}$
4. Permanent Extension: $\epsilon_p = \epsilon_2 - \sigma_2/E$.
5. True Stress-Stress Curve (see diagram)

6. In general, sand-cast aluminum parts have a larger grain structure compared to parts machined from wrought stock. This leads to a more brittle behavior with a lower strain to failure in the cast part. The stiffness of the two materials, however, would be identical. Forging causes metal flow and grain structure development, which usually leads to improved strength and toughness. Again, the stiffness of a forged and machined part of the same basic material (aluminum) would be the same.

b) Fluids

1. $F/A = \mu V/h$, hence $\mu = Fh/VA$.
2. In general, viscosity follows an Arrhenius temperature dependence of the type $\mu = Ae^{E/RT}$.
3. Hence, the viscosity changes at rate faster than a linear rate with change in temperature.
4. Consider the free body diagram of a “chunk” of fluid in a tube:

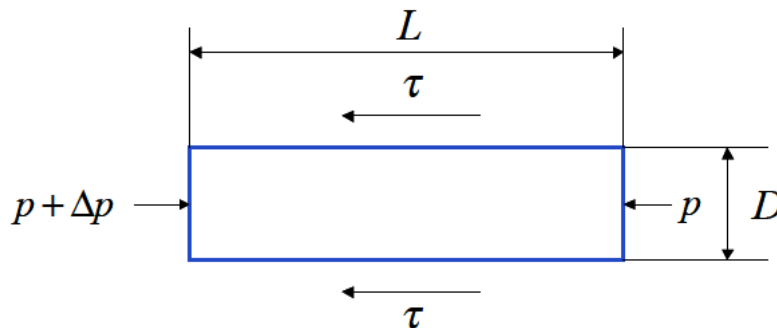


Figure 3: Free body diagram of an element of fluid

In steady state, viscous behavior, a force balance gives: $\Delta P \frac{\pi D^2}{4} = \tau \pi D L$. Hence, $\Delta P = 4\tau \frac{L}{D}$.

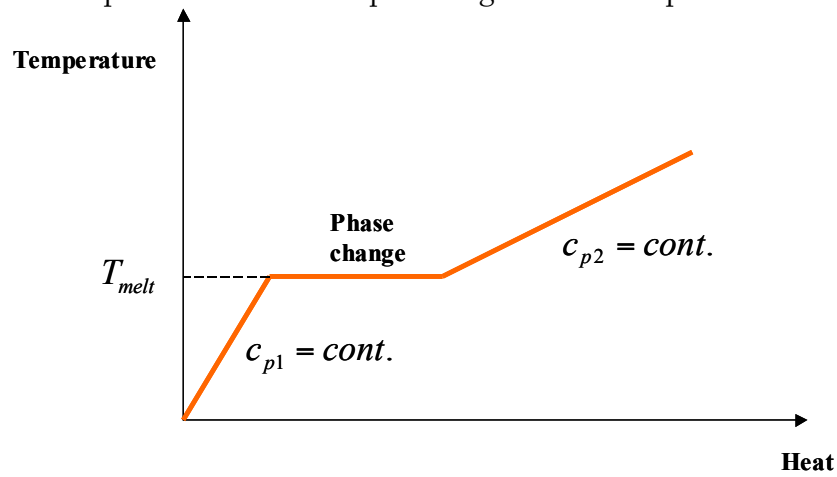
From part 1 above, we know that $\frac{F}{A} = \mu \frac{V}{h}$ for a thin gap, this translates to $\tau \sim \mu \frac{\bar{V}}{D}$ for our case. Substitution above yields

$$\Delta P \sim \mu \frac{\bar{V} L}{D^2}$$

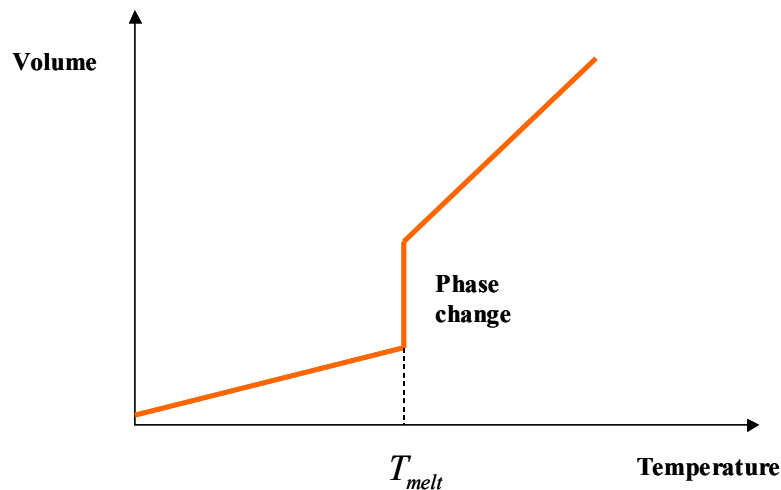
Hence, for a similar flow condition (average velocity $\bar{V}$ is constant, μ is constant and L is a constant), the pressure drop is inversely proportional to the square of D .

c) Heat

1. Conceptual Temperature vs. Heat Impact Diagram for a simple metal.

*Figure 4: Temperature vs. Heat for metals*

2. Volume vs. Temperature Diagram for a simple metal.

*Figure 5: Volume vs. Temperature for metals*

d) Process Classification (see paper by David Hardt)

Process	Type	Energy Source	Serial or Parallel
(a) extrusion	deformation	mechanical	serial
(b) sintering	addition	thermal	parallel
(c) stereo lithography	addition	chemical	serial
(d) swaging	deformation	mechanical	parallel
(e) arc welding	solidification	electrical	serial
(f) die casting	solidification	thermal	parallel

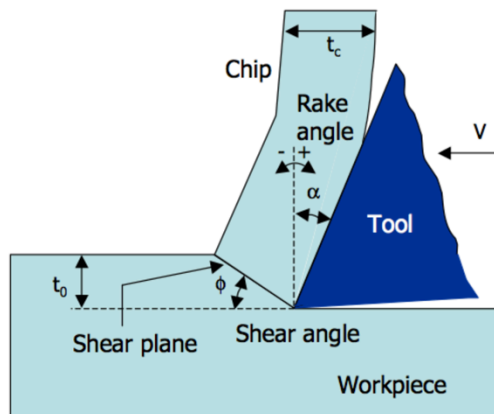
(g) forging	deformation	mechanical	parallel
(h) thermoforming	deformation	thermal/mech.	parallel
(i) injection molding	solidification	thermal	parallel
(j) reaming	removal	mechanical	serial

e) Energy Calculation

Order of magnitude estimates of the energy per unit volume required to remove material:

1. *Plastic deformation (e.g., turning, but ignoring friction):*

Assuming pure, constant shear, plastic deformation requires plastic work on the order of $\int \tau \gamma \, d\text{Vol}$. At yielding, $\tau \cong \frac{Y}{2}$ and γ can be of order 3 to 6 for turning. Hence, a rough estimate (using the larger value) of the work per unit volume in machining is $(Y/2) \cdot 6 = 3Y$, which, it turns out, is approximately equal to the material hardness expressed in the units of stress. This derivation is reviewed later when we discuss machining (see slide from Machining lecture reproduced below).



$$u_s = u_{\text{plastic work}} + u_{\text{friction}}$$

$$u_p \cong \frac{Y}{2} \cdot 3$$

if friction work

is on the order of plastic work

$$\text{then: } u_s \cong 3Y \cong H$$

Approximation

$$u_s \sim H \text{ (Hardness)}$$

In order to get specific energy in terms of J/mm^3 , we need to express Pascals with Joules:

$$1 \text{ Pa} = \text{N}/\text{m}^2 = \text{N} \times \text{m}/(\text{m}^2 \times \text{m}) = \text{J}/\text{m}^3.$$

Approximating the specific energy with $u_s \approx 3Y$ and using the yield strengths given in the table with material properties, the estimates are:

Steel:

$$u_s \approx 3(2000 \text{ MPa}) = 6000 \text{ MPa} = 6 \times 10^9 \text{ J}/\text{m}^3 = 6 \text{ J}/\text{mm}^3$$

Aluminum:

$$u_s \approx 3(330 \text{ MPa}) = 990 \text{ MPa} = 1 \times 10^9 \text{ J}/\text{m}^3 = 1 \text{ J}/\text{mm}^3$$

2. *Local melting:*

Performing an energy balance on a unit volume:

$$u_m = \int_{T_0}^{T_m} \rho C dT + \rho \Delta H_m$$

where

$\int_{T_0}^{T_m} \rho C dT$ = energy required to raise the temperature from T_0 to T_{melt} ,

ΔH_m = energy required for phase change (i.e., latent heat of melting).

If $C \approx$ constant with temperature, this can be simplified to:

$$u_m = \rho(C(T_m - T_0) + \Delta H_m)$$

Using the values given in the table, the estimates for specific energy due to melting are:

Steel:

$$u_m = 8000 \frac{\text{kg}}{\text{m}^3} \left(500 \frac{\text{J}}{\text{kg.K}} (1773 - 293 \text{ K}) + 270,000 \frac{\text{J}}{\text{kg}} \right) = 8.08 \times 10^9 \frac{\text{J}}{\text{m}^3} = 8.1 \frac{\text{J}}{\text{mm}^3}$$

Aluminum:

$$u_m = 2700 \frac{\text{kg}}{\text{m}^3} \left(900 \frac{\text{J}}{\text{kg.K}} (933 - 293 \text{ K}) + 400,000 \frac{\text{J}}{\text{kg}} \right) = 2.64 \times 10^9 \frac{\text{J}}{\text{m}^3} = 2.6 \frac{\text{J}}{\text{mm}^3}$$

3. *Local vaporization:*

Same as in (b), but with an additional temperature rise from T_{melt} to T_{vap} and an additional change of phase (vaporization):

$$u_v = \int_{T_0}^{T_m} \rho C dT + \rho \Delta H_m + \int_{T_m}^{T_v} \rho C dT + \rho \Delta H_v$$

For simplicity, we assume $C_{\text{solid}} = C_{\text{liquid}} = \text{constant}$ and $\rho_{\text{solid}} = \rho_{\text{liquid}} = \text{constant}$ (in order to do an order of magnitude calculation). Then,

$$u_v = \rho(C(T_m - T_0) + C(T_v - T_m) + \Delta H_m + \Delta H_v) = \rho(C(T_v - T_0) + \Delta H_m + \Delta H_v)$$

Using the values given in the table, the estimates for specific energy due to vaporization are:

Steel:

$$u_v = 8000 \frac{\text{kg}}{\text{m}^3} \left(500 \frac{\text{J}}{\text{kg.K}} (3134 - 293 \text{ K}) + 270,000 \frac{\text{J}}{\text{kg}} + 6,259,000 \frac{\text{J}}{\text{kg}} \right) = 63.6 \times 10^9 \frac{\text{J}}{\text{m}^3} = 63.6 \frac{\text{J}}{\text{mm}^3}$$

Aluminum:

$$u_v = 2700 \frac{\text{kg}}{\text{m}^3} \left(900 \frac{\text{J}}{\text{kg.K}} (2773 - 293 \text{ K}) + 400,000 \frac{\text{J}}{\text{kg}} + 10,900,000 \frac{\text{J}}{\text{kg}} \right) = 36.5 \times 10^9 \frac{\text{J}}{\text{m}^3} = 36.5 \frac{\text{J}}{\text{mm}^3}$$

In summary, the following order of magnitude estimates were obtained for the specific energy of the three processes:

Material	<i>Specific Energy (J/mm³)</i>		
	Plastic deformation	Melting	Vaporization
	u_s	u_m	u_v
Steel	6	8.1	63.6
Aluminum	1	2.6	36.5

As expected, the vaporization processes u_v are much more energy intensive compared to shearing (plastic work) u_s and melting u_m .

Order of magnitude estimates of the energy cost per unit weight:

The energy cost per unit weight can be found using the relation:

$$C_e = \left(\frac{\text{specific energy}}{\text{density}} \right) \cdot \text{Cost of energy}$$

Given that the cost of energy is \$0.10/kWh, and converting from kWh to Joules ($1 \text{ kWh} = 1000 \text{ W} \cdot \text{h} = 1000 \text{ W} \cdot 3600 \text{ s} = 1000 \text{ J/s} \cdot 3600 \text{ s} = 3,600,000 \text{ J} = 3.6 \cdot 10^6 \text{ J}$), we can calculate C_e for plastic deformation of aluminum ($u_s = 1 \times 10^9 \text{ J/m}^3$):

$$C_e = \frac{1 \times 10^9 \frac{\text{J}}{\text{m}^3}}{2700 \frac{\text{kg}}{\text{m}^3}} \left(0.10 \frac{\$}{\text{kWh}} \times \frac{1 \text{ kWh}}{3.6 \times 10^6 \text{ J}} \right) = 0.0103 \frac{\$}{\text{kg}} \times \frac{1 \text{ kg}}{2.2 \text{ lb}} = 0.0047 \frac{\$}{\text{lb}}$$

Repeating this calculation for all the other specific energy values, we get:

Material	<i>Energy Cost per Unit Weight (\$/lb)</i>		
	Plastic deformation	Melting	Vaporization
Steel	0.0095	0.013	0.100
Aluminum	0.0047	0.012	0.171

The costs of mild steel and aluminum are \$0.36/lb and \$2.27/lb, respectively. Note that in all three categories, the material costs far outweigh the energy costs of each of the processes.

Comment:

Specific energy calculations: The assumption of a constant specific heat, C , is dependent on the material under consideration. For our order of magnitude estimates with steel and aluminum, a constant C should be a reasonable approximation. The values of C for steel used in these calculations varied from 0.473 kJ/kg-K to 0.561 kJ/kg-K (i.e., less than 20% of the mean value) over the range of temperatures from T_o to T_m . In contrast, equivalent typical data for a ceramic material may vary by 80%.

Problem 3. Basic Understanding of Time, Rate, Cost, Quality and Flexibility

a) Serial vs. Parallel Processes

With Little's Law: $L = \lambda W$; process rate: λ , time in system: W

$$\begin{array}{lll} \text{Serial process: } W = 2t; & L = 2 & \Rightarrow \lambda = 1/t \\ \text{Parallel process: } W = t; & L = 2 & \Rightarrow \lambda = 2/t \end{array}$$

b) Parallel Process w/ n Die Cavities

$$W = t; \quad L = n \Rightarrow \lambda = n/t$$

c) Little's law suggests that to increase λ_f we can a) decrease the lead time W_f by making operations more efficient, or b) increase the inventory L_f e.g. by adding multiple identical production units, each with $W = W_f$.

d) Takt Time Calculation

$$\begin{aligned} \text{Def: } \text{Takt Time} &= \text{Avail. Time} / \text{Required Units} \\ \text{Takt Time} &= (3 \text{ months} * 22 \text{ days} * 8 \text{ hrs./day} * 60 \text{ min/hr.}) / 5000 \text{ units} \\ \text{Takt Time} &= 6.4 \text{ units / minute} \end{aligned}$$

e) Little's Law: fluid flow analogy

For example: average mass flow through a system = volume of the system residence time in the system.

f) Unsteady conditions lead to an accumulation or depletion of units in the system.

g) Costs

$$\begin{aligned} \text{Total costs: } C &= F + V * N \\ \text{Here: } C_{\text{steel}} &= (\$1\text{E}6/N) + (2 * \$25/\text{hr.} * 1/360 \text{ hrs.}) + (\$0.35/\text{lb.} * 7\text{lbs.}) \\ C_{\text{comp.}} &= (\$0.5\text{E}6/N) + (1 * \$25/\text{hr.} * 1/60 \text{ hrs.}) + (\$1/\text{lb.} * 7 \text{ lbs.}) \end{aligned}$$

For $N = 50,000$, we get,

$$\begin{aligned} C_{\text{steel}} &= \$22.58, \\ C_{\text{comp.}} &= \$17.41. \end{aligned}$$

And, for $N = 300,000$, we get,

$$\begin{aligned} C_{\text{steel}} &= \$5.92, \\ C_{\text{comp.}} &= \$9.08. \end{aligned}$$

Thus, we see that for larger volumes, stamped steel is the more economical choice.

h) Quality Assurance Calculation: Part 1

For a centered specification band C_p is defined as:

$$C_p = (USL - LSL) / 6\sigma.$$

Hence for $C_p = 0.5$, we get,

$$USL - LSL = 3\sigma.$$

For a mean-centered process, $USL - \mu = \mu - LSL$. Therefore, we get,

$$USL - \mu = \mu - LSL = 1.5\sigma.$$

The process is depicted in Figure 6.

To determine the percentage of parts out-of-spec, we convert this process into a standard normal curve (i.e., a normal curve with $\mu = 0$ and $\sigma = 1$) by defining a variable z as

$$z = (x - \mu) / \sigma.$$

Thus, for the LSL and USL, we get,

$$z_{LSL} = (\mu - 1.5\sigma - \mu) / \sigma = -1.5,$$

$$z_{USL} = (\mu + 1.5\sigma - \mu) / \sigma = 1.5.$$

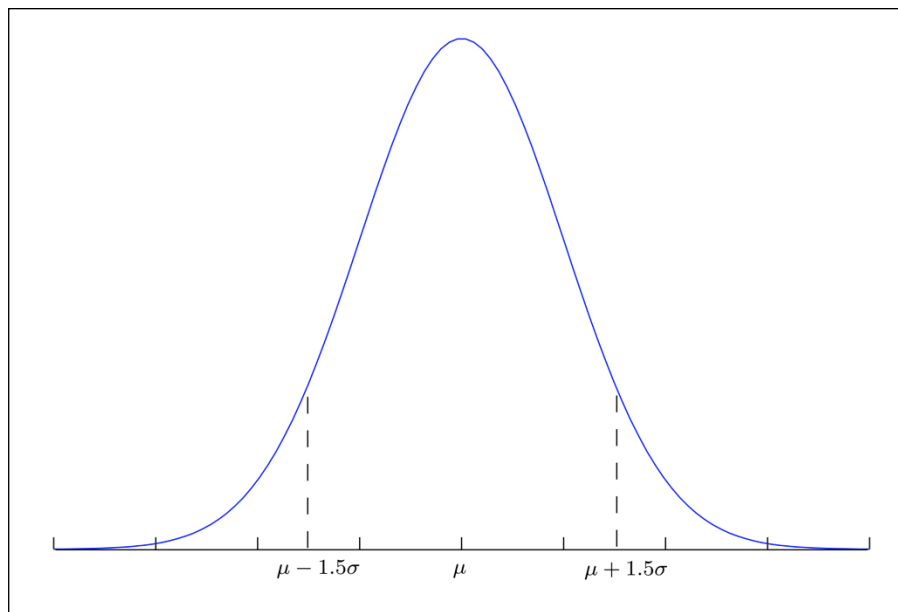


Figure 6: A mean-centered process with specification limits at $\pm 1.5\sigma$

The standard normal curve is shown in Figure 7 below.

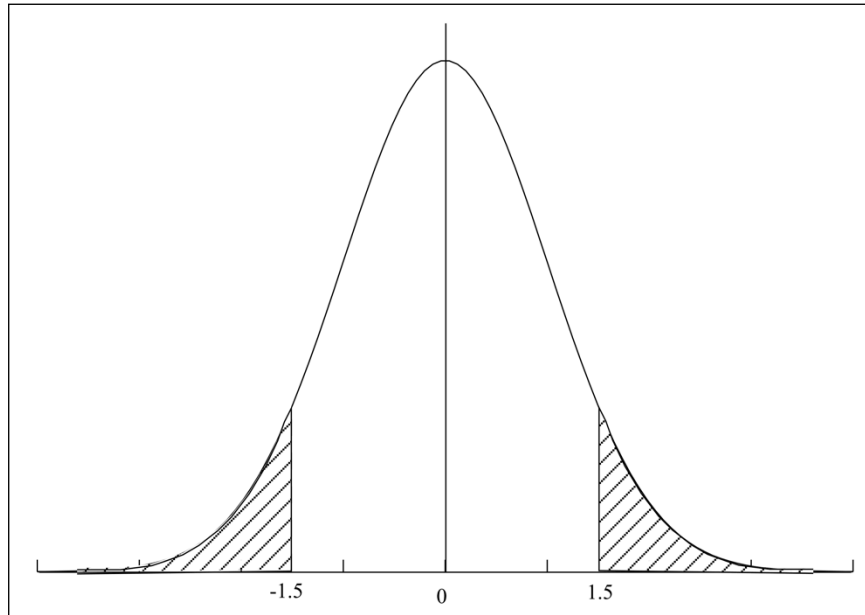


Figure 7: Standard normal curve

The curve extends from $-\infty$ to ∞ . The area under the curve from $-\infty$ to a certain z is the cumulative distribution function of Z , written as $\Phi(z)$. It is the probability that the random variable Z takes on a value less than or equal to z . The area under the curve is equal to one, since it is a probability of a dimension having a certain value.

Because of the symmetry of the standard normal curve, $\Phi(-z) = 1 - \Phi(z)$. That is, the shaded areas are equal.

The probability of parts being in the interval $[-1.5, 1.5]$ is $P[-z, +z] = 2 * \Phi(z) - 1$

For parts which are out of spec: $P_{\text{out}}[-z, +z] = 1 - P[-z, +z] = 2 - 2 * \Phi(z)$

$$\begin{aligned} \text{Hence for } P_{\text{outCp}0.5} &= 2 - 2 * 0.9332 = 13.36\%, \\ P_{\text{outCp}0.9} &= 2 - 2 * 0.9965 = 0.7\%. \end{aligned}$$

i) Quality Assurance Calculation: Part 2

For non-centered specific bands C_{pk} is defined as:

$$C_{pk} = \min(USL - \sigma, \sigma - LSL) / 3 \sigma.$$

$$\text{Hence, } C_{pk} = 1/3$$

The lower spec limit is at 1σ from the mean and the USL is at 3σ from the mean. For a normal distribution, the area beyond the 3σ from the mean is negligible. In order to obtain the desired value we need to find $\Phi(1)$.

$$\Rightarrow P_{\text{OUT}} = 1 - \Phi(Z) = 15.9\%.$$

The selection of C_p does not make any sense since the process is not mean-centered. C_p would give an inflated value of percentage of parts in spec. C_p represents the potential

process capability that could be achieved if the process was mean-centered. The term C_{pk} represents the actual process capability.

Note: the standard normal curve areas given in the table are for the area Φ .
The cumulative area $\Phi = \Phi + 0.5$.

j) Machining Costs

$$\text{Cost}_1 = C_{su} + N * V + (N * T * I)/2$$

$$\begin{aligned} \text{Cost}_2 &= 2 * C_{su} + 2 * (N * V)/2 + 2 * ((N/2 * T/2 * I) / 2) \\ &= 2 * C_{su} + N * V + (N * T * I) / 4 \end{aligned}$$

In general, where n = number of batches, $\text{Cost}_n = n C_{su} + N * V + (N * T * I) / 2n$

$$\text{Find minimum: } dC_n/dn = C_{su} - (N * T * I) / 2n^2 = 0$$

$$n = \text{square root } (N * T * I) / 2 C_{su} \text{) and with batch size } B = N/n$$

$$\Rightarrow B_{\text{best}} = \text{square root } ((2 * C_{su} * N) / (T * I))$$

Problem 4. Process Control

a) Control Regimes as defined by David Hardt:

Regime 1 Sensitivity and Parameter Optimization: minimize $\partial Y / \partial \Delta \alpha$

Regime 2 Minimum of Disturbances: minimize $\Delta \alpha$

Regime 3 Feedback Control of Processes

b) Classification of Control Events

- | | |
|--|----------|
| 1) inspection of incoming material | Regime 2 |
| 2) monitoring for tool breakage | Regime 2 |
| 3) controlling the ambient temperature in the machine shop | Regime 2 |
| 4) warranty repair | Regime 2 |
| 5) poke-yoke inspection | Regime 2 |
| 6) SPC | Regime 2 |
| 7) process simulation | Regime 1 |
| 8) design of experiments parameter optimization | Regime 1 |