

22.02 Fall 2006 Problem Set #9 Solutions

1. Krane 10.5 : (a) $4^+ \rightarrow 2^+ \Rightarrow E2, M3, E4, M5, E6$

$$E = 0.2995 \text{ MeV} - 0.0914 \text{ MeV} = 0.2081 \text{ MeV}, A = 164$$

$$\lambda(E2) = 7.3 \times 10^7 A^{4/3} E^5 = 2.6 \times 10^7 \text{ s}^{-1}$$

$$\lambda(M3) = 16 A^{4/3} E^7 = 2.4 \times 10^7 \text{ s}^{-1}$$

$$\lambda(E4) = 1.1 \times 10^{-5} A^{8/3} E^9 = 6.5 \times 10^{-6} \text{ s}^{-1}$$

$$\frac{\lambda(E2)}{\lambda(M3)} = 1.1 \times 10^8, \quad \frac{\lambda(E2)}{\lambda(E4)} = 3.9 \times 10^{12} \Rightarrow {}^{164}\text{Er} \text{ will decay by } E2 \text{ from } 4^+ \rightarrow 2^+ \text{ predominantly}$$

(b) $4^+ \rightarrow 0^+ \Rightarrow E4$, $\lambda(E4) = 1.1 \times 10^{-5} A^{8/3} E^9$, $E = 0.2995 \text{ MeV} \Rightarrow$

$$\lambda(E4) = 1.7 \times 10^{-4} \text{ s}^{-1}$$

$$\frac{\lambda(E2)|_{4^+ \rightarrow 2^+}}{\lambda(E4)|_{4^+ \rightarrow 0^+}} \approx 1.5 \times 10^{11} \Rightarrow E2 \text{ from } 4^+ \rightarrow 2^+ \text{ will dominate}$$

Krane 10.6 :

(a) $\frac{9}{2}^- \rightarrow \frac{7}{2}^+ \Rightarrow 1 \rightarrow 8$, Parity : $(-)(+) = (-)$

Permitted : E1, M2, E3, M4, E5, M6, E7, M8, Most intense : E1

(b) $\frac{1}{2}^- \rightarrow \frac{7}{2}^- \Rightarrow 3 \rightarrow 4$, Parity : $(-)(-) = (+)$

Permitted : M3, E4, Most intense : M3

(c) $1^- \rightarrow 2^+ \Rightarrow 1 \rightarrow 3$, Parity : $(-)(+) = (-)$

Permitted : E1, M2, E3, Most intense : E1

(d) $4^+ \rightarrow 2^+ \Rightarrow 2 \rightarrow 6$, Parity : $(+)(+) = (+)$

Permitted : E2, M3, E4, M5, E6, Most intense : E2

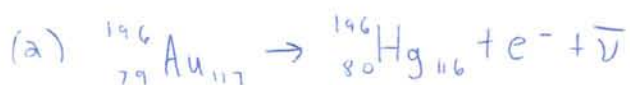
(e) $\frac{11}{2}^- \rightarrow \frac{3}{2}^+ \Rightarrow 4 \rightarrow 7$, Parity : $(-)(+) = (-)$

Permitted : M4, E5, M6, E7, Most intense : M4

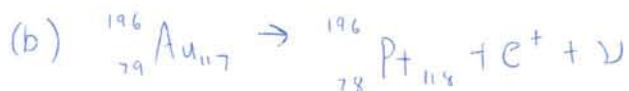
(f) $3^+ \rightarrow 3^+ \Rightarrow 0 \rightarrow 6$, Parity : $(+)(+) = (+)$

Permitted : M1, E2, M3, E4, M5, E6, Most intense : M1

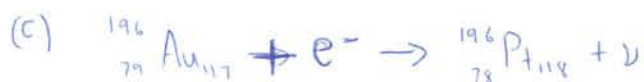
2. Krane 9.3:



$$Q_{\beta^{-}} = [m({}^{196}\text{Au}) - m({}^{196}\text{Hg})]c^2 = 0.6865 \text{ MeV}$$



$$Q_{\beta^{+}} = [m({}^{196}\text{Au}) - m({}^{196}\text{Pt}) - 2m_e]c^2 = 0.485 \text{ MeV}$$

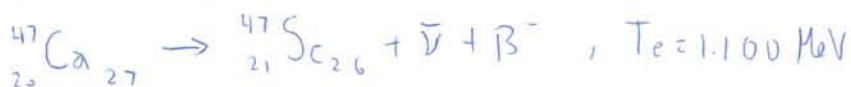


$$Q_E = [m({}^{196}\text{Au}) - m({}^{196}\text{Pt})]c^2 - B_n$$

$$K \text{ shell} \rightarrow B_K = 0.080725 \text{ MeV}$$

$$Q_E = 1.4264 \text{ MeV}$$

Krane 9.6:



$$Q_{\beta^{-}} = T_e + E_{\bar{\nu}}$$

$$Q_{\beta^{-}} = [m({}^{47}\text{Ca}) - m({}^{47}\text{Sc})]c^2$$

$$E_{\bar{\nu}} = Q_{\beta^{-}} - T_e = [m({}^{47}\text{Ca}) - m({}^{47}\text{Sc})]c^2 - T_e$$

$$m({}^{47}\text{Ca}) = 46.954543 \text{ amu}, \quad m({}^{47}\text{Sc}) = 46.952409 \text{ amu}$$

$$c^2 = 931.502 \text{ MeV/amu}$$

$$E_{\bar{\nu}} = [(46.954543 - 46.952409) \text{ amu}] (931.502 \text{ MeV/amu}) - 1.100 \text{ MeV} = 0.8878 \text{ MeV}$$

$$E_{\bar{\nu}} = 0.8878 \text{ MeV}$$

Krane 9.9:

$$(a) \bar{\nu} + {}^3\text{He} \rightarrow e^+ + {}^3_1\text{H}$$

$$(b) {}^6_2\text{He} \rightarrow {}^6_3\text{Li} + e^- + \bar{\nu}$$

$$(c) e^- + {}^8_5\text{B} \rightarrow \nu + {}^8_4\text{Be}$$

$$(d) \nu + {}^{12}_6\text{C} \rightarrow e^- + {}^{12}_7\text{N}$$

$$(e) {}^{40}_{19}\text{K} \rightarrow \nu + e^+ + {}^{40}_{18}\text{Ar}$$

$$(f) {}^{40}_{19}\text{K} \rightarrow \bar{\nu} + e^- + {}^{40}_{20}\text{Ca}$$

Krane 9.11:

$${}^7_4\text{Be} + e^- \rightarrow {}^7_3\text{Li} + \nu$$

$$Q_e = [m({}^7_4\text{Be}) - m({}^7_3\text{Li})]c^2 - B_n$$

$$B_n = B_{K-shell} = 111.5 \text{ eV}$$

$$Q_e = [7.016928 \text{ amu} - 7.016003 \text{ amu}] (931.502 \text{ MeV}/c^2) - 1.115 \times 10^{-4} \text{ MeV}$$

$$Q_e = 0.8615 \text{ MeV}$$

$$Q_e = T_{\text{Li}} + T_{\nu}$$

The mass of the neutrino is so much smaller than mass of the Li nucleus that the recoil energy of the nucleus can be assumed to be zero.

$$T_{\text{Li}} \approx 0, T_{\nu} = 0.8615 \text{ MeV}$$

3. $^{99m}_{43}\text{Tc}_{56} \Rightarrow$ unpaired proton in $1g_{7/2} \Rightarrow \frac{9}{2}^+$

$L=4$ & $L=5$, odd: $(+)(-) = (-) \Rightarrow M4, E5$, M4 dominates

$$\lambda(M4) = 4.5 \times 10^{-6} A^2 E^9, A=99, E=0.140 \text{ MeV}$$

$$\lambda(M4) = 9.1 \times 10^{-10} \text{ s}^{-1}$$

$$t_{1/2} = \ln 2 / \lambda \approx 24 \text{ years (not close to 6 hours, approximate formula)}$$

^{98}Mo bombarded by a neutron

0^+ nucleus bombarded by a $\frac{1}{2}^+$ neutron $\Rightarrow \frac{1}{2}^+$

^{99}Mo decays to ^{99m}Tc (excited state) instead of ^{99}Tc (ground state) because of smaller change in angular momentum is required ($\frac{1}{2} \rightarrow \frac{1}{2}$ instead of $\frac{1}{2} \rightarrow \frac{9}{2}$)