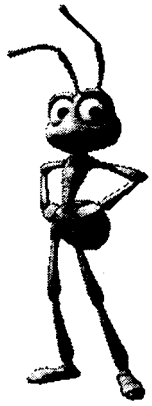
6.002 Circuits and Systems

Final Review



Jason Kim

6.002 Circuits and Systems: Outline of Topics

- Resistor Networks
  concepts: Node Method, KCL, KVL, Superposition, Thevenin and Norton equivalent circuit models, Multiport Networks, Transformers, Power.

- 1st Order Circuits
  concepts: Consituitive Laws, RC netwroks, RL networks, Homogenous and Particular solution, Time constant τ, response to an impulse, power, Low/High/Band/Notch pass filters, sinusodial steady-state, transients, Impedance Model, Bode Plots(Magnitude and Phase).

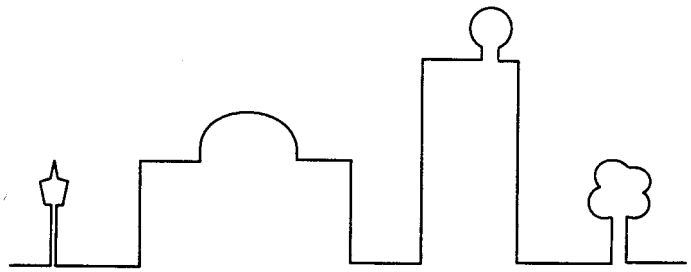
- 2nd Order Circuits
  concepts: LC networks, LRC networks, damping coefficient α, natural frequency ω_o, Underdamped/Critical/Overdamped Systems, Resonance, Q factor, Half-power point, Sinusodial steady-state, transients, Power(real/reactive), Impedance Model, ELI ICE, Bode Plots(Magnitude and Phase).

- Digital Abstraction
  concepts: Boolean Logic(truth table, formula, gates, and transistor level), primitive laws, DeMorgan's Law(formula and gate equivalent), MSP(minumum sum of products), MPS(minimum product of sums).

- MOSFET Transistors
  concepts: Large Signal Model(S, SR, SCS, SVR), Small Signal Model, Saturation/Linear Region, R_{on}, Loadline, Operating Point, Input/Output Resistence, Current Gain, Power Gain, Noise Margin.

- Op-Amp Circuits
  concepts: Single/Multiple input op-amp circuits, differentiator, integrator, adder, subtractor, op-amp with R,L,C, negative feedback, positive feedback, Bode Plots(Magnitude and Phase), Input/Output resistence, Schmitt Trigger, Hysteresis, Cascaded stages.

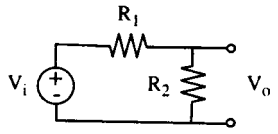
- Diodes
  concepts: *i-v* characteristics, model, diode w/ R, L, C, and op-amp, Peak detection, Clipper circuits, Incremental Analysis.



1. Primitive Elements

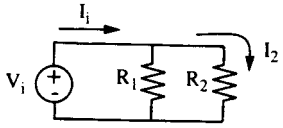
Resistor: time domain: $V = IR$ freq. domain: $Z = R$

Series: $R_1 + R_2$ Parallel: $R_1 \| R_2 = \dfrac{R_1 R_2}{R_1 + R_2}$



$V_o = \dfrac{R_2}{R_1 + R_2} V_i$

Voltage Divider

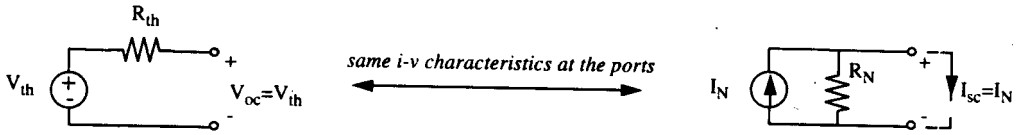


$I_2 = \dfrac{R_1}{R_1 + R_2} I_i$

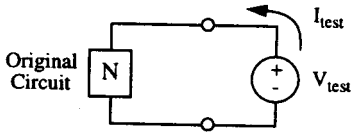
Current Divider

- Resistor Network
  - N-unknowns & N-equations (most primitive approach)
  - Node Analysis (KCL, KVL)
    - *KCL: Sum of all currents entering and leaving a node is zero.
    - *KVL: Sum of all voltages around a closed loop path is zero. (be careful with polarities)
    - 1) LABEL all current directions and voltage polarities. (Remember, currents flow from + to -)
    - 2) write KCL & KVL Equation.
    - 3) substitute and solve.
  - Simplification (by inspection)

- Thevenin Equivalent Circuit Model — Norton Equivalent Circuit Model



  - * Three variables: $V_{th}=V_{oc}$, $R_{th}=R_N$, and $I_N=I_{sc}$. They are related by $V_{oc}=I_{sc}*R_{th}$.
  - * $V_{th}=V_{oc}$: Leave the port open(thus no current flow at the port) and solve for V_{oc}. For a resistive network, this gives you a point on the V-axis of the i-v plot.
  - * $I_N=I_{sc}$: Short the port(thus no voltage across the port) and solve for I_{sc} flowing out of + and into -. For a resistive network, this gives you a point on the I-axis of the i-v plot.
  - * R_{th}: Set all sources to zero, except dependent sources. Solve the resistive network. When setting sources to zero, V source becomes short and I source becomes open. R_{th} may also be found by attaching I_{test} and V_{test} and setting $R_{th} = (V_{test})/(I_{test})$. If dependent sources are present, set only the independent sources to zero and attach I_{test} and V_{test}. Use KCL and KVL to find the expression $(V_{test})/(I_{test})=R_{th}$.



note the direction of I_{test} and polarity of V_{test}

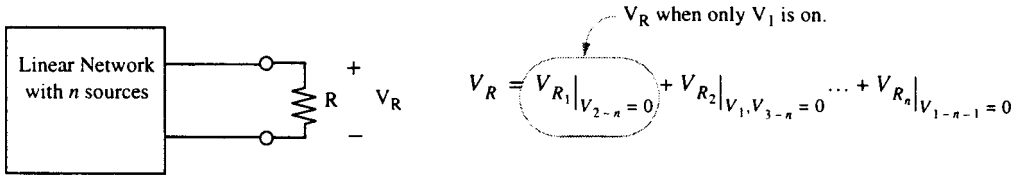
$R_{th} = \dfrac{V_{test}}{I_{test}}$

  - * For both Thevenin & Norton E.C.M.'s, they have the same power consumption *at the port* as the original circuit, but *not* for its individual components. Power dissipated at R_{th} does *not* equal power dissipated at the resistors of the original circuit. Same holds for the power delivered by the sources in the Thevenin model and the original circuit. Thevenin and Norton E.C.M.'s are for terminal i-v characteristics *only*.

- Power = Real Power = $IV = I^2R = V^2/R$ (energy dissipated as heat)
- $\langle Power \rangle = \dfrac{1}{T}\int_0^T I(t)V(t)dt$

- Superposition

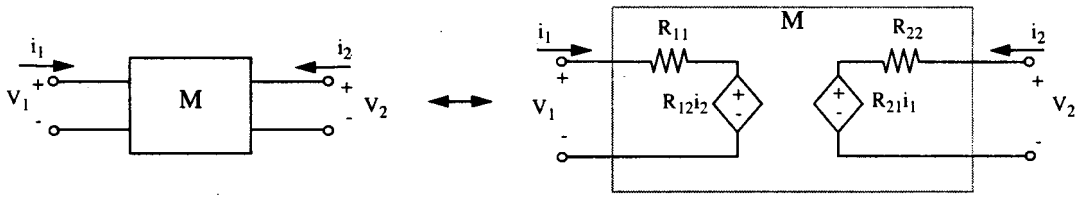
"In a linear network with a number of independent sources, the response can be found by summing the response to each independent sources acting alone, with all other independent sources set to zero."



1) Leave one source on and turn off all other sources.
   (Voltage source "off" = short & Current source "off" = open)
2) Find the effect from the "on" source.
3) Repeat for each sources.
4) Sum the effect from each sources to obtain the total effect.

For cases where a linear dependent source is present along with multiple independent sources, DO NOT turn off the dependent source. Leave the dependent source on and carry it in your expressions. Tackle the dependent source term last by solving linear equations. Remember that the variable which the dependent source is depended on is affected by the individual independent sources that you are turning on and off.

- Multiport Network



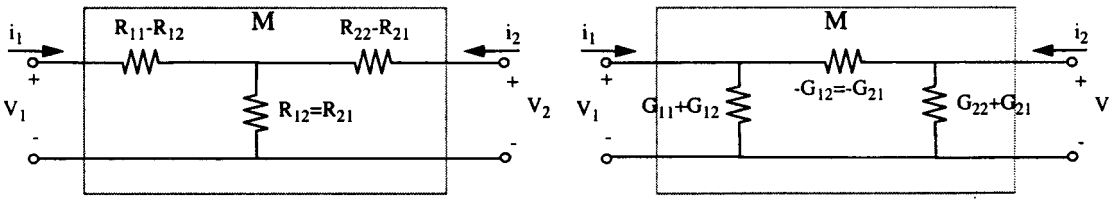
z-parameter model

* By attaching V_1, I_1 and V_2,I_2 at the ports,

$$V_1 = R_{11}I_1 + R_{12}I_2$$
$$V_2 = R_{21}I_1 + R_{22}I_2$$

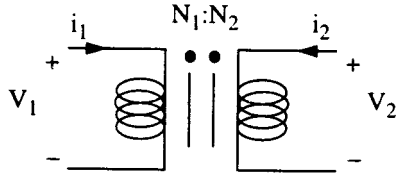
The first subscript denote the place(port) of voltage measurement.
The second subscript denote the place(port) of current source.
Thus, R_{12} means the resistance V_1/I_2 when voltage V_1 is measured at open port 1 and I_2 current source is placed at port 2.

* If all the Z-parameters(R_{xx}'s) are identical, then the networks can't be differentiated using only the *i-v* characteristics at the ports.
* Reciprocity: $R_{12}=R_{21}$.
* Using the definitions from above,



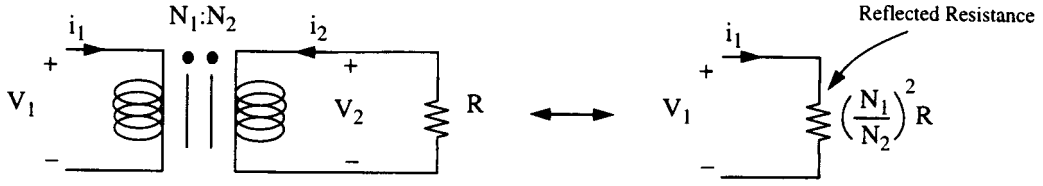
T model *π model*

- Transformer



$\frac{V_1}{N_1} = \frac{V_2}{N_2}$ & $N_1 i_1 = -N_2 i_2$. Turns Ratio $= \frac{N_1}{N_2}$

Reflected Resistance: the equiv. resistance seen at port 1, of the resistance on the other side, port2.



Transformer vs. Voltage Divider : Voltage divider uses a series of resistors to divide and obtain a scaled-down voltage. In doing so, power is dissipated not only at the load but also in the series resistors as well, resulting in unwanted power dissipation. On the other hand, a transformer converts its voltage through magnetic coupling and power is dissipated only in the load. Thus, no unwanted power dissipation. (assuming ideal transformers)

Capacitor: time domain: $I = C\frac{dV}{dt}$ freq. domain: $Z = \frac{1}{Cs} = \frac{1}{jwC}$

High Frequency = Short & Low Frequency = Open

Series: $\frac{C_1 C_2}{C_1 + C_2}$ Parallel: $C_1 + C_2$

$V_c(0^-)=V_c(0^+)$ except when the input source is an impulse.

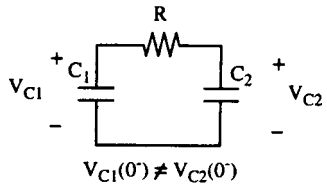
$V_c(t)$ is continuous while $I_c(t)$ may be discontinuous.

"I C E" : Current(I) *LEADS* Voltage(EMF) by 90°.

Energy Conservation: $\int I dt = Q = CV$.

Energy Stored: $E = \frac{1}{2}CV^2$. (no energy dissipation)

Note: Two identical capacitors, C_1 and C_2, in series may act like an open circuit after a long time where the total voltage across the capacitors are split between the two cap's. For this case, V_c does not discharge to zero but $V_{C1}=V_{C2}$.



Inductor: time domain: $V = L\frac{dI}{dt}$ freq. domain: $Z = Ls = jwL$

High Frequency = Open & Low Frequency = Short

Series: $L_1 + L_2$ Parallel: $\frac{L_1 L_2}{L_1 + L_2}$

$I_L(0^-)=I_L(0^+)$ except when the input source is an impulse.

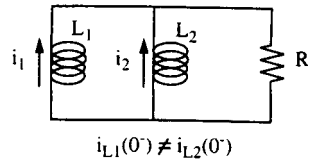
$I_L(t)$ is continuous while $V_L(t)$ may be discontinuous.

"E L I" : Voltage(EMF) *LEADS* Current(I) by 90°.

Energy Conservation: $\int V dt = \lambda = LI$.

Energy Stored: $E = \frac{1}{2}LI^2$. (no energy dissipation)

Note: Two identical inductors, L_1 and L_2, in parallel may act like a short circuit after a long time and have current flow through this loop indefinitely. For this case, i_L is not zero but $i_{L1}=-i_{L2}$.



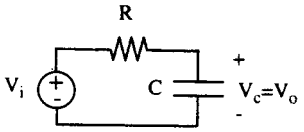
LC

Series@ ω_o = Short & Parallel@ ω_o = Open

Energy transfers between inductors and capacitors sinusodially (180° out of phase). Also, V_c peaks when I_L=0 and I_L peaks when V_c=0.

2. First Order Circuits

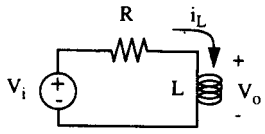
RC Network



$$\frac{dV_c}{dt} + \frac{V_c}{RC} = \frac{V_i}{RC}$$

Time Constant: $\tau = RC$

RL Network

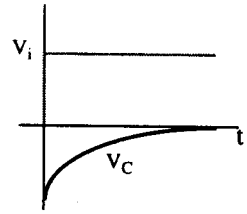


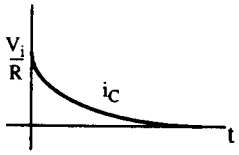
$$\frac{di_L}{dt} + \frac{i_L}{\left(\frac{L}{R}\right)} = \frac{V_i}{L}$$

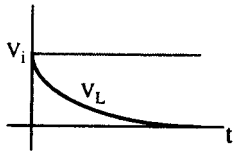
Time Constant: $\tau = \frac{L}{R}$

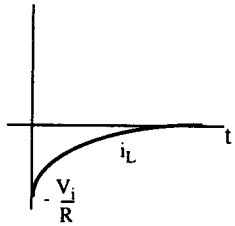
*one time constant τ charges/discharges 63% of the maximum value. Small time constant shows the effect of the exponential curve sooner, while big time constant shows a linear line for a long time before the exponential effect becomes visible.

Homogeneous:

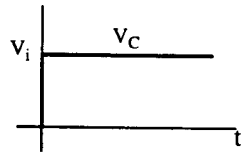


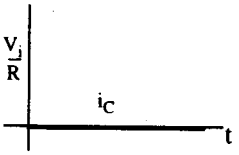


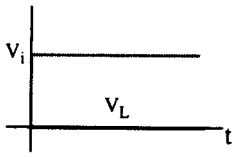


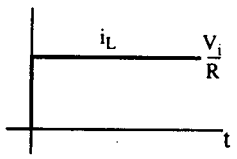


Particular:









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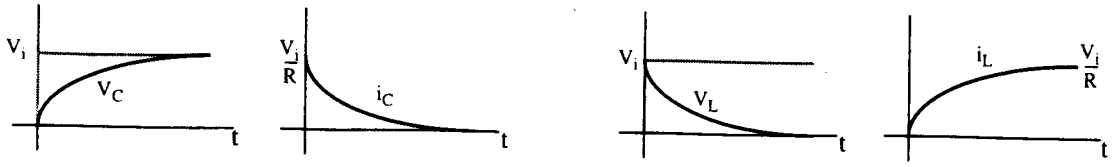


* Notice the *Duality* between V_c and I_L & I_c and V_L.

Approach to First Order Circuits

1) Set up *differential equations*, using KCL and KVL. Use the constitutive laws for C and L.

$$\frac{dV_c}{dt} + \frac{V_c}{RC} = \frac{V_i}{RC}$$

2) Find Homogeneous solution by setting input to zero and substituting $V_h=Ae^{st}$ as a solution.

$$Ase^{st} + \frac{1}{RC}Ae^{st} = 0 \qquad \therefore s = -\frac{1}{RC}$$

3) Find Particular solution. Remember the output follows the form of the input.

$$V_p = V_i \;\; ;t>0$$

4) Combine Homogeneous and Particular solutions, and use *initial conditions* to find missing variables. (Be careful when input is an impulse!)

$$V_o = V_h + V_p = Ae^{\frac{-t}{RC}} + V_i$$

$$V_o = A + V_i = 0 \;\text{ at } t=0^+. \;\; \therefore A = -V_i$$

$$V_o = V_i\left(1 - e^{\frac{-t}{RC}}\right) \qquad \text{at } t=0^+,\ V_o=0 \text{ and } t=\infty,\ V_o=V_i.$$

*After a long time, transients(homogeneous solution) die away, leaving only the particular solution. Thus, the output may look different from the input in the beginning but slowly catches up to follow the input.

Impulse as Input

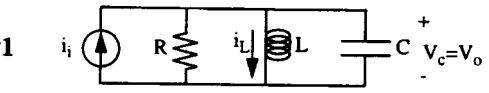
ex. RC circuit: $V_i = Q\delta t$ and $V_o(0^{minus}) = 0$ and $\frac{dV_o}{dt} + \frac{V_o}{RC} = \frac{V_i}{RC}$.

$$dV_o + \frac{1}{RC}V_o dt = \frac{Q}{RC}\delta t dt \rightarrow \int_{0^{minus}}^{0^{plus}} dV_o + \frac{1}{RC}\int_{0^{minus}}^{0^{plus}} V_o dt = \frac{Q}{RC}\int_{0^{minus}}^{0^{plus}} \delta t dt \rightarrow V_o(0^{plus}) - V_o(0^{minus}) = \frac{Q}{RC}$$

$$V_o(0^{plus}) - 0 = \frac{Q}{RC} \rightarrow \;\; \therefore V_o(0^{plus}) = \frac{Q}{RC}$$

3. Second Order Circuits

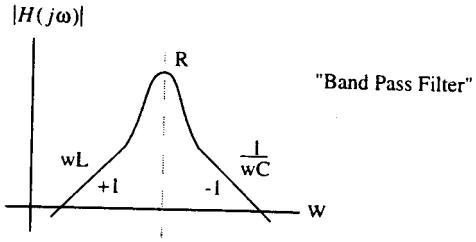
LRC Network

#1 

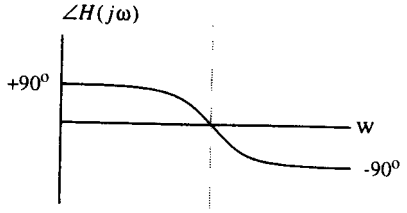
$$\frac{\partial^2 V_o}{\partial t^2} + \frac{1}{RC}\frac{\partial V_o}{\partial t} + \frac{V_o}{LC} = \frac{1}{C}\frac{\partial i_i}{\partial t}$$

$$\frac{V_o}{i_i} = \frac{j\omega LR}{(j\omega)^2 LCR + j\omega L + R}$$

Time Constant: $\tau = \frac{1}{\sqrt{LC}}$

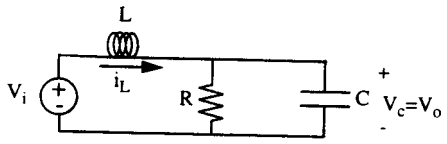


$$\omega_o = \frac{1}{\sqrt{LC}}$$



$$\omega_o = \frac{1}{\sqrt{LC}}$$

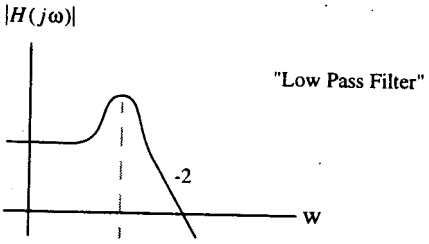
#2



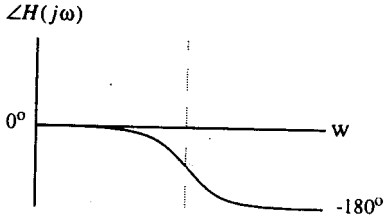
$$\frac{\partial^2 V_o}{\partial t^2} + \frac{1}{RC}\frac{\partial V_o}{\partial t} + \frac{V_o}{LC} = \frac{V_i}{LC}$$

$$\frac{V_o}{V_i} = \frac{R}{(j\omega)^2 LRC + j\omega L + R}$$

Time Constant: $\tau = \frac{1}{\sqrt{LC}}$

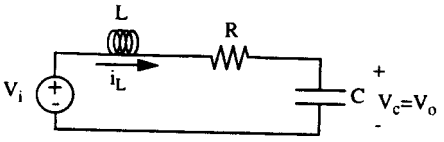


$$\omega_o = \frac{1}{\sqrt{LC}}$$



$$\omega_o = \frac{1}{\sqrt{LC}}$$

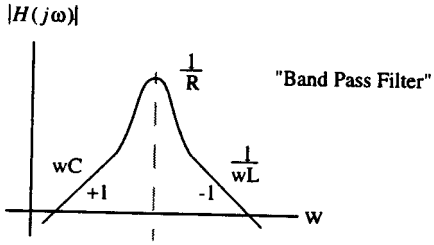
#3



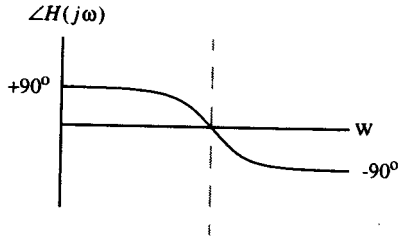
$$\frac{\partial^2 i_L}{\partial t^2} + \frac{R}{L}\frac{\partial i_L}{\partial t} + \frac{i_L}{LC} = \frac{1}{L}\frac{dV_i}{dt}$$

$$\frac{i_L}{V_i} = \frac{j\omega C}{(j\omega)^2 LC + j\omega RC + 1}$$

Time Constant: $\tau = \frac{1}{\sqrt{LC}}$

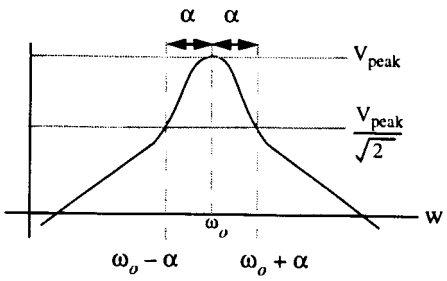


$$\omega_o = \frac{1}{\sqrt{LC}}$$



$$\omega_o = \frac{1}{\sqrt{LC}}$$

- "peakiness" $Q = \frac{\omega_o}{2\alpha}$ & Half-Power Point = $\omega_o \pm \alpha$



$$Power = \left(\frac{V_{peak}}{\sqrt{2}}\right)^2 \frac{1}{R} = \frac{V_{peak}^2}{2R} = \frac{Power_{peak}}{2}$$

Hence, Half-power Point.

Characteristic Equation (Homogeneous Solution)

Obtain a characteristic equation by setting input to zero and replacing $\frac{d}{dt}$ with s to get into the form:

$$s^2 + 2\alpha s + \omega_o^2 = 0$$

where ω_o is the "resonant" or "natural" frequency and α is the damping coefficient.

solving s with quadratic formula,

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -\alpha \pm j\sqrt{\omega_o^2 - \alpha^2} = -\alpha \pm j\omega_d \quad ; \ \omega_d \text{ is the damped natural frequency}$$

1) "Overdamped"	$\alpha > \omega_o$	$s_1 \neq s_2$	$s_{1,2}$ are real
2) "Critically Damped"	$\alpha = \omega_o$	$s_1 = s_2$	$s_{1,2}$ are real
3) "Underdamped"	$\alpha < \omega_o$	$s_1 \neq s_2$	$s_{1,2}$ are complex

Approach (Transients)

1) Set up 2^{nd} *order differential equations.*

$$\frac{\partial^2 i_L}{\partial t^2} + \frac{R}{L}\frac{\partial i_L}{\partial t} + \frac{i_L}{LC} = \frac{1}{L}\frac{dV_i}{dt} \text{ (ex. circuit \#3)}$$

2) Obtain characteristic equation and identify ω_o and α.

$$s^2 + 2\left(\frac{R}{2L}\right)s + \left(\frac{1}{\sqrt{LC}}\right)^2 = 0; \quad \therefore \omega_o = \frac{1}{\sqrt{LC}} \text{ and } \therefore \alpha = \frac{R}{2L}$$

3) Find Homogeneous solution.(for the appropriate system type)

Overdamped: $V_h = Ae^{-(\alpha - \sqrt{\alpha^2 - \omega_o^2})t} + Be^{-(\alpha + \sqrt{\alpha^2 - \omega_o^2})t}$

Critically Damped: $V_h = Ae^{-\alpha t} + Bte^{-\alpha t}$

Underdamped: $V_h = Ae^{-\alpha t}\cos\omega_d t + Be^{-\alpha t}\sin\omega_d t$

4) Find Particular solution. Remember the output follows the form of the input.

$V_p = V_i$ (usually a step)

5) Combine Homogeneous and Particular solutions, and use initial conditions($v(0)$, $i(0)$) to find missing variables. (Be careful when input is an impulse!)

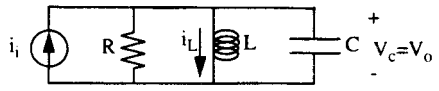
$V_o = V_p + V_h = V_i + Ae^{-\alpha t}\cos\omega_d t + Be^{-\alpha t}\sin\omega_d t$ $\qquad$ $i_L = Ce^{-\alpha t}(-A\omega_d \sin\omega_d t + B\omega_d \cos\omega_d t)$

$V_o(0) = V_i + A = 0 \quad \therefore A = -V_i$ $\qquad$ $i_L(0) = CB\omega_d = 0 \quad \therefore B = 0$

Thus, $V_o = V_i(1 - e^{-\alpha t}\cos\omega_d t)$ & $i_L = V_i C\omega_d e^{-\alpha t}\sin\omega_d t$

Approach (Sinusodial Steady-State)

1) Replace the primitive elements with their impedance models and solve the network like a resistor network.



$i_i = I\cos(wt)$ $\qquad V_o = V\cos(\omega t + \phi)$

$$\tilde{V}_o = \frac{1}{\left(\frac{1}{sL}\right) + sC + \frac{1}{R}}\tilde{i}_i = \frac{j\omega L}{1 - \omega^2 LC + j\omega\frac{L}{R}}\tilde{i}_i$$

2) Put the equation into the form of a real amplitude and a complex phase.

$$\tilde{V}_o = \frac{\omega L}{\sqrt{(1-\omega^2 LC)^2 + \left(\omega\frac{L}{R}\right)^2}} I e^{j\left[0 - \operatorname{atan}\left(\frac{\left(\omega\frac{L}{R}\right)}{(1-\omega^2 LC)}\right)\right]}$$

imaginary part

real part

3) Match the coefficient V and phase shift ϕ

$$V = \frac{\omega L}{\sqrt{(1-\omega^2 LC)^2 + \left(\omega\frac{L}{R}\right)^2}} I \qquad \text{and} \qquad \phi = -\operatorname{atan}\left(\frac{\left(\omega\frac{L}{R}\right)}{(1-\omega^2 LC)}\right)$$

*notice that at the resonant frequency $\omega_o = \frac{1}{\sqrt{LC}}$, the magnitude of the output is just R, and the LC circuit just behaves as if nothing is there (=open circuit). See circuit #1.

If the input was a sine wave instead of a cosine wave, then we would add an additional phase shift of 90 degrees, since $\sin(\omega t)=\cos(\omega t-\frac{\pi}{2})$. Thus, our new phase shift would be $\phi' = \frac{\pi}{2} - \phi$

4. Bode Plots

Poles & Zeros

obtain a transfer function of the system by replacing *d/dt* with *s*, which is also equal to $j\omega$,

$$H(j\omega) = \frac{(10 + 100s\tau)}{s\tau(10 + s\tau)} = \frac{10 + j(100\omega RC)}{j\omega\tau(10 + j\omega RC)}$$

roots of the numerator are "zeros"

roots of the denominator are "poles"

Break Point Frequency: ω_{BP}

Poles and Zeros are break point frequencies. Equate real and imaginary parts for each term in the numerator and denominator (i.e., factor and find the roots) to get the break point frequencies:

numerator: $\omega_{z1} = \frac{1}{10RC}$

denominator: $\omega_{p1} = 0$ and $\omega_{p2} = \frac{10}{RC}$

Magnitude Plot

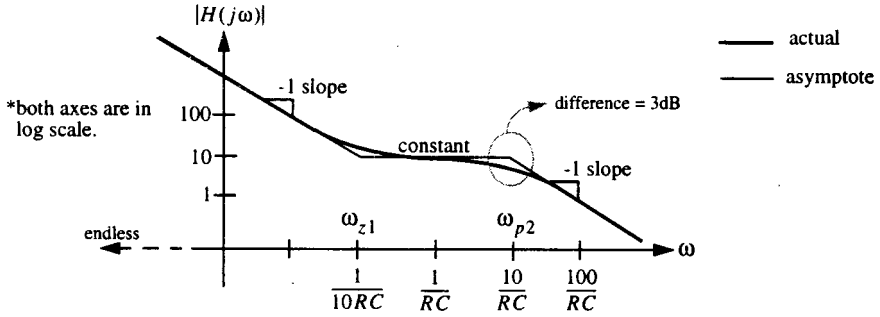
short cut: "zeros" give you a +1 slope. (constant before ω_z, +1 slope after ω_z ; in log scale)

"poles" give you a -1 slope. (constant before ω_p, -1 slope after ω_p ; in log scale)

Thus,

Z1:	0	+1	+1
P1:	0	0	-1
P2:	-1	-1	-1
$\sum$:	-1	0	-1

Since $\omega_{p1} = 0$, which is the left most end of the frequency axis (in log scale), the magnitude plot starts off at -1 slope.



Approach to Plotting Magnitude

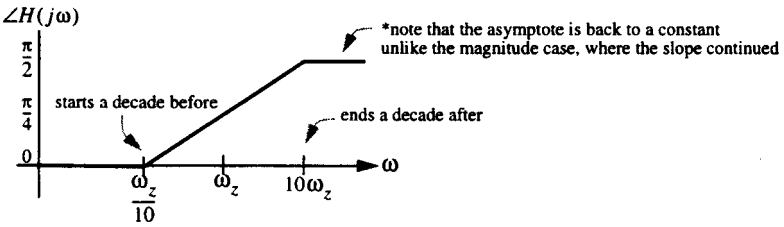
1) Find all ZEROs and POLEs. (i.e., ω_{BP}'s)
2) Draw and Label Axes. Denote break point frequencies.
3) Beginning at the far end of the ω-axis, plot for ω very small
   if ZERO at ω=0, start with +1 slope, (+2 slope if double Zero@ ω=0, and +3 for triple, ...)
   POLE at ω=0, start with -1 slope, (-2 slope if double Pole@ ω=0, and -3 for triple, ...)
   otherwise, start with a constant. (label!)
4) As you come up on the frequency axis, for each POLE you hit, go down by -1 slope, for each ZERO you hit, go up by +1 slope.
5) At ω_{BP}, your actual curve will be off from the sharp asymptote corner by 3dB.

Phase Plot

short cut: "zeros" give you a $+\frac{\pi}{2}$ phase over ±decade.

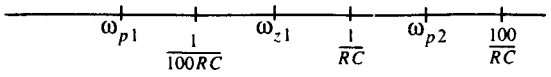
"poles" give you a $-\frac{\pi}{2}$ phase over ±decade.

example,

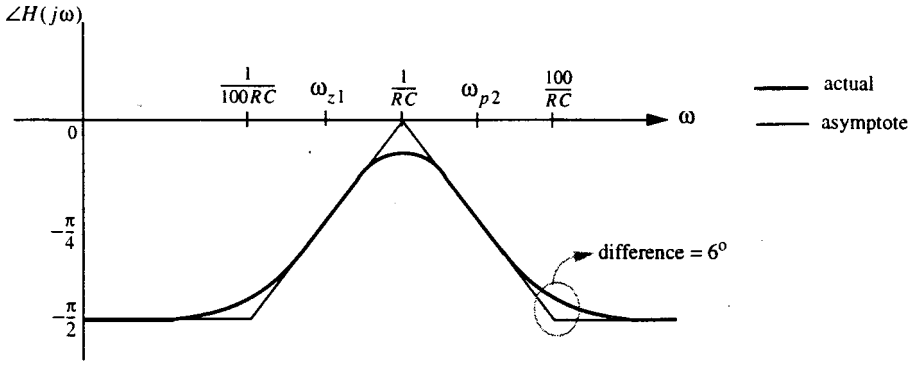


Thus,

Z1:	0	0	$+\frac{\pi}{4}$	$+\frac{\pi}{2}$	$+\frac{\pi}{2}$	$+\frac{\pi}{2}$
P1:	$-\frac{\pi}{2}$	$-\frac{\pi}{2}$	$-\frac{\pi}{2}$	$-\frac{\pi}{2}$	$-\frac{\pi}{2}$	$-\frac{\pi}{2}$
P2:	0	0	0	0	$-\frac{\pi}{4}$	$-\frac{\pi}{2}$



$\sum$:	$-\frac{\pi}{2}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$-\frac{\pi}{4}$	$-\frac{\pi}{2}$



Approach to Plotting Phase

1) Find all ZEROs and POLEs. (i.e., ω_{BP}'s)
2) Draw and label all axes and break point frequencies, including their +/- decades.
3) Beginning at the far left of the ω-axis, plot for ω very small

   if ZERO at $\omega=0$, start with $+\frac{\pi}{2}$ phase, ($+\pi$ slope if double Zero@ $\omega=0$, and $+\frac{3\pi}{2}$ for triple, ...)

   POLE at $\omega=0$, start with $-\frac{\pi}{2}$ phase, ($-\pi$ slope if double Zero@ $\omega=0$, and $-\frac{3\pi}{2}$ for triple, ...)

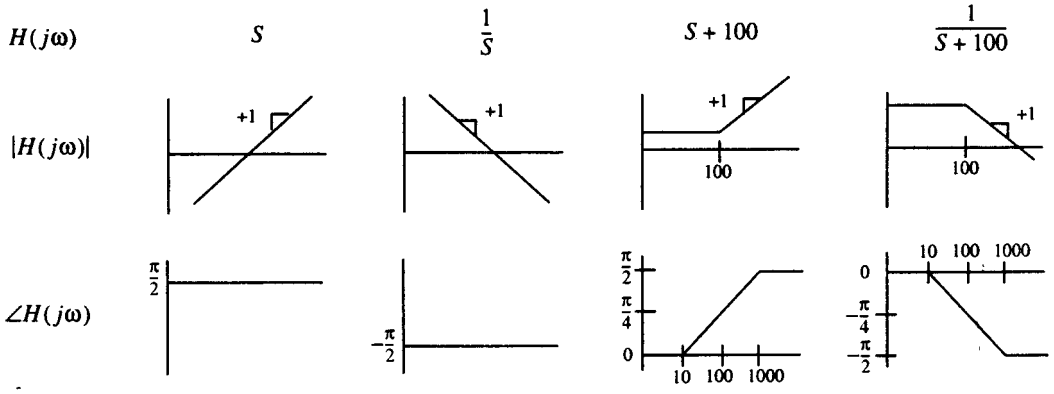
   otherwise, start at 0 phase.

4) As you come up the ω-axis, for each POLE you hit, make a change of $-\frac{\pi}{2}$ phase over a ±decade.

   ($\frac{\omega_p}{10} \sim \omega_p \sim 10\omega_p$). For each ZERO, make a change of $+\frac{\pi}{2}$ phase over a ±decade.

5) Be careful when the regions overlap, the effect from each pole and zero may cancel each other out.
6) At ω_{BP}, the actual curve is off by 6°.

Building Blocks



5. Digital Abstraction

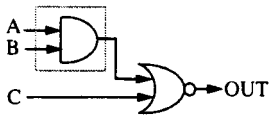
Boolean Logic

*become comfortable expressing in different forms: truth table ↔ formula ↔ gates ↔ transistor level.

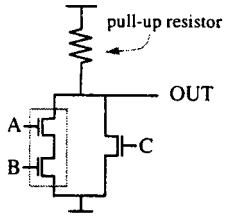
A	B	C	OUT
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

↔

$OUT = \overline{(A \cdot B) + C}$



↔



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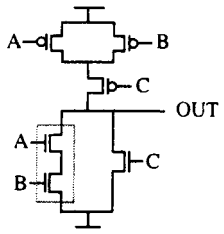


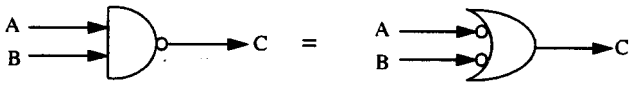
*Note that in the transistor-level implementation, transistors in series form an AND expression and those in parallel form an OR expression. The final output is always inverted in the pull-up implementation, equivalent to adding a NOT(inverter) to the output.

Primitive Rules

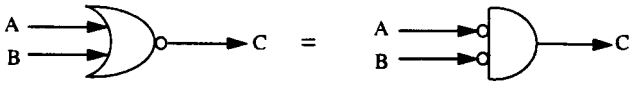
$A + (B + C) = (A + B) + C$	$A \cdot (B \cdot C) = (A \cdot B) \cdot C$	(associative)
$A + B = B + A$	$A \cdot B = B \cdot A$	(communative)
$A + (B \cdot C) = (A + B) \cdot (A + C)$	$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$	(distributive)
$A + 1 = 1$	$A \cdot 0 = 0$	
$A + 0 = A$	$A \cdot 1 = A$	
$A + \overline{A} = 1$	$A \cdot \overline{A} = 0$	(complement)
$A + A = A$	$A \cdot A = A$	(idempotence)
$A + (A \cdot B) = A$	$A \cdot (A + B) = A$	(absorption)
$A + (\overline{A} \cdot B) = A + B$	$A \cdot (\overline{A} + B) = A \cdot B$	(absorption)

DeMorgan's Laws

1) $\overline{A \cdot B} = \overline{A} + \overline{B}$ Gate Equivalent:

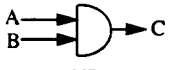


2) $\overline{A + B} = \overline{A} \cdot \overline{B}$ Gate Equivalent:



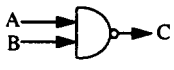
Minimum Sum of Products(MSP): (1)first level AND's, (2) combine with OR's. ex) $(A \cdot B) + (B \cdot C)$
Minimum Product of Sums(MPS): (1)first level OR's, (2) combine with AND's. ex) $(A+B) \cdot (B+C)$

Common Gates



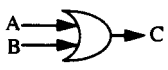
AND

A	B	C
0	0	0
0	1	0
1	0	0
1	1	1



NAND

A	B	C
0	0	1
0	1	1
1	0	1
1	1	0



OR

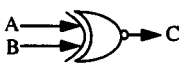
A	B	C
0	0	0
0	1	1
1	0	1
1	1	1

NOR

A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

XOR

A	B	C
0	0	0
0	1	1
1	0	1
1	1	0



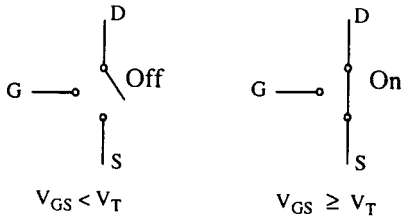
XNOR

A	B	C
0	0	1
0	1	0
1	0	0
1	1	1

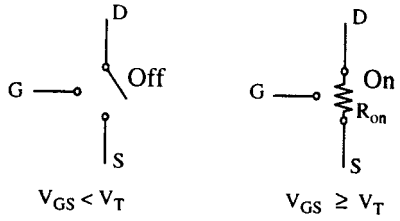
6. MOSFET Transistor (n-channel)

Large Signal Model

S Model:

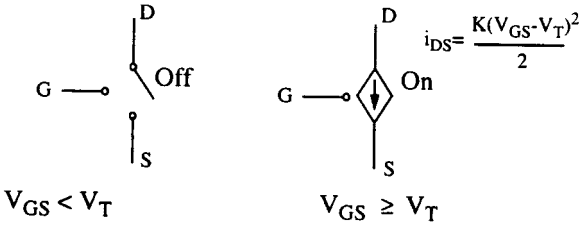


SR Model:



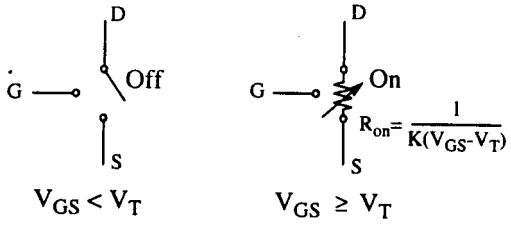
When ($V_{DS} \geq V_{GS} - V_T$),

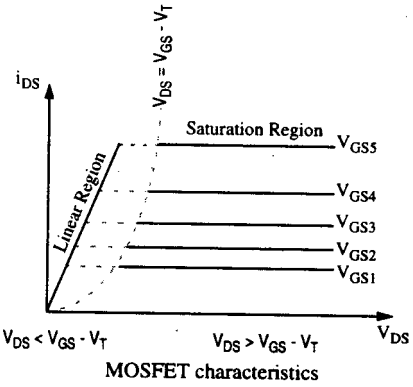
SCS Model *(Saturation)*:



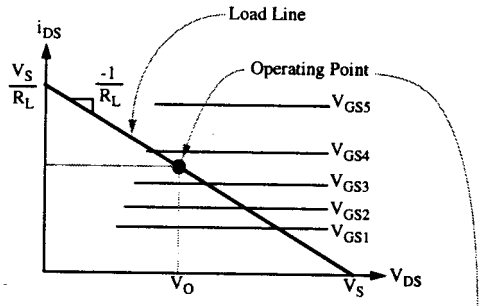
When (VDS < VGS - VT),

SVR Model (Linear):



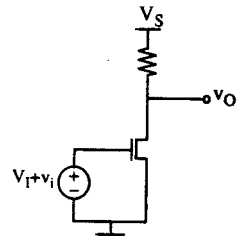


MOSFET characteristics

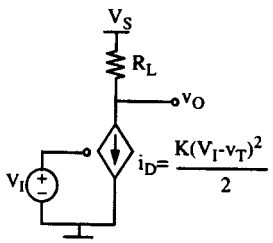


Load Line superimposed on MOS characteristics for operating point analysis of MOSFET Amplifier

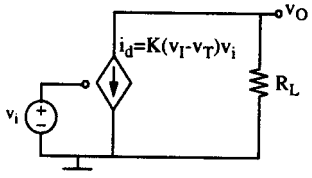
7. MOSFET Amplifier



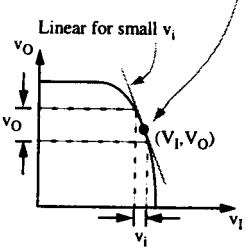
MOSFET Amplifier



Large Signal Model(SCS)



Small Signal Model

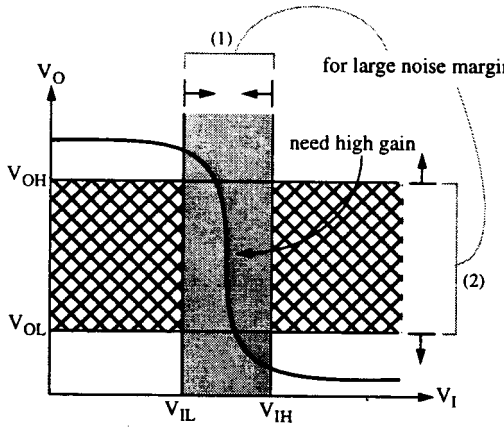


Operating Point@V_I

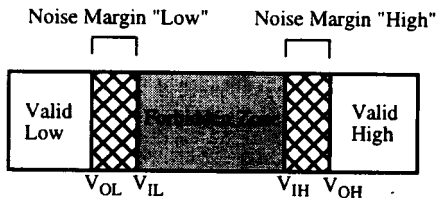
Small Signal gain: $\frac{v_o}{v_i} = -K(V_I - v_T)R_L = -g_m R_L$ *Note that the output is a function of both V_I(DC) and v_i(AC).

Parameter	Value (for small signal model)	Parameter	Value (for small signal model)
Input Resistance	∞	Current Gain	$K(V_I - v_T)(R_L \parallel R_O)\frac{R_i}{R_O}$; = ∞ if R_i=∞
Output Resistance	R_L	Power Gain	$[K(V_I - v_T)(R_L \parallel R_O)]^2\frac{R_i}{R_O}$; = ∞ if R_i=∞

8. Noise Margins



Inverter characteristic curve



Noise Margins

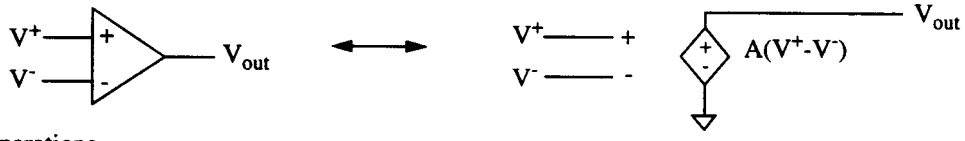
To improve noise margins: (1) increase V_{IL} and lower V_{IH}, and/or (2) increase V_{OH} and lower V_{OL}.

(1) : increase v_T of the MOSFET. This would make the transistor to turn on at a higher threshold voltage, increasing V_{IL} and lowering V_{IH}.

(2): increase load resistance R_L or (W/L) ratio of the MOSFET since $V_O \propto \frac{R_{on}}{R_{on} + R_L}$ and $R_{on} \propto \frac{L}{W}$.

9. Op-Amp Circuits

Model

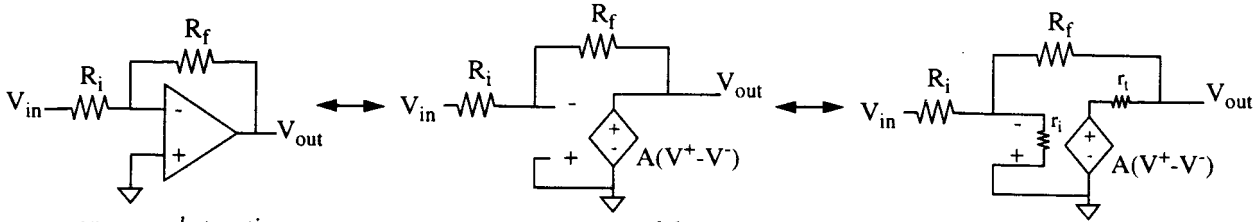


- Operations
  * The Difference (V^+ - V^-) is amplified by A(~10^6) as V_{out}. The + and - terminal inputs have infinite impedance, so no current flows into the terminals. Most Op-Amp circuits makes use of negative feedback where output tries to follow the input. A typical op-amp has output limits imposed by the power supply of +/- 15V.

- Approach
  1) Set V^+=V^-. (ideal op-amp w/ negative feedback)
  2) Do KCL at + and - terminal. (no current flows into the + and - terminals)
  3) Solve for the expression V_o/V_i.

Negative Feedback

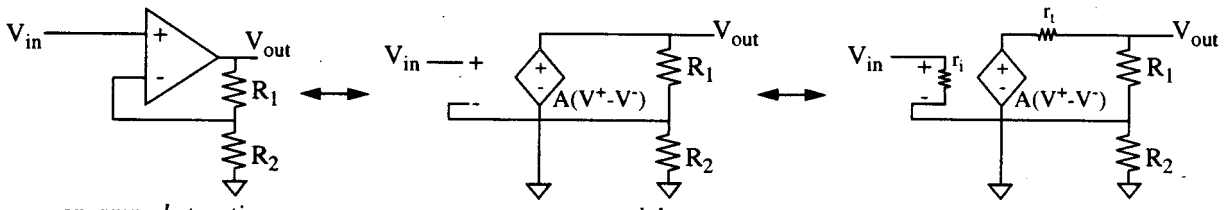
- Inverting Op Amp:



op-amp abstraction *op-amp model* *more accurate op-amp model*

* No current flows into the + and - terminals.
* $V_{out}=A(V^+-V^-) \Rightarrow V^+-V^- = V_{out}/A \sim 0$. Thus, $V^+=V^-$.
* KCL @ V^-, $V_{out} = -\frac{R_f}{R_i}V_{in}$. In General, $V_{out} = -\frac{Z_f}{Z_i}V_{in}$ thus, $\frac{V_{out}}{V_{in}} = H(j\omega) = -\frac{Z_f}{Z_i}$
* Input Resistance: $R_i = r_i \| (R_f + r_t) \| \left(\frac{R_f + r_t}{A}\right) \approx \frac{R_f + r_t}{A}$
* Output Resistance: $R_o = \frac{r_t}{1 + A\frac{R_s}{R_s + R_f}} \approx \frac{r_t}{A\frac{R_s}{R_s + R_f}}$

- Non-inverting Op Amp:

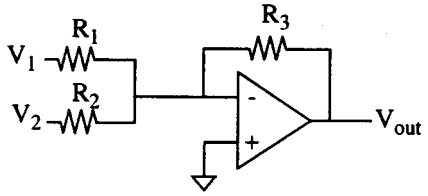


op-amp abstraction *op-amp model* *more accurate op-amp model*

* $V_{out} = \frac{R_1 + R_2}{R_2}V_{in}$
* Input Resistance: $R_i \approx r_i\left[\frac{AR_2}{R_1 + r_t + R_2}\right]$
* Output Resistance: $R_o \approx \frac{r_t}{A\frac{R_2}{R_1 + R_2}}$

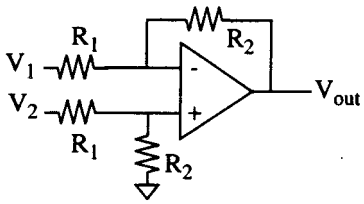
Important Circuits

Adder



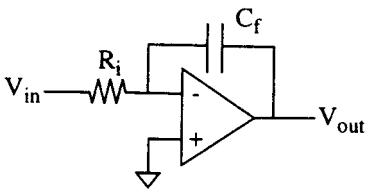
$V_o = -\left(\frac{R_3}{R_1}V_1 + \frac{R_3}{R_2}V_2\right)$

Subtractor



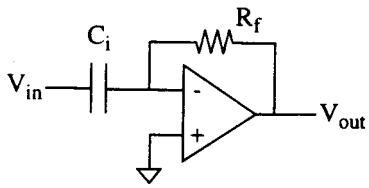
$V_o = \frac{R_2}{R_1}(V_2 - V_1)$

Integrator



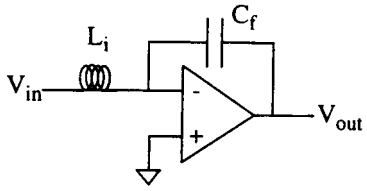
$$V_o = \frac{1}{RC}\int V_i dt$$

Differentiator



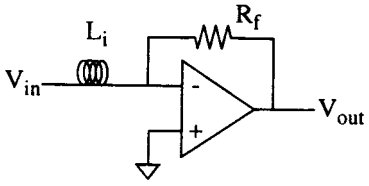
$$V_o = RC\frac{dV_i}{dt}$$

Double Integrator



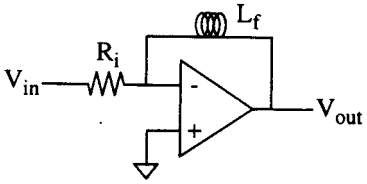
$$V_o = \frac{1}{LC}\int\int V_i dt dt$$

Integrator



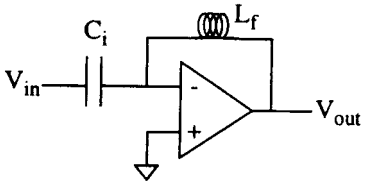
$$V_o = \frac{R}{L}\int V_i dt$$

Differentiator



$$V_o = \frac{L}{R}\frac{dV_i}{dt}$$

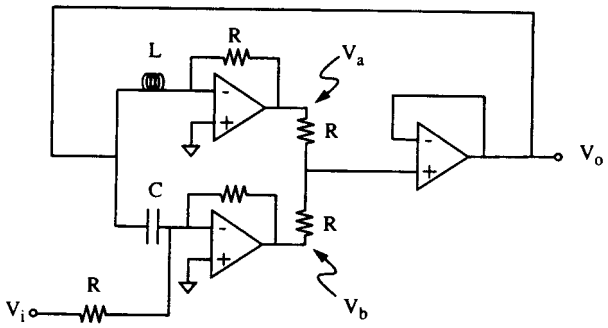
Double Differentiator



$$V_o = LC\frac{d^2V_i}{dt^2}$$

Several of these op-amp circuits may be cascaded in stages. For such cases, you may find the V_o/V_i relationship for each stage using impedances and multiply them together to obtain the total V_o/V_i.

example:



$$V_a = -\frac{R}{L}\int V_o dt$$

$$V_b = -V_i - RC\frac{dV_o}{dt}$$

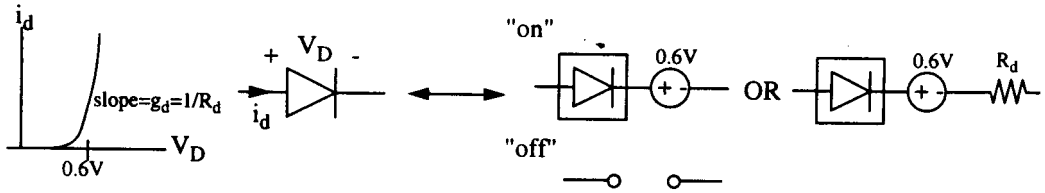
$$V_o = \frac{V_a + V_b}{2}$$

Thus, $$\frac{dV_i}{dt} = -RC\frac{d^2V_o}{dt^2} - 2\frac{dV_o}{dt} - \frac{R}{L}V_o$$

10. Non-linear Circuits

Diode

- Model



DC Analysis: For the large signal model, the diode acts like a short with a voltage drop of 0.6V when the diode is on(V_d>0.6V), and like an open when the diode is off(V_d<0.6V):

$$i_D = I_s\left(e^{\frac{qV_D}{kT}} - 1\right) \quad \text{where } I_s \sim 10^{-12} \text{ Amps for Silicon diodes.}$$

AC Analysis: For the incremental model, the diode acts like a current dependent resistor. The resistor value is set by the nominal(DC) operating current I_d. $r_d = \frac{KT}{qI_d} = \frac{26mV}{I_d}$

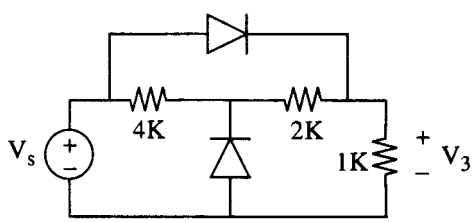
- Circuit Analysis

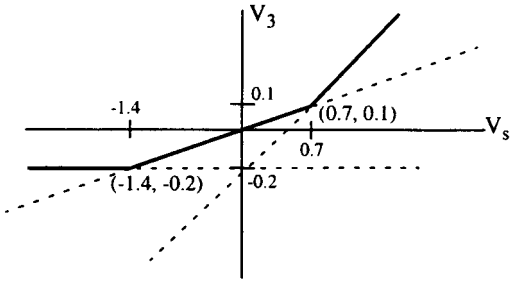
  One Diode: subst. the model for "on" and "off" state and solve using KCL and KVL.

  Multiple Diodes:

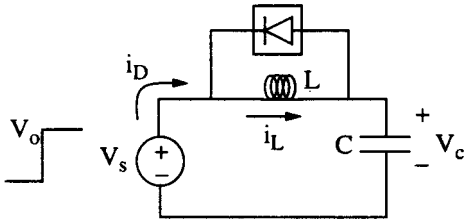
  1) Setup cases of "on" and "off" states for each diode. Total of 2^n cases for n diodes.
  2) For each case, subst. the model and solve using KCL and KVL.
  3) Obtain asymptote for each case.
  4) determine which case (i.e., asymptote) is appropriate for each region of operation.
     * ex) For asymptote crossing the V=0 axis, see which case is valid for V=0 and choose the asymptote for that case. repeat for other regions of operation.

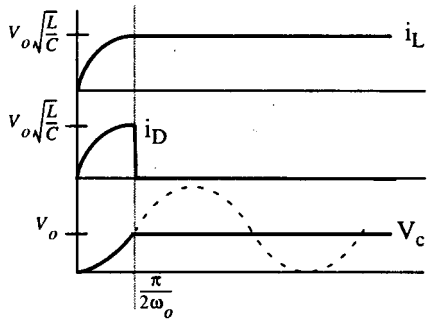
example,



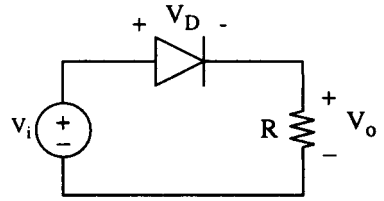


another example,

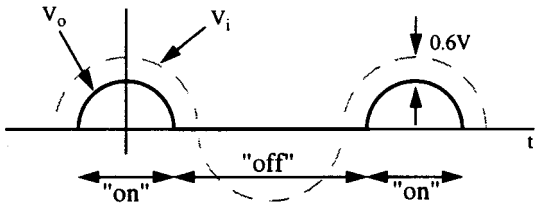




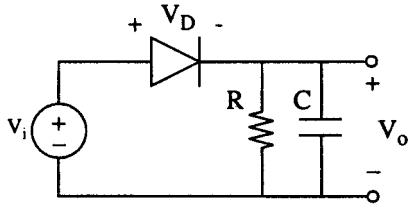
- Half-wave Rectifier: Clips the negative portion of the input wave and passes only the positive portion of the input wave with a diode drop of 0.6V.

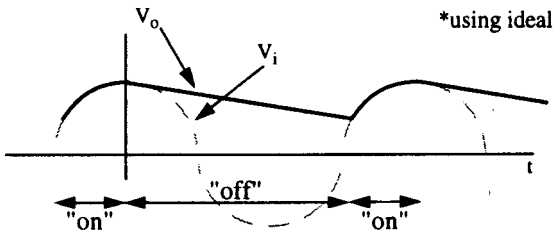


$$-V_i + i_D R + V_D = 0$$



- Peak Detector

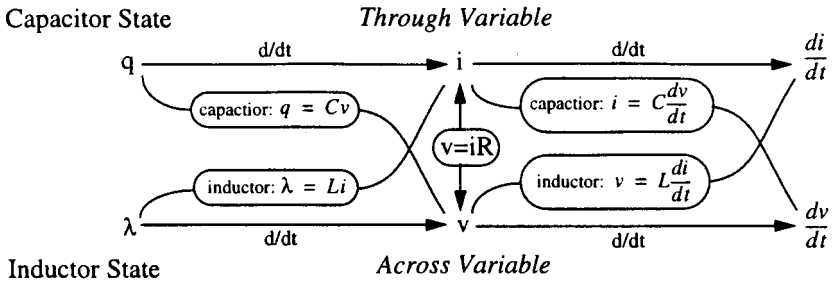




*using ideal diode

11. Electrical/Mechanical Analogs

Electrical

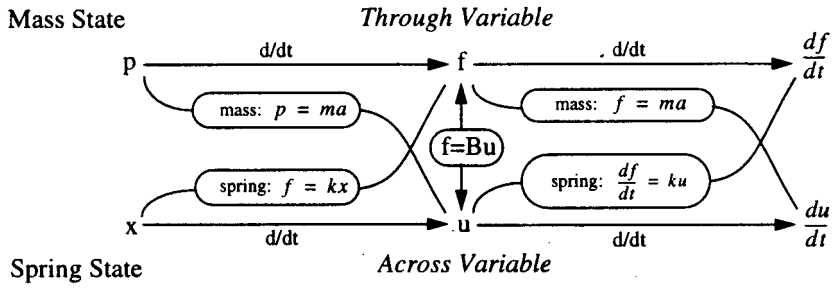


Power=iv

KVL: $\sum_{loop} v = 0$

KCL: $\sum_{node} i = 0$

Mechanical



Power=uf

$\sum_{loop} u = 0$

$\sum_{node} f = 0$

Electrical/Mechanical Pairs

Capacitor/Mass	Inductor/Spring	Resistor/Damper
$q = Cv$ vs. $p = mu$	$\lambda = Li$ vs. $x = \frac{1}{k}f$	$v = Ri$ vs. $u = \frac{1}{B}f$

References

Agarwal, Anant and Jeff Lang. 6.002 Notes (1998)
Leeb, Steve. 6.002 Recitation Notes (fall '97)
Schlecht, Martin. 6.002 Lecture Notes (fall '97)
Searle, Campbell. 6.002 Notes (1991)

Good Luck!

12. 8. 1998. JK.