6.037 Lecture 7B

Scheme Variants
Normal Order
Lazy Evaluation
Streams

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Further Variations on a Scheme

Beyond Scheme – more language variants

Lazy evaluation
- Complete conversion – normal order evaluator
- Selective Laziness: Streams

Punchline: Small edits to the interpreter give us a *new programming language*

Environment model

Rules of evaluation:
- If expression is *self-evaluating* (e.g. a number), just return value
- If expression is a *name*, look up value associated with that name in environment
- If expression is a *lambda*, create procedure and return
- If expression is *special form* (e.g. if) follow specific rules for evaluating subexpressions
- If expression is a *compound expression*
  - Evaluate subexpressions in any order
  - If first subexpression is primitive (or built-in) procedure, just apply it to values of other subexpressions
  - If first subexpression is compound procedure (created by lambda), evaluate the body of the procedure in a new environment, which extends the environment of the procedure with a new frame in which the procedure's parameters are bound to the supplied arguments

Alternative models for computation

- Applicative Order (aka Eager evaluation):
  - evaluate all arguments, then apply operator
- Normal Order (aka Lazy evaluation:
  - go ahead and apply operator with unevaluated argument subexpressions
  - evaluate a subexpression only when value is *needed*
    - to print
    - by primitive procedure (that is, primitive procedures are "*strict*" in their arguments)
    - to test (if predicate)
    - to apply (operator)

Making Order of Evaluation Visible

- (define (notice x)
      (display "noticed")
    x)
- (+ (notice 52) (notice (+ 4 4))
  noticed
  noticed
  => 60

Applicative Order Example

```
(define (foo x)
  (display "inside foo")
  (+ x x))

(foo (notice 222))
    => (notice 222)
        => 222
=> (begin (display "inside foo")
           (+ 222 222))

noticed
 inside foo
 => 444
```

We first evaluated argument, then substituted value into the body of the procedure

Normal Order Example

```
(define (foo x)
  (display "inside foo")
  (+ x x))

(foo (notice 222))

 => (begin (display "inside foo")
           (+ (notice 222)
              (notice 222)))
```

From body of foo

As if we substituted the *unevaluated expression* in the body of the procedure

```
inside foo
noticed
noticed
=> 444
```

Applicative Order vs. Normal Order

```
(define (foo x)
  (display "inside foo")
  (+ x x))

(foo (notice 222))
```

Applicative order

```
noticed
inside foo
```

Think of as substituting values for variables in expressions

Normal order

```
inside foo
noticed
noticed
```

Think of as expanding expressions until only involve primitive operations and data structures

Normal order (lazy evaluation) versus applicative order

- How can we change our evaluator to use normal order?
  - Create "promises" – expressions whose evaluation has been delayed
  - Change the evaluator to force evaluation only when needed
- Why is normal order useful?
  - What kinds of computations does it make easier?

m-apply – the original version

```
(define (m-apply procedure arguments)
  (cond ((primitive-procedure? procedure)
         (apply-primitive-procedure
           procedure
           arguments))
        ((compound-procedure? procedure)
         (eval-sequence
          (procedure-body procedure)
          (extend-environment
              (procedure-parameters procedure)
              arguments
              (procedure-environment procedure))))
        (else (error "Unknown procedure" procedure))))
```

Actual values

Actual values

How can we implement lazy evaluation?

```
(define (l-apply procedure arguments env)   ; changed
  (cond ((primitive-procedure? procedure)
         (apply-primitive-procedure
          procedure
          (list-of-arg-values arguments env)))
        ((compound-procedure? procedure)
         (l-eval-sequence
          (procedure-body procedure)
          (extend-environment
           (procedure-parameters procedure)
           (list-of-delayed-args arguments env)
           (procedure-environment procedure))))
        (else (error "Unknown proc" procedure))))
```

Delayed expressions

Need to convert to actual values

Delayed Expressions

Need to create delayed version of arguments that will lead to values

Lazy Evaluation – `l-eval`

- Most of the work is in **`l-apply`**; need to call it with:
  - actual value for the operator
  - just expressions for the operands
  - the environment...

```
(define (l-eval exp env)
  (cond ((self-evaluating? exp) exp)
        ...
        ((application? exp
         (l-apply (actual-value (operator exp) env)
                  (operands exp)
                  env))
        (else (error "Unknown expression" exp))))
```

m-eval versus l-Eval

```
(define (m-eval exp env)
  (cond ((self-evaluating? exp) exp)
        …
        ((cond? exp) (m-eval (cond->if exp) env))
        ((application? exp)
         (m-apply (m-eval (operator exp) env)
                  (list-of-values (operands exp) env)))
        (else (error "Unknown expression type -- EVAL" exp))))
```

```
(define (l-eval exp env)
  (cond ((self-evaluating? exp) exp)
        ...
        ((cond? exp)
        ((application? exp
         (l-apply (actual-value (operator exp) env)
                  (operands exp) env))
        (else (error "Unknown expression" exp))))
```

Actual vs. Delayed Values

```
(define (actual-value exp env)
  (force-it (l-eval exp env)))
```

```
(define (list-of-arg-values exps env)
  (if (no-operands? exps) '()
      (cons (actual-value (first-operand exps) env)
            (list-of-arg-values (rest-operands exps)
                                env))))
```

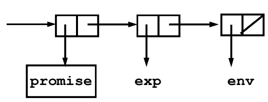
Used when applying a primitive procedure

```
(define (list-of-delayed-args exps env)
  (if (no-operands? exps)
      '()
      (cons (delay-it (first-operand exps) env)
            (list-of-delayed-args (rest-operands exps)
                                  env))))
```

Used when applying a compound procedure

Representing Promises

- *Abstractly* –a "promise" to return a value when later needed ("forced")
  - *Concretely* – our representation:



  - Book calls it a *thunk*, which means procedure with no arguments.
  - Structure looks very similar.

Promises – delay-it and force-it

```
(define (delay-it exp env) (list 'promise exp env))
(define (promise? obj) (tagged-list? obj 'promise))
(define (promise-exp promise) (cadr promise))
(define (promise-env promise) (caddr promise))

(define (force-it obj)
  (cond ((promise? obj)
         (actual-value (promise-exp obj)
                       (promise-env obj)))
        (else obj)))

(define (actual-value exp env)
  (force-it (l-eval exp env)))
```

Lazy Evaluation – other changes needed

- Example: Need actual predicate value in conditional if...

```
(define (l-eval-if exp env)
  (if (true? (actual-value (if-predicate exp) env))
      (l-eval (if-consequent exp) env)
      (l-eval (if-alternative exp) env)))
```

- Example: Don't need actual value in assignment...

```
(define (l-eval-assignment exp env)
  (set-variable-value!
     (assignment-variable exp)
     (l-eval (assignment-value exp) env)
     env)
  'ok)
```

Examples

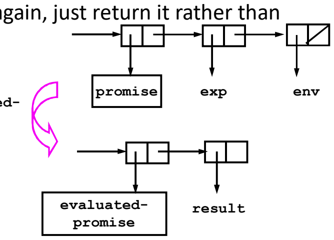
- (define identity (lambda (x) x)) identity: <proc>
- (define a (notice 3)) a: promise 3 Noticed!
- (define b (identity (notice 3))) b: promise (notice 3)
- (define c b) c:
- (define d (+ b c)) d: 6 Noticed! Noticed!
- (define plus (identity +)) plus: promise +
- (plus a b) => 6 Noticed!
- c => 3 Noticed!

Memo-izing evaluation

- In lazy evaluation, if we reuse an argument, have to reevaluate each time
- In usual (applicative) evaluation, argument is evaluated once, and just referenced
- Can we keep track of values once we've obtained them, and avoid cost of re-evaluation?

Memo-izing Promises

- *Idea*: once promise **`exp`** has been evaluated, remember it
- If value is needed again, just return it rather than recompute
  - *Concretely* – mutate a `promise` into an `evaluated-promise`



Why mutuate? – because other names or data structures may point to this promise!

Promises – Memoizing Implementation

```
(define (evaluated-promise? obj)
  (tagged-list? obj 'evaluated-promise))
(define (promise-value evaluated-promise)
  (cadr evaluated-promise))

(define (force-it obj)
  (cond ((promise? obj)
         (let ((result (actual-value (promise-exp obj)
                                     (promise-env obj))))
           (set-car! obj 'evaluated-promise)
           (set-car! (cdr obj) result)
           (set-cdr! (cdr obj) '())
           result))
        ((evaluated-promise? obj)  (promise-value obj))
        (else obj)))
```

Examples - Memoized

- (define identity (lambda (x) x))   identity: <proc>
- (define a (notice 3))   a: promise 3   Noticed!
- (define b (identity (notice 3))) b: promise (notice 3)
- (define c b)   c:
- (define d (+ b c)) d: 6   Noticed! *CHANGE*
- (define plus (identity +)) plus: promise +
- (plus a b) => 6   *CHANGE*
- c => 3 *CHANGE*

Summary of lazy evaluation

- This completes changes to evaluator
  - Apply takes a set of expressions for arguments and an environment
    - Forces evaluation of arguments for primitive procedure application
    - Else defers evaluation and unwinds computation further
    - Need to pass in environment since don't know when it will be needed
  - Need to force evaluation on branching operations (e.g. if)
  - Otherwise small number of changes make big change in behavior of language

Laziness and Language Design

- We have a dilemma with lazy evaluation
  - Advantage: only do work when value actually needed
  - Disadvantages
    - not sure when expression will be evaluated; can be very big issue in a language with side effects
    - may evaluate same expression more than once
- Memoization doesn't fully resolve our dilemma
  - Advantage: Evaluate expression at most once
  - Disadvantage: What if we *want* evaluation on each use?
- Alternative approach: **Selective Laziness**

Choose via Parameter Declarations

- Handle lazy and lazy-memo extensions in an upward-compatible fashion.

```
(lambda (a (b lazy) c (d lazy-memo)) ...)
```

- "a", "c" are usual variables (evaluated before procedure application)
- "b" is lazy; it gets (re)-evaluated each time its value is actually needed
- "d" is lazy-memo; it gets evaluated the first time its value is needed, and then that value is returned again any other time it is needed

Streams – the lazy way

Beyond Scheme – designing language variants:

- Streams – an alternative programming style

to infinity, and beyond…

Decoupling computation from description

- Can separate order of events in computer from apparent order of events in procedure description

```
(list-ref
  (filter (lambda (x) (prime? x))
          (enumerate-interval 1 1000000))
  100)
```

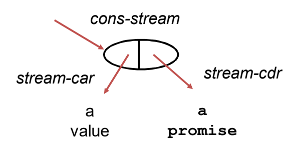
Creates 100K elements

Creates 1M elements

Generate only what you actually need…

Stream Object

- A pair-like object, except the cdr part is *lazy* (not evaluated until needed):



- Example

```
(define x (cons-stream 99 (/ 1 0)))
(stream-car x) => 99
(stream-cdr x) => error – divide by zero
```

Stream-cdr forces the promise wrapped around (/ 1 0), resulting in an error

Implementing Streams

- Stream is a data structure with the following contract:
  - (cons-stream *a b*) – cons together *a* with promise to compute *b*
  - (stream-car *s*) – Returns car of *s*
  - (stream-cdr *s*) – Forces and returns value of cdr of *s*
- Implement in regular evaluator with a little syntactic sugar
  - (define (cons-stream->cons exp)
    `(cons ,(second exp) (lambda () ,(third exp))))
  - In m-eval, add to cond:
    ((cons-stream? exp) (m-eval (cons-stream->cons exp) env))
  - And the following regular definitions (inside m-eval!)
    - (define stream-car car)
    - (define (stream-cdr s) ((cdr s)))
- Streams can be done in lazy eval
  - (define (cons-stream a b) (cons a b)) ← doesn't work! (Why?)
    (define (cons-stream a b) (cons a (lambda () b)))

Ints-starting-with

- (define (ints-starting-with i)
  (cons-stream i (ints-starting-with (+ i 1))))

  Delayed!
- Recursive procedure with no base case!
  - Why does it work?

Stream-ref

```
(define (stream-ref s i)
   (if (= i 0)
       (stream-car s)
       (stream-ref (stream-cdr s) (- i 1))))
```

- Like list-ref, but cdr's down stream, forcing

Stream-filter

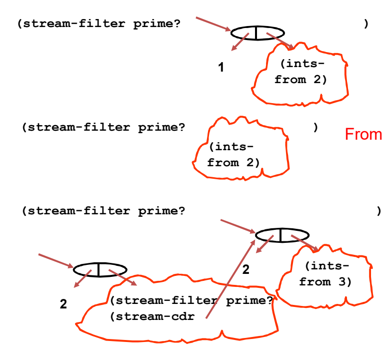
```
(define (stream-filter pred str)
    (if (pred (stream-car str))
        (cons-stream (stream-car str)
                     (stream-filter pred
                          (stream-cdr str)))
        (stream-filter pred
                     (stream-cdr str))))
```

Decoupling Order of Evaluation

```
(define (stream-filter pred str)
    (if (pred (stream-car str))
        (cons-stream (stream-car str)
                     (stream-filter pred
                          (stream-cdr str)))
        (stream-filter pred
                     (stream-cdr str))))
(stream-ref
   (stream-filter (lambda (x) (prime? x))
                  (ints-starting-with 2))
   4)
```

Decoupling Order of Evaluation

```
(stream-filter prime? (ints-from 1))
```

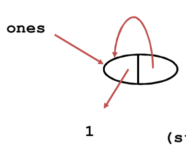


One Possibility: Infinite Data Structures!

- Some very interesting behavior

```
(define ones (cons 1 ones))

(define ones (cons-stream 1 ones))
(stream-car (stream-cdr ones)) => 1
```



The infinite stream of 1's!

```
ones: 1 1 1 1 1 1 ....
(stream-ref ones 1) → 1
(stream-ref ones 1000) → 1
(stream-ref ones 10000000) → 1
```

Finite list procs turn into infinite stream procs

```
(define (add-streams s1 s2)
  (cons-stream
    (+ (stream-car s1) (stream-car s2))
    (add-streams (stream-cdr s1)
                 (stream-cdr s2)))))
(define ints
  (cons-stream 1 (add-streams ones ints)))
```

```
ones: 1 1 1 1 1 1 ....
ints: 1 2 3 ...
```

add-streams ones ints

add-streams (str-cdr ones) (str-cdr ints)

Finding all the primes

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	60
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Using a sieve

```
(define (sieve str)
  (cons-stream
    (stream-car str)
    (sieve (stream-filter
             (lambda (x)
               (not (divisible? x (stream-car str))))
             (stream-cdr str)))))

(define primes
    (sieve (stream-cdr ints)))
```

(2 sieve (filter ints 2))

(2 3 sieve (filter
sieve (filter ints 2)
3))

Interleave

Produce a stream that has all the elements of two input streams:

(define (interleave s1 s2)
(cons-stream (stream-car s1)
(interleave s2 (stream-cdr s1))))

Rationals

1/1	1/2	1/3	1/4	1/5	…
2/1	2/2	2/3	2/4	2/5	…
3/1	3/2	3/3	3/4	3/5	…
4/1	4/2	4/3	4/4	4/5	…
5/1	5/2	5/3	5/4	5/5	…
…	…	…	…	…	…

```
(define (div-by-stream s n)
  (cons-stream (/ n (stream-car s))
               (div-by-stream (stream-cdr s) n))))

(define (make-rats n)
  (cons-stream n
     (interleave (div-by-streams (stream-cdr ints) n)
                 (make-rats (+ n 1)))))

(define rats (make-rats 1))
```

Power Series

- Approximate function by summation of infinite polynomial
- Great application for streams!

<We'll do this in recitation!>