Fall Term 2006

22.02 Introduction to APPLIED NUCLEAR PHYSICS
Problem Set #4

Prof. Molvig

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1. Liboff, problem 5.12

2. The parity operation flips the sign of the spatial coordinates. This can be formalized by defining the *parity operator* (in one dimension), $\widehat{P}$,

$$\widehat{P}f(x) = f(-x)$$

Now consider a free particle in one dimension with Hamiltonian, $\widehat{H} = \widehat{p}^2/2m$. Prove that parity commutes with this Hamiltonian, $\left[\widehat{H}, \widehat{P}\right] = 0$. Now there must be a set of eigenfunctions common to both energy and parity. Starting from the two independent energy eigenfunctions,

$$\begin{aligned} \phi_1(x) &= e^{ikx} \\ \phi_2(x) &= e^{-ikx} \end{aligned}$$

construct two linear combinations of ϕ_1 and ϕ_2,

$$\begin{aligned} \psi_1 &= T_{11}\phi_1 + T_{12}\phi_2 \\ \psi_2 &= T_{21}\phi_1 + T_{22}\phi_2 \end{aligned}$$

that are common eigenfunctions to both energy and parity. State all the eigenvalues for both operators.

3. For a quantum particle moving in a 3D potential, $V(\mathbf{x}) = V(x, y, z)$, state which directions, if any, of the particle's momentum commute with the Hamiltonian if the potential takes the following form:

   (a) $V = ax + by^2 + cz^3$, where the constants, a, b, and c are all positive.

   (b) $V = a\arctan(x/L)$

   (c) $V = b\exp(-y^2/d^2)$

4. For which systems characterized by the potentials listed below, do the energy and parity (in 3D) observables have common eigenfunctions?

   (a) $V = e^2/r$; where $r = \sqrt{x^2 + y^2 + z^2}$.

   (b) $V = ax + by^2 + cz^3$

   (c) $V = axy$