MASSACHVSETTS INSTITVTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.002 – Electronic Circuits
Fall 2001

Problem Set 1 Solutions

Problems:

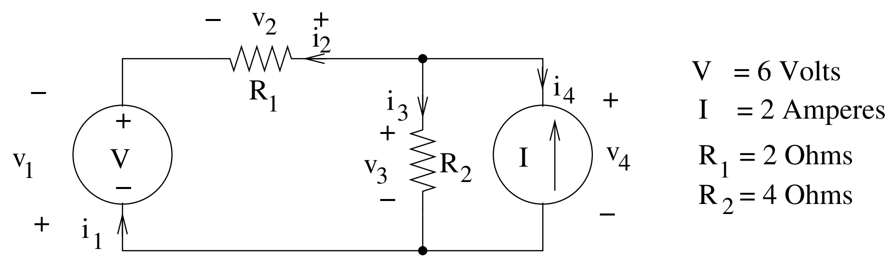

Figure 1: Circuit for Problem 1

1. Figure 1 shows a circuit with four elements: two resistors, a voltage source, and a current source. The resistances of the resistors and the strengths of the sources are given. Branch voltage circuit variables (v_k) and branch current circuit variables (i_k) are defined for each element, in associated reference directions.

   (a) How many nodes are there in the circuit? Write a KCL equation for each node. How many of the KCL equations are independent? (Independent means, "Cannot be derived from the other equations.")
   There are 3 nodes.

$$i_1 + i_2 = 0 \tag{1}$$
$$-i_2 - i_3 - i_4 = 0 \tag{2}$$
$$-i_1 + i_3 + i_4 = 0 \tag{3}$$

   Only two are independent. Equation (3) can be derived by adding equations (1) and (2).

   (b) How many loops are there in the circuit? Write a KVL equation for each loop. How many of these equations are independent?
   There are 3 loops.

$$v_1 - v_2 + v_3 = 0 \tag{4}$$
$$v_1 - v_2 + v_4 = 0 \tag{5}$$
$$v_3 - v_4 = 0 \tag{6}$$

   Only two are independent. Equation (6) can be derived by substracting equation (5) from (4).

(c) Write an equation expressing the v-i constraint for each element.

$$v_1 = -V$$

$$v_2 = i_2 R_1 \Leftrightarrow i_2 = \frac{v_2}{R_1}$$

$$v_3 = i_3 R_2 \Leftrightarrow i_3 = \frac{v_3}{R_2}$$

$$i_4 = -I$$

(d) You should now have a set of linear equations in the branch voltage and current variables. If you count only the independent equations, you should have one equation for each unknown branch voltage or current. Solve the equations and fill in the table of results on the next page. Compute the power ($v_k i_k$) leaving the circuit by each branch and enter it into your table.

$$i_1 + i_2 = 0 \Rightarrow i_2 = -i_1 \Rightarrow i_1 = -\frac{v_2}{R_1} \Rightarrow v_2 = -i_1 R_1$$

$$i_2 + i_3 + i_4 = 0 \Rightarrow \frac{v_2}{R_1} + \frac{v_3}{R_2} - I = 0 \Rightarrow \frac{v_2}{2\Omega} + \frac{v_3}{4\Omega} - 2A = 0$$

$$\Rightarrow 2v_2 + v_3 = 8V \tag{7}$$

$$v_1 - v_2 + v_3 = 0 \Rightarrow -V - v_2 + v_3 = 0$$

$$\Rightarrow -v_2 + v_3 = 6V \tag{8}$$

Adding equation (7) and twice equation (8),

$$3v_3 = 20V \Rightarrow v_3 = \frac{20}{3}V \Rightarrow v_2 = \frac{2}{3}V$$

Table:

$v_1 = -6V$	$i_1 = -1/3A$	$v_1 i_1 = 2W$
$v_2 = 2/3V$	$i_2 = 1/3A$	$v_2 i_2 = 2/9W$
$v_3 = 20/3V$	$i_3 = 5/3A$	$v_3 i_3 = 100/9W$
$v_4 = 20/3V$	$i_4 = -2A$	$v_4 i_4 = -40/3W$

(e) Now compute the sum of the four powers you computed. It should be exactly zero, or you made an error!

$v_1 i_1 + v_2 i_2 + v_3 i_3 + v_4 i_4 = \frac{18+2+100-120}{9} W \stackrel{\checkmark}{=} 0$

2. Repeat the process you did in Problem 1 for the circuit of Figure 2 below. Notice that the sum of the powers is zero here too.

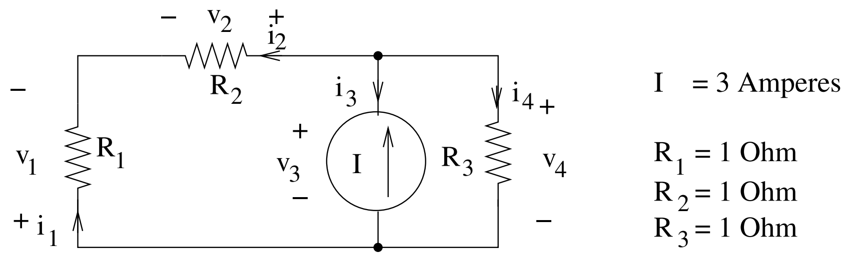

Figure 2: Circuit for Problem 2

KCL:

$$i_1 + i_2 = 0$$
$$-i_2 - i_3 - i_4 = 0$$
$$-i_1 + i_3 + i_4 = 0$$

KVL:

$$-v_1 + v_2 - v_3 = 0$$
$$-v_1 + v_2 - v_4 = 0$$
$$v_3 - v_4 = 0$$

v-i constraints:

$$v_1 = i_1 R_1$$
$$v_2 = i_2 R_2$$
$$i_3 = -I$$
$$v_4 = i_4 R_3$$

Solving the equations:

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} = 0 \Rightarrow \frac{v_1}{1\Omega} + \frac{v_2}{1\Omega} = 0 \Rightarrow v_1 = -v_2$$

$$-\frac{v_2}{R_2} + I - \frac{v_4}{R_3} = 0 \Rightarrow \frac{v_2}{1\Omega} + \frac{v_4}{1\Omega} - 3A = 0$$

$$\Rightarrow v_2 + v_4 = 3V \tag{9}$$

$$v_1 - v_2 + v_4 = 0 \Rightarrow -2v_2 + v_4 = 0 \tag{10}$$

Equation (9) minus equation (10),

$$3v_2 = 3V \Rightarrow v_2 = 1V \Rightarrow v_1 = -1V \Rightarrow v_4 = 2V \Rightarrow v_3 = 2V$$

Table:

$v_1 = -1V$	$i_1 = -1A$	$v_1 i_1 = 1W$
$v_2 = 1V$	$i_2 = 1A$	$v_2 i_2 = 1W$
$v_3 = 2V$	$i_3 = -3A$	$v_3 i_3 = -6W$
$v_4 = 2V$	$i_4 = 2A$	$v_4 i_4 = 4W$

$$v_1 i_1 + v_2 i_2 + v_3 i_3 + v_4 i_4 = 1W + 1W - 6W + 4W \stackrel{\checkmark}{=} 0$$

3. Now we want you to do something really weird: Make a table of the voltages you computed from Problem 1 paired with the corresponding currents you got in Problem 2. (That is right... get v_1 from the first table and i_1 from the second table...) Now, what is the sum of the products of these voltages and the currents? More precisely, compute $v_1 i_1 + v_2 i_2 + v_3 i_3 + v_4 i_4$. We promise that we didn't choose the numbers so that this would work out! You get a *gold star* if you can explain this result.

$v_1 = -6V$	$i_1 = -1A$	$v_1 i_1 = 6W$
$v_2 = 2/3V$	$i_2 = 1A$	$v_2 i_2 = 2/3W$
$v_3 = 20/3V$	$i_3 = -3A$	$v_3 i_3 = -20W$
$v_4 = 20/3V$	$i_4 = 2A$	$v_4 i_4 = 40/3W$

$$v_1 i_1 + v_2 i_2 + v_3 i_3 + v_4 i_4 = \frac{18+2-60+40}{3} W \stackrel{\checkmark}{=} 0$$

This result is part of a larger theorem called *Tellegen's Theorem*. This theorem states that for any network, for all vectors of voltage v and all vectors of current i, $v^T i = 0$, as long as associates reference directions are used (Chua, Desoer & Kuh; *Linear And Nonlinear Circuits*).