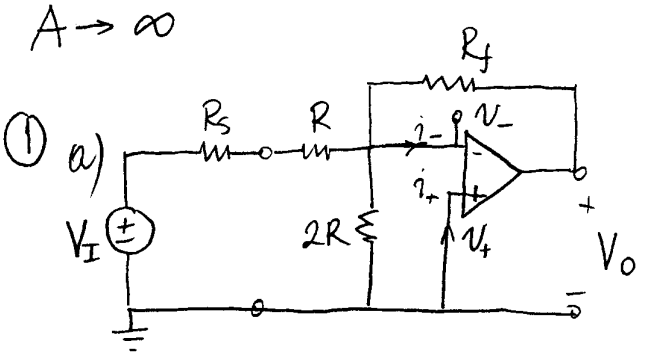
Problem Set 10

$A \to \infty$

① a)



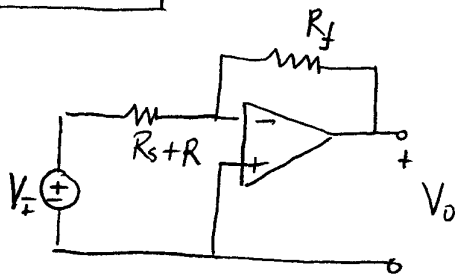
$V_o = A(v^+ - v^-)$

$\boxed{v^+ = 0}$

$\Rightarrow \boxed{v_- = -\frac{V_o}{A} = -\frac{V_o}{\infty} = 0}$ $\Big\}$ => no voltage drop across 2R

$\boxed{i_- = i_+ = 0}$

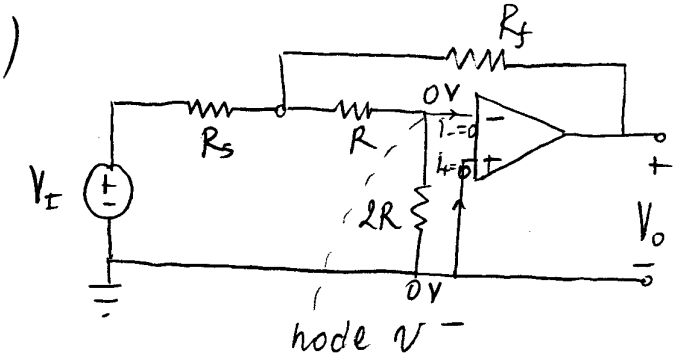
=>



$V_o = -\frac{R_f}{R+R_S} V_I$

$\boxed{\frac{V_o}{V_I} = -\frac{R_f}{R+R_S}}$

b)



node v^-

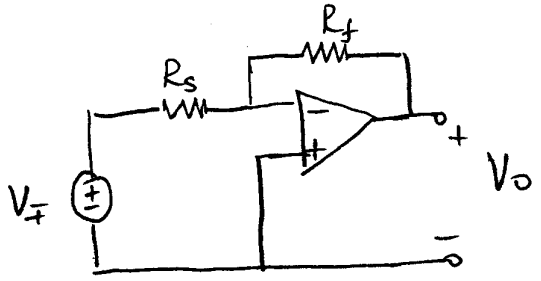
Since the op-amp model assumes $\boxed{i_- = 0}$ and

$$v_o = A(v^+ - v^-) = -Av^- \Rightarrow v^- = \frac{v_o}{A} = \frac{v_o}{\infty} = \boxed{0 = v^-},$$

$v^+ = 0$

we conclude there is no voltage drop across 2R resistor. To satisfy KCL at node v^-, current through R resistor must be 0 and thus voltage drop across R must be 0.

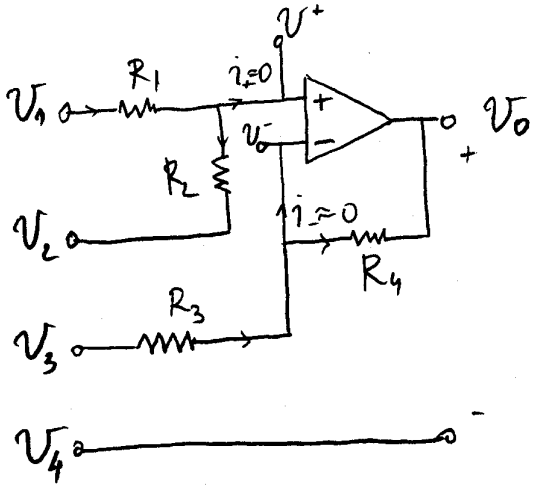
$\Rightarrow$ the circuit can be redrawn as



$$\Rightarrow V_o = -\frac{R_f}{R_s} V_I$$

$$\boxed{\frac{V_o}{V_I} = -\frac{R_f}{R_s}}$$

$\Rightarrow$ The magnitude of the gain found here is larger than the magnitude of the gain from part a)

② 

$$v_o = A(v^+ - v^-)$$

$$v^+ - v^- = \frac{v_o}{A} = \frac{v_o}{\infty} = 0$$

$$\Rightarrow \boxed{v^+ = v^-}$$

KCL @ V^+

$$\frac{V_1 - V^+}{R_1} = \frac{V^+ - V_2}{R_2}$$

$$R_2 V_1 - R_2 V^+ = R_1 V^+ - R_1 V_2$$

$$R_2 V_1 + R_1 V_2 = V^+ (R_1 + R_2)$$

$$\Rightarrow \boxed{V^+ = \frac{R_2 V_1 + R_1 V_2}{R_1 + R_2}}$$

KCL @ V^-

$$\frac{V_3 - V^-}{R_3} = \frac{V^- - V_0}{R_4}$$

$$R_4 V_3 - R_4 V^- = R_3 V^- - R_3 V_0$$

$$R_4 V_3 + R_3 V_0 = V^- (R_3 + R_4)$$

$$\Rightarrow \boxed{V^- = \frac{R_4 V_3 + R_3 V_0}{R_3 + R_4}}$$

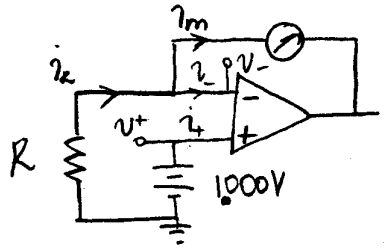
$$V^+ = V^-$$

$$\Rightarrow \frac{R_2 V_1 + R_1 V_2}{R_1 + R_2} = \frac{R_4 V_3 + R_3 V_0}{R_3 + R_4}$$

$$\frac{R_3 + R_4}{R_1 + R_2} (R_2 V_1 + R_1 V_2) = R_4 V_3 + R_3 V_0$$

$$\Rightarrow \boxed{V_0 = \frac{R_3 + R_4}{R_3 (R_1 + R_2)} (R_2 V_1 + R_1 V_2) - \frac{R_4}{R_3} V_3}$$

③



a) For ideal op-amp:

$i_- = 0$, $i_+ = 0$

$V^+ = V^-$ (as shown in problem 2)

From KCL @ node V^-:

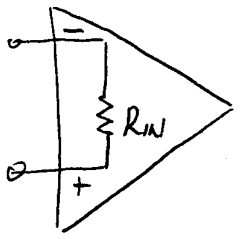
$i_R = i_m$

$-\frac{V^-}{R} = i_m$

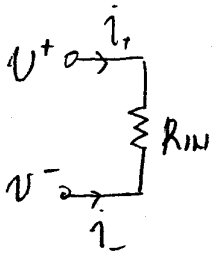
$v^- = v^+ = 1000\,V$

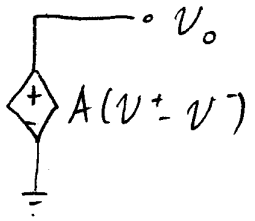
$$\Rightarrow \boxed{i_m = -\frac{1000}{R}}$$

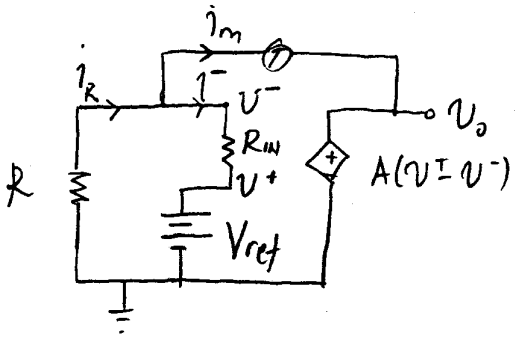
e)



non-ideal op-amp model →







$v_o = A(v^+ - v^-)$

$\Downarrow$

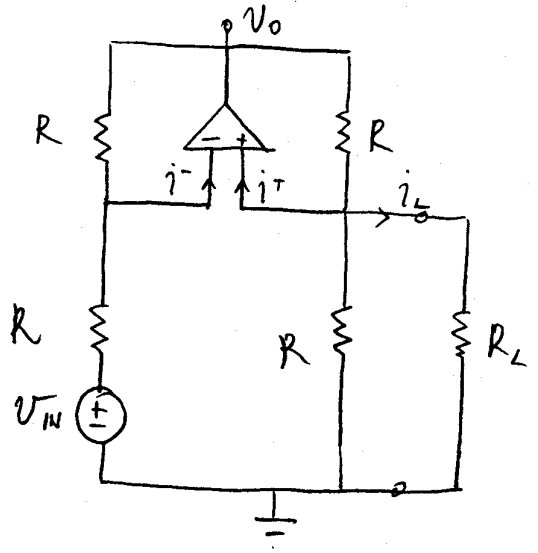
again $\boxed{v^+ = v^-}$

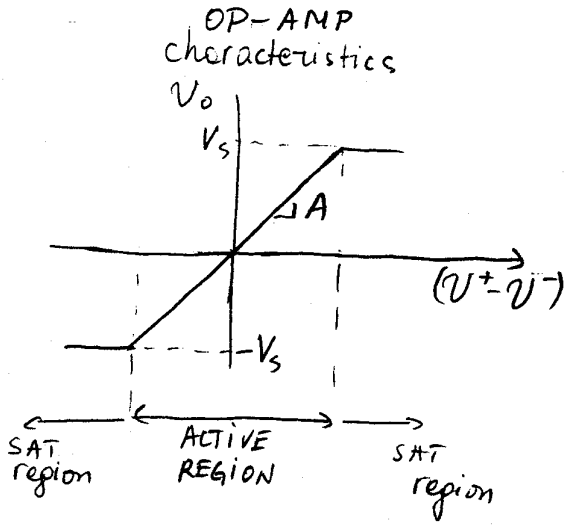
$\Rightarrow i_- = 0$

$\Rightarrow i_R = i_m$

$$\boxed{i_m = -\frac{1000}{R}}$$

④



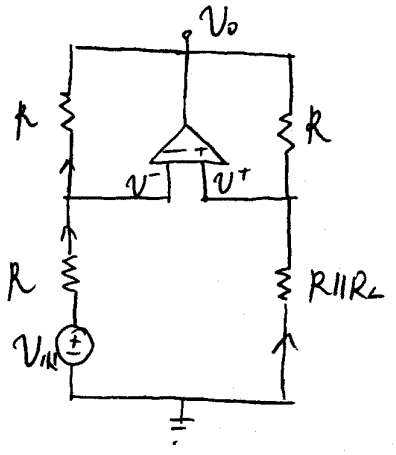


a) assume op-amp is ideal, i.e.

$i^+ = i^- = 0$

$V^+ = V^-$

$i_L = \frac{V^+}{R_L}$ ⇒ we need to solve for V^+ as a function of V_{IN}:



$\frac{V_{IN} - V^-}{R} = \frac{V^- - V_o}{R}$ → KCL @ V^-

$V_o = 2V^- - V_{IN}$

$-\frac{V^+}{R \| R_L} = \frac{V^+ - V_o}{R}$ → KCL @ V^+

$V^+ = V^-$ ⇒ $-RV^- = \frac{R \cdot R_L}{R + R_L}(V^- - V_o)$

$V^-\left(R + \frac{RR_L}{R+R_L}\right) = V_o \cdot \frac{R \cdot R_L}{R+R_L}$

($V_o = 2V^- - V_{in}$)

$$v^-\left(R+\frac{RR_L}{R+R_L}\right)=2v^-\frac{R\cdot R_L}{R+R_L}-v_{IN}\frac{R\,R_L}{R+R_L}$$

$$v^-\left(R-\frac{RR_L}{R+R_L}\right)=-v_{IN}\frac{R\cdot R_L}{R+R_L} \quad / \cdot \frac{R+R_L}{R_L}$$

$$v^-\left(\frac{R+R_L}{R_L}-1\right)=-v_{IN} \qquad \frac{R+R_L}{R_L}=\frac{R}{R_L}+1$$

$$v^-=-\frac{v_{IN}}{\frac{R}{R_L}+1-1}=\boxed{-\frac{R_L}{R}\cdot v_{IN}=v^-}$$

$$\Rightarrow \boxed{v^+=-\frac{R_L}{R}v_{IN}}$$

$$\Rightarrow i_L=-\frac{R_L}{R}v_{IN}\cdot\frac{1}{R_L}$$

$$\Rightarrow \boxed{i_L=-\frac{v_{IN}}{R}}$$

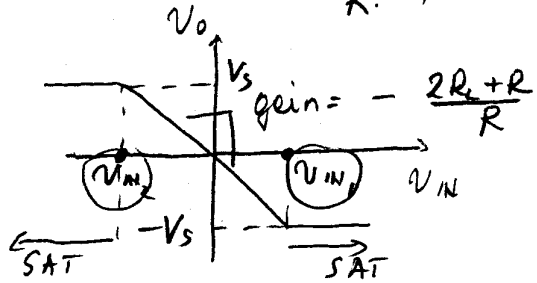
6) OP-AMP is no longer in active region when

$v_o = 15V$ or $v_o = -15V$

$$v_o=2v^--v_{IN}=-\frac{2R_L}{R}v_{IN}-v_{IN}=-v_{IN}\left(\frac{2R_L+R}{R}\right)$$

$$\Rightarrow v_{IN}=-\frac{R}{2R_L+R}\cdot v_o \Rightarrow$$

$$\boxed{\frac{v_o}{v_{IN}}=-\frac{2R_L+R}{R}}$$



INPUT - OUTPUT CHARACTERISTICS

OP-AMP IS IN SAT REGION FOR

$V_{IN} > V_{IN_1}$ V $V_{IN} < V_{IN_2}$

$$V_{IN_1} = + \frac{R}{2R_L+R} \cdot (+V_S) = \boxed{15 \cdot \frac{R}{2R_L+R} = V_{IN_1}}$$

$$V_{IN_2} = - \frac{R}{2R_L+R} (+V_S) = \boxed{-15 \frac{R}{2R_L+R} = V_{IN_2}}$$