

2.810 Manufacturing Processes and Systems

2016 Practice Quiz 2 Solutions

Open book, open notes, calculators and computers with internet turned off.
Present your work clearly and state all assumptions.

Problems:

1. Gas Shortage
2. Toyota Production Cell Design
3. Unreliable Machines
4. Fused Deposition Modeling
5. Literature

Problem 1. Gas Shortage

- (a) Suppose that all car owners fill up their tanks when they are exactly half full. At the present time, an average of 6 customers per hour arrive at a single pump gas station. It takes an average of 5 minutes to service a car. Please estimate the total time each car is in the station, including the waiting time, and the number of cars in the queue. You may assume that the time distributions are exponential.

We can use the M/M/1 queue model (since we are told that the time distributions are exponential) combined with Little's Law ($L = \lambda W$) to find the required quantities.

$$\text{Average arrival rate: } \lambda = 6 \frac{\text{cars}}{\text{hr}} = 0.1 \frac{\text{cars}}{\text{min}}$$

$$\text{Average service rate: } \mu = \frac{1 \text{ cars}}{5 \text{ min}} = 0.2 \frac{\text{cars}}{\text{min}}$$

Then, from M/M/1 queue, the average number of cars in the queue is:

$$L = \frac{\lambda}{\mu - \lambda} = \frac{0.1}{0.2 - 0.1} = 1 \text{ car}$$

From Little's Law, the average waiting time is:

$$W = \frac{L}{\lambda} = \frac{1}{0.1} = 10 \text{ min}$$

- (b) Suppose there is a gasoline shortage and panic buying takes place. To model this phenomenon, suppose that all car owners now purchase gas when their tanks are exactly 3/4 full. Now, since each car owner is putting less gas into her tank during each visit to the station, we assume that the average service time has been reduced to 4.5 minutes. How has panic buying affected the wait at the gas station and the size of the queue?

The drivers are now filling up when their tanks are 3/4 full, which is twice as often as before. Therefore, the arrival rate doubles:

$$\text{Average arrival rate: } \lambda = 12 \frac{\text{cars}}{\text{hr}} = 0.2 \frac{\text{cars}}{\text{min}}$$

The service rate increases slightly:

$$\text{Average service rate: } \mu = \frac{1 \text{ cars}}{4.5 \text{ min}} = 0.222 \frac{\text{cars}}{\text{min}}$$

As before, using M/M/1 queue and Little's Law:

$$L = \frac{\lambda}{\mu - \lambda} = \frac{0.2}{0.222 - 0.2} = 9 \text{ cars}$$

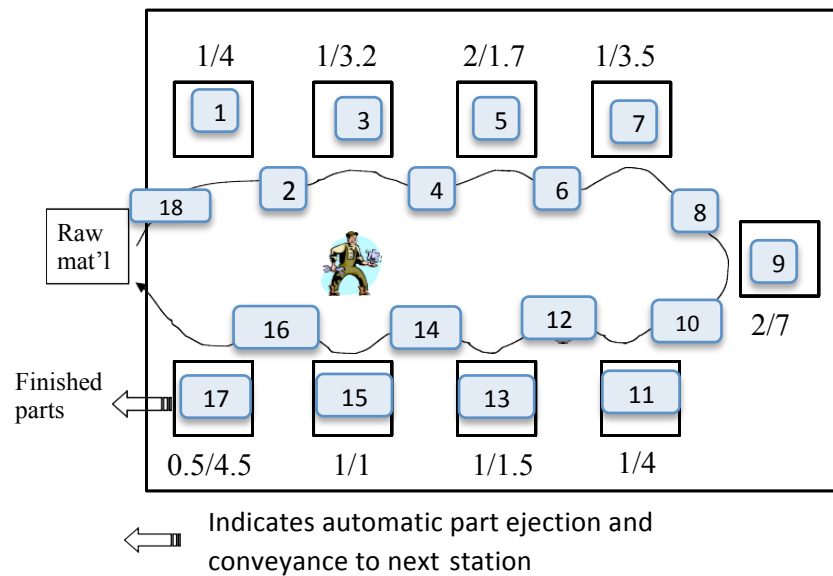
$$W = \frac{L}{\lambda} = \frac{9}{0.2} = 45 \text{ min}$$

We can see that the both the number of cars in the queue and the waiting time increased significantly.

Problem 2. Toyota Production Cell Designs

Consider the cell design shown in the diagram below and answer the following questions. Note that a box represents a process and holds one part at a time, and a darkened circle represents a “decoupler” which can hold one part. The times given by each process are in minutes as manual time/machine time. Assume a 3 second walking time between machines (including raw materials as a “machine”). State all assumptions.

- (a) What is the current production rate, inventory, and time in the system for the system shown below with one operator?



The blue squares represent parts in the system. Part 18 is the raw material on its way to machine 1.

3 seconds = 0.05 min

Segment	Manual Time (min)	Walking Time (min)	Process Time* (min)
1 (Raw Mat'l to Machine 1)	1	0.05	4
2	1	0.05	3.2
3	2	0.05	1.7
4	1	0.05	3.5
5	2	0.05	7
6	1	0.05	4
7	1	0.05	1.5
8	1	0.05	1
9	0.5	0.05	4.5
Finished Parts to Raw Mat'l's		0.05	
	10.5 min	0.5 min	

*For any machine i , the Machine Time $MT_i = (\text{Process Time})_i + (\text{Manual Time})_i$

Cycle Time (CT) = Total Manual Time + Walking Time = 11 min

Check that $CT > MT_i$ for all machines. If true, then Cycle Time determines the production rate.

Little's Law is $L = \lambda W$, where

L = units in system (inventory)

λ = arrival rate (which for steady state = production rate)

W = time in system

$$\text{Production Rate: } \lambda = \frac{1 \text{ part}}{11 \text{ min}} = 0.091 \frac{\text{parts}}{\text{min}}$$

For most of the 11 minute cycle time, there are 18 parts in the system (one at each machine, one at each decoupler between the machines, and one in the operator's hand). For the time between when operator removes the finished part (part 17 in the diagram) from the last machine and puts it in the finished parts collection (not part of the system) till when she arrives at the raw materials station, she does not have a part in her hand, so for 0.05 min the total number of parts in the system is only 17. This yields the following equation:

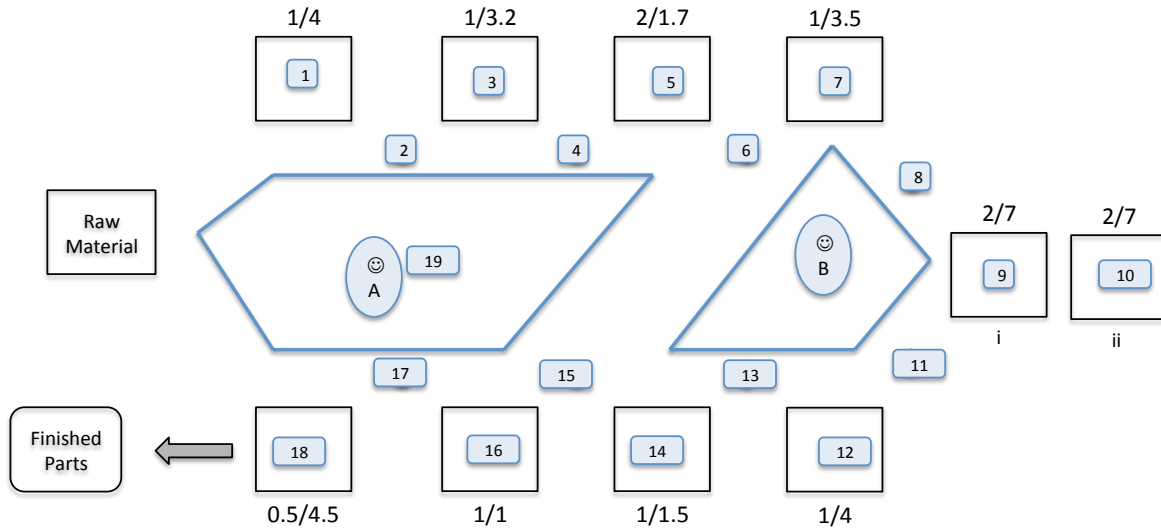
$$\text{Inventory: } L = 17 \left(\frac{0.05 \text{ min}}{11 \text{ min}} \right) + 18 \left(\frac{10.95 \text{ min}}{11 \text{ min}} \right) = 17.995 \approx 18 \text{ parts}$$

$$\text{Production Rate: } W = \frac{\lambda}{L} = (17.995 \text{ parts}) \left(\frac{1 \text{ part}}{11 \text{ min}} \right) = 197.95 \text{ min}$$

(b) Modify the system so that it produces one part every 6 minutes. Mark your changes clearly on the diagram below. Also separately list what you have changed to meet this goal. Estimate:

- 1) The production rate
- 2) Inventory in the system
- 3) Time in the system for your new system after you make your modifications.

Times are again given in minutes. Walking time between machines is as in part (a).



Changes to System:

1. Add an additional machine: The process time for the 5th machine is greater than the desired cycle time (7 min > 6 min). To alleviate this problem, simply add a second, identical machine. Worker B will alternate between machines i and ii, so the apparent process time for this machine is $7/2 = 3.5$ min. The Machine Time is now $3.5 + 2 = 5.5$ min.

2. Add an additional worker: The original cycle time with one worker is 11 minutes. To reduce this to about 6 minutes, add a second worker, and divide the cell into two parts.

Worker A:

$$\text{Cycle Time} = (1+1+2+1+0.5)\text{min} + (0.05 \times 6)\text{min} = 5.8 \text{ min}$$

Worker B:

$$\text{Cycle Time} = (1+2+1+1)\text{min} + (0.05 \times 4) = 5.2 \text{ min}$$

Since the cycle time for worker B is 5.2 min, and the largest machine time is 5.5 min, the new cycle time (the time to make one part) is the cycle time for worker 1, 5.8 minutes.

Using the same Little's Law equation as in part a, but now with 19 total parts instead of 18 due to the duplicated machine:

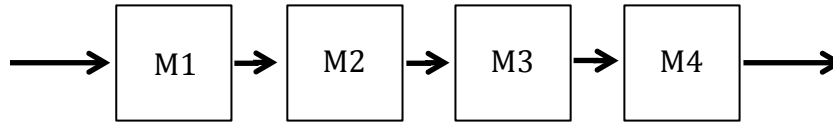
$$\text{Production Rate: } \lambda = \frac{1 \text{ part}}{5.8 \text{ min}} = 1.72 \frac{\text{parts}}{\text{min}}$$

$$\text{Inventory: } L = 18 \left(\frac{0.05 \text{ min}}{5.8 \text{ min}} \right) + 19 \left(\frac{5.75 \text{ min}}{5.8 \text{ min}} \right) = 18.99 \text{ parts}$$

$$\text{Production Rate: } W = \frac{\lambda}{L} = (18.99 \text{ parts}) \left(\frac{1 \text{ part}}{5.8 \text{ min}} \right) = 110.1 \text{ min}$$

Problem 3. Unreliable Machines

You are making parts on a transfer line consisting of 4 machines. Each machine has a different mean time to failure and mean time to repair, listed in the table (in minutes). The operation time on every machine is 1 minute per part. There is no buffer between any of the machines.



	M1	M2	M3	M4
MTTF	500	100	200	250
MTTR	20	10	10	20

- (a) Assume that machine failures occur as a result of tool breakage, which only happens when the machines are operating. What is the average production rate of the transfer line?

This question assumes the operation-dependent failure model, i.e., the machine can only fail when it is operating. Thus, Buzacott's formula for the production rate of a line with no buffers can be used.

$$P = \frac{1}{\tau} \frac{1}{1 + \sum \frac{MTTR}{MTTF}}$$

$$P = 1 \frac{1}{1 + (0.04 + 0.10 + 0.05 + 0.08)} = 0.787 \frac{\text{parts}}{\text{min}}$$

- (b) Using the same operation-dependent failure model as in part a, you can now place one infinite buffer between any two machines. Where would you place it in order to achieve a minimum production rate of 0.85 parts/minute?

Assume the original (operation-dependent) failure model applies. You would generally want to place buffers around the bottleneck machine, which in this case is machine 2 (it has the lowest production rate on its own). If we try placing an infinite buffer after this machine, we get a line of machines 1+2 with no buffer, followed by the line of machines 3+4. Using Buzacott's formula for each of the two lines:

$$P_{1,2} = \frac{1}{1 + 0.04 + 0.10} = \frac{1}{1.14} = 0.8772 \frac{\text{parts}}{\text{min}}$$

$$P_{3,4} = \frac{1}{1 + 0.05 + 0.08} = \frac{1}{1.13} = 0.8850 \frac{\text{parts}}{\text{min}}$$

With the infinite buffer, the production rate of the entire line will be the bottleneck (minimum production rate) among these two parts:

$$P = \min(P_{1,2}, P_{3,4}) = \min(0.877, 0.885) = 0.877 \frac{\text{parts}}{\text{min}},$$

which exceeds the requirement of 0.85 parts/minute.

Note: If you place the buffer between machines 1 and 2, you get a production rate of 0.813 parts/min. If you place it between machines 3 and 4, you get a production rate of 0.84 parts/min.

- (c) Assume you are using machines with the same MTTF and MTTR as above, but now their failures are caused only by the failure of the control system (which is always on whether or not the machine is operating). What is the average production rate of the transfer line?

This problem asks to use a different failure model, i.e. time-dependent failure, rather than operation-dependent failure as in part (a), which was presented on slides 27-29 of the "Time Analysis for Manufacturing Systems" lecture. The equation for production rate with this model is slightly different:

$$P = \frac{1}{\tau} P_1 P_2 P_3 P_4 = \frac{1}{\tau} \left(\frac{1}{1 + \frac{MTTR_1}{MTTF_1}} \right) \left(\frac{1}{1 + \frac{MTTR_2}{MTTF_2}} \right) \left(\frac{1}{1 + \frac{MTTR_3}{MTTF_3}} \right) \left(\frac{1}{1 + \frac{MTTR_4}{MTTF_4}} \right)$$

$$P = 1 \left(\frac{1}{1.04} \right) \left(\frac{1}{1.1} \right) \left(\frac{1}{1.05} \right) \left(\frac{1}{1.08} \right) = (0.9615)(0.9091)(0.9524)(0.9259) = 0.771 \frac{\text{parts}}{\text{min}}$$

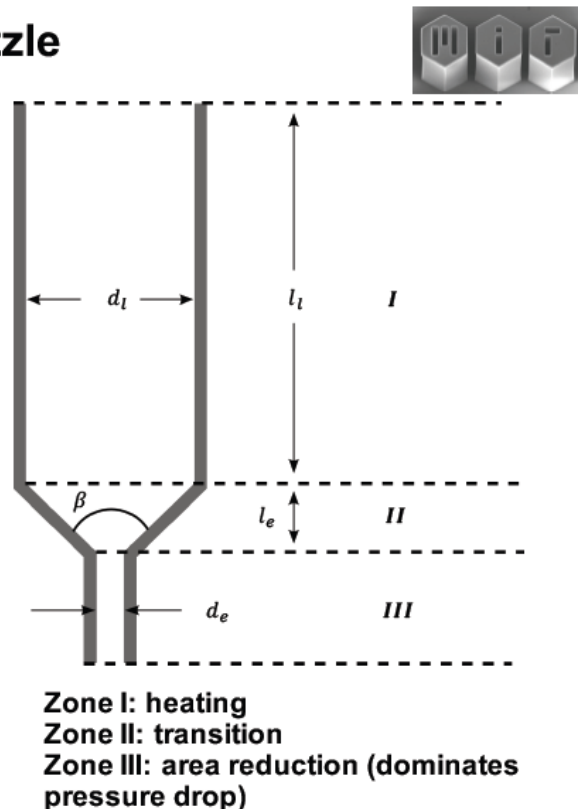
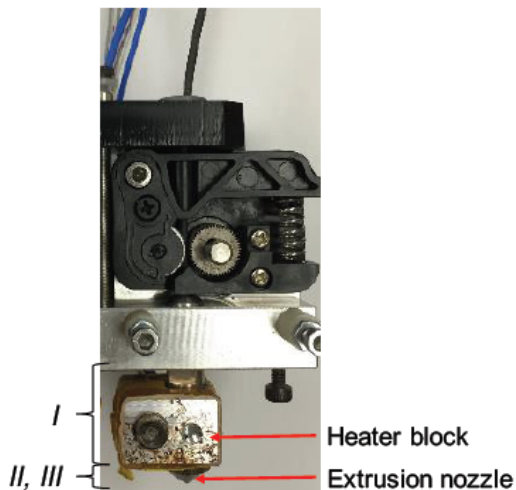
Problem 4. Maximum Extrusion Rate for Fused Deposition Modeling

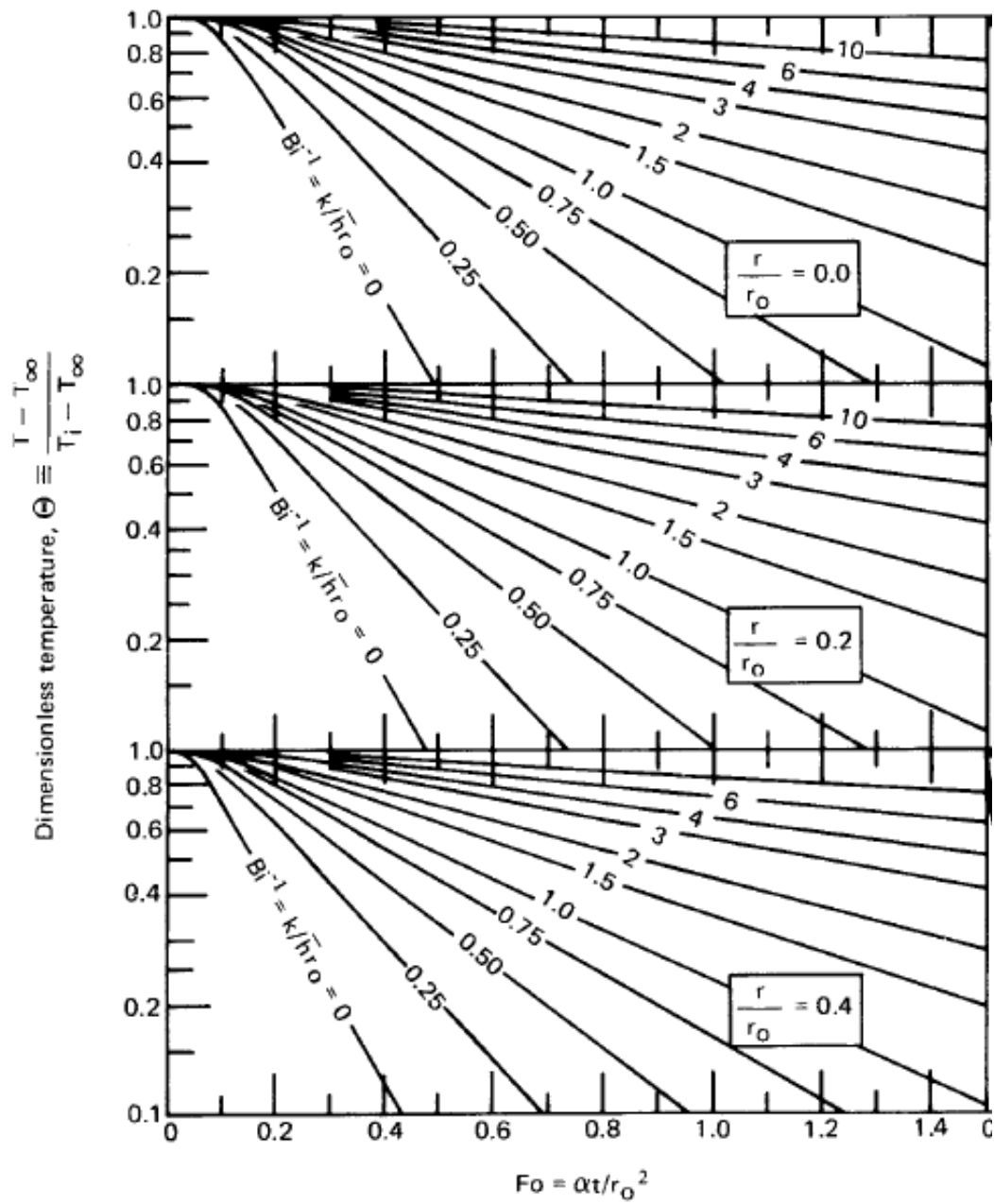
The figure below, based on Professor John Hart's lecture slides, shows the nozzle for the Fused Deposition Modeling (FDM) Additive Manufacturing process for thermoplastics. An ABS filament approximately 1 mm in diameter is pushed through the heating zone (Zone I) where it melts, and is then reduced in area for printing. We are concerned with the **maximum rate** at which plastic can be pushed through this zone and still be guaranteed that the output is fully melted (at the point where it enters Zone II).

Referring to the figure, assume the diameter of the heating zone is $d_t = 1.0$ mm and its length is $l_t = 1.25$ cm. Its walls are maintained at 260°C with an electric resistance heater. Please make an estimate for the maximum rate at which the ABS filament can be fed into the heating zone (in meters/second).

Note: The chart from the textbook by Lienhard for conductive heat transfer in cylinders is included at the end of the exam for reference.

Geometry of the FDM nozzle





Transient temperature distributions in a long cylinder of radius r_o at three positions. $r/r_o = 0$ is the centerline. (Lienhard, Fig 5.8)

This problem can be approached by considering the ABS filament as a cylinder that's being heated by the nozzle walls that are held at constant temperature. Initially, the filament is at room temperature ($T_i = 25^\circ\text{C}$). The walls remain at $T_w = T_\infty = 260^\circ\text{C}$. In order to ensure that the ABS melts before it enters the transition zone, we can look at the time needed for the center of the filament to reach melting temperature.

Because ABS is an amorphous polymer, it does not have a well-defined melting point where it changes phase and thus no latent heat of fusion; instead, it changes from solid to rubbery after it passes the glass transition temperature and then to a fluid with decreasing viscosity as it rises beyond the melting point. From John Hart's other slides on Fused Deposition Modeling and from Boothroyd's Ch. 8, Table 8.5 on injection molding parameters, the recommended forming temperature for ABS is in the range of $240\text{--}260^\circ\text{C}$. (This solution assumes $T = 250^\circ\text{C}$; other θ 's in the range 0-0.1 are OK.)

The transient temperature distribution chart provided above provides a graphical way to estimate the time needed to reach a given temperature at a given radial position within the cylinder. We need to calculate the dimensionless temperature (θ) and Biot number (Bi), and use the top chart (for $r_0/r = 0$, since we are looking at the center of the cylinder) to find the Fourier number (Fo). Note that the charts are provided for a body that's being *cooled*; in order to adapt them to a body being heated, we can just change the signs in the temperature calculation so that θ remains between 0 and 1:

$$\theta = \frac{T_\infty - T}{T_\infty - T_i} = \frac{260 - T}{260 - 25} = \frac{260 - 250}{235} = 0.043$$

For the inverse Biot number, $Bi^{-1} = k/hr_0$, knowing that thermal conductivity k is very low (~ 0.2 W/mK for polymers), we can assume that $Bi^{-1} \sim 0$. At the intersection of this θ and Bi^{-1} , the Fourier number on the x-axis is approximately $Fo = 0.45$, which we can use to calculate time (using typical thermal diffusivity α for polymers):

$$t = \frac{r_0^2 Fo}{\alpha} = \frac{(0.05 \text{ cm})^2 (0.45)}{10^{-3} \frac{\text{cm}^2}{\text{s}}} = 1.1 \text{ s}$$

Thus the filament needs to spend a minimum of 1.1 seconds inside the heating element. Then the maximum feed rate can be calculated from (length of heating zone/time):

$$v = \frac{l}{t} = \frac{1.25 \text{ cm}}{1.1 \text{ s}} = 1.13 \frac{\text{cm}}{\text{s}} = 0.01 \frac{\text{m}}{\text{s}}$$

Alternatively: The simplest way to solve this problem is by analogy with our one-dimensional analysis for injection molding. The same equation applies for heating and for cooling. Remember

we got that $\frac{1}{t} \sim \frac{\alpha}{(\frac{h}{2})^2}$ and that $t = \frac{(\frac{h}{2})^2}{\alpha}$ was the exact solution when $\theta = 0.1$ for constant wall

temperature and $Bi^{-1} = 0$ (Injection Molding lecture, Slide 14). We have a very similar situation here, except that the geometry is now a cylinder.

Problem 5. Literature

- (a) According to Maccoby, which jobs are most prized by factory workers?

From Maccoby, page 162: "Even when supervisors have allowed teamwork and opportunities for problem solving and participation (all of which workers appreciate), assembly work remains monotonous and stressful... The jobs most prized by employees are the ones that allow the most autonomy, ranging from high-paying jobs such as programming robots to lower-paying janitorial work."

- (b) Please list *four* reasons explaining why Ford's changeover from the Model T to the Model A was so difficult.

Hounshell's Chapter 7 (pp. 280- 292) identifies multiple challenges encountered during the changeover to the Model A in 1927, which required a 6-month complete shutdown of the plant. Some examples:

- (1) Henry Ford wanted to stop using stampings for the car body and use forgings instead. Forging was used only minimally on the previous Model T, so most forging capability had been eliminated from the Rouge plant and they were purchased from outside suppliers. This caused procurement delays and a large cost increase.
- (2) Delays in design due to disagreements between Ford and his engineers.
- (3) Ford's decision to make the gasoline tank integral with the cowl of the Model A, which caused issues in production with seam welding.
- (4) Ford decided to increase the precision of all machining work in order to ensure the highest possible quality for the automobile. Smaller tolerances on most parts required more frequent use of gauging, scaling, and balancing in production and thus more time.

- (c) Please list *three* possible sources of waste in a manufacturing system and describe how the Toyota Production System addressed each one.

Referring to the Monden reading, several sources of waste can be pointed out, such as: (1) excessive workforce, (2) excessive inventory, (3) excessive capital investment. The TPS addresses these as follows:

- a. Excessive workforce can be eliminated by re-allocating worker operations and ensuring a flexible multi-functional workforce.
- b. Excessive inventory is controlled by implementing Just-in-time production, i.e., production regulated by the velocity of sales, which is maintained using the pull system.
- c. Excessive capital investment can be reduced by eliminating excessive inventory, since extra inventory requires investment in extra warehouses and additional transport equipment.