22.02 Fall 2006 Problem Set #9 Solutions

1. Krane 10.5: (a) $4^+ \to 2^+ \Rightarrow$ E2, M3, E4, M5, E6

$E = 0.2995\ \text{MeV} - 0.0914\ \text{MeV} = 0.2081\ \text{MeV}$, $A = 164$

$\lambda(E2) = 7.3\times10^{7} A^{4/3} E^{5} = 2.6\times10^{7}\ s^{-1}$

$\lambda(M3) = 16\ A^{4/3} E^{7} = 2.4\times10^{-1}\ s^{-1}$

$\lambda(E4) = 1.1\times10^{-5} A^{8/3} E^{9} = 6.5\times10^{-6}\ s^{-1}$

$\frac{\lambda(E2)}{\lambda(M3)} = 1.1\times10^{8}$, $\frac{\lambda(E2)}{\lambda(E4)} = 3.9\times10^{12} \Rightarrow$ ^{164}Er will decay by E2 from $4^+ \to 2^+$ predominantly

(b) $4^+ \to 0^+ \Rightarrow$ E4, $\lambda(E4) = 1.1\times10^{-5} A^{8/3} E^{9}$, $E = 0.2995$ MeV $\Rightarrow$

$\lambda(E4) = 1.7\times10^{-4}\ s^{-1}$

$\frac{\lambda(E2)|_{4^+\to2^+}}{\lambda(E4)|_{4^+\to0^+}} \cong 1.5\times10^{11} \Rightarrow$ E2 from $4^+ \to 2^+$ will dominate

Krane 10.6:

(a) $\frac{9}{2}^- \to \frac{7}{2}^+ \Rightarrow 1 \to 8$, Parity: $(-)\cdot(+) = (-)$

Permitted: E1, M2, E3, M4, E5, M6, E7, M8, Most intense: E1

(b) $\frac{1}{2}^- \to \frac{7}{2}^- \Rightarrow 3 \to 4$, Parity: $(-)\cdot(-) = (+)$

Permitted: M3, E4, Most intense: M3

(c) $1^- \to 2^+ \Rightarrow 1 \to 3$, Parity: $(-)(+) = (-)$

Permitted: E1, M2, E3, Most intense: E1

(d) $4^+ \to 2^+ \Rightarrow 2 \to 6$, Parity: $(+)(+) = (+)$

Permitted: E2, M3, E4, M5, E6, Most intense: E2

(e) $\frac{11}{2}^- \to \frac{3}{2}^+ \Rightarrow 4 \to 7$, Parity: $(-)(+) = (-)$

Permitted: M4, E5, M6, E7, Most intense: M4

(f) $3^+ \to 3^+ \Rightarrow 0 \to 6$, Parity: $(+)(+) = (+)$

Permitted: M1, E2, M3, E4, M5, E6, Most intense: M1

2. Krane 9.3:

(a) ${}^{196}_{79}Au_{117} \rightarrow {}^{196}_{80}Hg_{116} + e^- + \bar{\nu}$

$Q_{\beta^-} = [m({}^{196}Au) - m({}^{196}Hg)]c^2 = 0.6865 \text{ MeV}$

(b) ${}^{196}_{79}Au_{117} \rightarrow {}^{196}_{78}Pt_{118} + e^+ + \nu$

$Q_{\beta^+} = [m({}^{196}Au) - m({}^{196}Pt) - 2m_e]c^2 = 0.485 \text{ MeV}$

(c) ${}^{196}_{79}Au_{117} + e^- \rightarrow {}^{196}_{78}Pt_{118} + \nu$

$Q_\varepsilon = [m({}^{196}Au) - m({}^{196}Pt)]c^2 - B_n$

K shell $\Rightarrow B_K = 0.080725 \text{ MeV}$

$Q_\varepsilon = 1.4264 \text{ MeV}$

Krane 9.6:

${}^{47}_{20}Ca_{27} \rightarrow {}^{47}_{21}Sc_{26} + \bar{\nu} + \beta^-$, $T_e = 1.100 \text{ MeV}$

$Q_{\beta^-} = T_e + E_{\bar{\nu}}$

$Q_{\beta^-} = [m({}^{47}Ca) - m({}^{47}Sc)]c^2$

$E_{\bar{\nu}} = Q_{\beta^-} - T_e = [m({}^{47}Ca) - m({}^{47}Sc)]c^2 - T_e$

$m({}^{47}Ca) = 46.954543 \text{ amu}$, $m({}^{47}Sc) = 46.952409 \text{ amu}$

$c^2 = 931.502 \text{ MeV/amu}$

$E_{\bar{\nu}} = [(46.954543 - 46.952409) \text{ amu}]\, 931.502 \text{ MeV/amu} - 1.100 \text{ MeV} = 0.8878 \text{ MeV}$

$E_{\bar{\nu}} = 0.8878 \text{ MeV}$

Krane 9.9:

(a) $\bar{\nu} + {}^{3}He \rightarrow e^{+} + {}^{3}_{1}H$

(b) ${}^{6}_{2}He \rightarrow {}^{6}_{3}Li + e^{-} + \bar{\nu}$

(c) $e^{-} + {}^{8}_{5}B \rightarrow \nu + {}^{8}_{4}Be$

(d) $\nu + {}^{12}_{6}C \rightarrow e^{-} + {}^{12}_{7}N$

(e) ${}^{40}_{19}K \rightarrow \nu + e^{+} + {}^{40}_{18}Ar$

(f) ${}^{40}_{19}K \rightarrow \bar{\nu} + e^{-} + {}^{40}_{20}Ca$

Krane 9.11:

${}^{7}_{4}Be + e^{-} \rightarrow {}^{7}_{3}Li + \nu$

$Q_{\epsilon} = \left[m({}^{7}_{4}Be) - m({}^{7}_{3}Li) \right] c^{2} - B_{n}$

$B_{n} = B_{k\text{-shell}} = 111.5 \text{ eV}$

$Q_{\epsilon} = [7.016928 \text{ amu} - 7.016003 \text{ amu}](931.502 \text{ MeV}/c^{2}) - 1.115 \times 10^{-4} \text{ MeV}$

$Q_{\epsilon} = 0.8615 \text{ MeV}$

$Q_{\epsilon} = T_{Li} + T_{\nu}$

The mass of the neutrino is so much smaller than mass of the Li nucleus that the recoil energy of the nucleus can be assumed to be zero.

$T_{Li} \approx 0 \;,\; T_{\nu} = 0.8615 \text{ MeV}$

3. ${}^{99m}_{43}Tc_{56}$ ⇒ unpaired proton in $1g_{9/2}$ ⇒ $\frac{9}{2}^{+}$

$L=4$ & $L=5$, odd : $(+)(-)=(-)$ ⇒ M4, E5, M4 dominates

$\lambda(M4) = 4.5 \times 10^{-6} A^2 E^9$, $A = 99$, $E = 0.140$ MeV

$\lambda(M4) = 9.1 \times 10^{-10}\ s^{-1}$

$t_{1/2} = \ln 2/\lambda \approx 24$ years (not close to 6 hours, approximate formula)

${}^{98}Mo$ bombarded by a neutron

0^{+} nucleus bombarded by a $\frac{1}{2}^{+}$ neutron ⇒ $\frac{1}{2}^{+}$

${}^{99}Mo$ decays to ${}^{99m}Tc$ (excited state) instead of ${}^{99}Tc$ (ground state) because of smaller change in angular momentum is required ($\frac{1}{2} \to \frac{1}{2}$ instead of $\frac{1}{2} \to \frac{9}{2}$)