MASSACHVSETTS INSTITVTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.002 – Electronic Circuits
Fall 2001

Lab 3: Second-Order Networks

Introduction

The purpose of this lab is to give you experience with second-order networks, and to illustrate that real network elements do not always behave in an ideal manner. All exercises in this lab focus on the behavior of the network and network elements shown in Figure 1. You should complete the pre-lab exercises in your lab notebook before coming to lab. Then, carry out the in-lab exercises between November 5 and November 9. After completing the in-lab exercises, have a TA check your work and sign your lab notebook. Finally, complete the post-lab exercises in your lab notebook, and turn in your lab notebook on or before November 21.

Bring in your favorite CD for In-Lab Exercise 3-5; it is meant to be a fun experiment and its results will not be needed for the post-lab exercises.

Pre-Lab Exercises

You are encouraged to use math software to generate the graphs for Exercises 3-2 and 3-5. It will not only save you time, but will also help you generate graphs which are extremely accurate and precise. See the appendix for further details.

(3-1) Assume that the network in Figure 1 is initially at rest. At $t = 0$, the input voltage $v_{IN}(t)$ steps from 0 V to V_{TI}. Given this input, determine the transient response of $v_{OUT}(t)$. Note that $v_{OUT}(t)$ takes the form $v_{OUT}(t) = V_{TO}e^{-\alpha_T t}\sin(\omega_T t + \phi_T)$.

(3-2) Let $L = 47$ mH, $C = 0.0047$ μF, $R = 220$ Ω and $V_{TI} = 10$ V. Under these conditions, graph the transient response of $v_{OUT}(t)$ for $0 \leq t \leq 0.3$ ms. Graphing the peaks and zero crossings of the response and a few points in between each peak and zero crossing should be sufficient. On a separate graph, repeat this exercise for $R = 1000$ Ω.

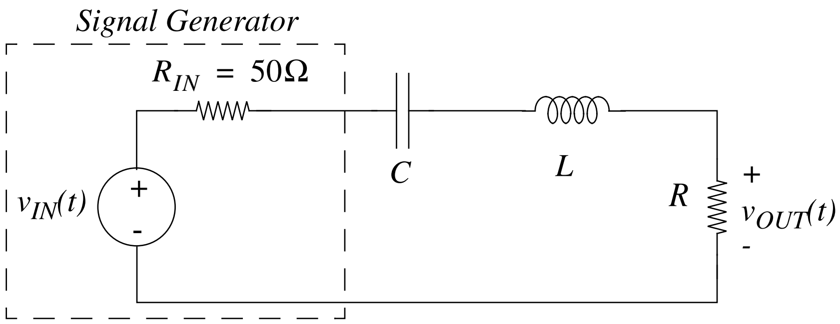

Figure 1: second-order network.

(3-3) For both values of R, compute the voltage V_{TP} at the first peak of the transient response, the frequency ω_T at which the transient response oscillates, and the rate α_T at which the transient response decays. Note that peaks of the transient response occur at times such that $\tan(\omega_T t + \phi_T) = \omega_T/\alpha_T$; you should verify this fact.

(3-4) Assume that the network is in sinusoidal steady state. Determine the response of $v_{OUT}(t)$ to the input $v_{IN}(t) = V_{SI}\cos(\omega_S t)$. Note that $v_{OUT}(t)$ will take the form $v_{OUT}(t) = V_{SO}(\omega_S)\cos(\omega_S t + \phi_S(\omega_S))$.

(3-5) Let $L = 47$ mH, $C = 0.0047$ μF and $R = 220$ Ω. On separate graphs, graph $\log_{10}|H_S(\omega_S)|$ and $\phi_S(\omega_S)$ versus $\log_{10}(\omega_S/(2\pi \times 10 \text{ kHz}))$ for $2\pi \times 1 \text{ kHz} \leq \omega_S \leq 2\pi \times 100 \text{ kHz}$ where $H_S(\omega_S) \equiv V_{SO}(\omega_S)/V_{SI}$. Ten to fifteen points per graph should be sufficient to clearly outline H_S if you space the points more closely near the peak of $\log_{10}|H_S|$. On a separate graph, repeat this exercise for $R = 1000$ Ω. You may find it easiest to use log-log graph paper for the graph of $\log_{10}|H_S|$ and linear-log graph paper for the graph of ϕ_S.

(3-6) For both values of R compute the peak value H_{SP} of $|H_S|$, the frequency ω_{SP} at which the peak occurs, and Q. Note that Q is defined as $Q \equiv \omega_{SP}/2\alpha_T$, and that $|H_S|$ will have fallen from its peak value of H_{SP} by a factor of $\sqrt{2}$ at $\omega_S \approx \omega_{SP} \pm \alpha_T$.

In-lab Exercises

The in-lab exercises involve measuring both the step response and sinusoidal response of the network shown in Figure 1 for two values of R. Afterwards, you will use the same network to filter a signal from a CD player.

Real network elements do not always behave the way we model them in 6.002. For example, a real inductor might be better modeled as an ideal inductor in series with a resistor as shown in Figure 2. The resistor is a parasitic element, meaning that it is undesired, but unavoidable. The resistor accounts for the resistance of the wire used to wind the inductor. Yet more complex models could account for core losses and the capacitance between winding turns. For this reason, the model shown in Figure 2 is not the only possible model. In a similar way, a real capacitor might be better modeled as an ideal capacitor in parallel with a parasitic conductance which models leakage through the dielectric of the capacitor. This is also shown in Figure 2.

In the exercises which follow, the network in Figure 1 will be exposed to inputs that vary at high enough frequencies that you can ignore the parasitic parallel conductance of the capacitor. Therefore, we need only be concerned with the parasitic series resistance of the inductor.

(3-1) Take a 47 mH inductor, a 0.0047 μF capacitor, a 220 Ω resistor, and a 1000 Ω resistor

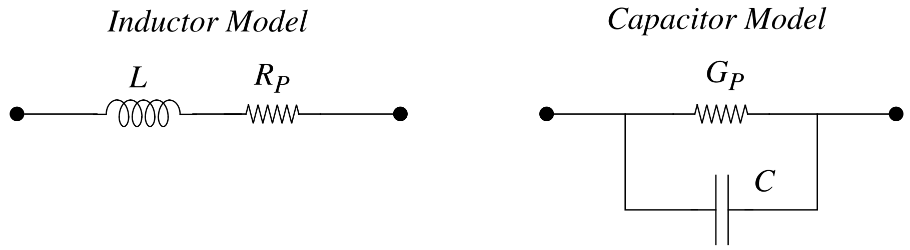

Figure 2: improved models for a real inductor and a real capacitor.

from your lab kit to the instrument desk and use the GenRad impedance meter to measure these elements and determine the parasitic resistance and conductance of the inductor and capacitor, respectively.

To measure the inductor, set the meter for 1 kHz, the series model, and the appropriate element type and value range. The meter will directly read the inductor value. It will also read Q from which you can determine R_P from $Q = \omega L/R_P$, where $\omega = 2\pi\times$ 1 kHz.

To measure the capacitor, set the meter for 1 kHz, the parallel model, and the appropriate element type and value range. The meter will now directly read the capacitor value. It will also read D from which you can determine G_P from $D = G_P/\omega C$, where $\omega = 2\pi\times$ 1 kHz.

(3-2) Construct the second-order network shown in Figure 1 using the measured inductor, capacitor and 220 Ω resistor.

(3-3) Set the signal generator to produce a 10 V peak-to-peak square wave at 50 Hz with a 5 V offset so that its open-circuit output voltage steps between 0 V and 10 V. Connect the SYNC output of the generator to one oscilloscope channel, and trigger the oscilloscope off that channel. Use the other oscilloscope channel to measure the transient response of the resistor voltage $v_{OUT}(t)$ to the positive going step, and compare the response to that plotted during the pre-lab exercises. Measuring data at times which correspond to the points graphed during Pre-Lab Exercise 3-2 is sufficient. Also measure the voltage V_{TP}, the oscillation frequency ω_T, and the rate of decay α_T. To measure ω_T, consider the time between zero crossings of the step response. To measure α_T, consider the ratio of the amplitudes of the first two peaks. Strictly speaking, the exponential envelope does not intersect the centers of the peaks. However, for the purposes of this measurement, it is sufficiently accurate to assume that it does.

Replace the 220 Ω resistor with the measured 1000 Ω resistor and repeat the exercise. Replace the 1000 Ω resistor with the measured 220 Ω resistor for the next exercise.

(3-4) Set the signal generator to produce an open-circuit 10 V peak-to-peak sinusoidal voltage with zero offset; the open-circuit voltage is $v_{IN}(t)$. Connect the SYNC output of the generator to one oscilloscope channel, and trigger the oscilloscope off that channel. Use the other oscilloscope channel to measure the resistor voltage $v_{OUT}(t)$ over the frequency range of 1 kHz to 100 kHz. In particular, measure the peak-to-peak amplitude of v_{OUT}, and the difference in time between neighboring zero crossings of the SYNC output and v_{OUT}; note that the SYNC output has the same zero crossings as v_{IN}. From this data you will determine $|H_S|$ and ϕ_S during the post-lab exercises. Measuring data at frequencies which correspond to the points graphed during Pre-Lab Exercise 3-5 is sufficient. Also measure the peak voltage ratio H_{SP} of $|H_S|$, the frequency ω_{SP} at which the peak occurs, and Q. Q is most easily measured by first measuring the difference between the two frequencies at which $|H_S|$ falls from its peak value of H_{SP} by a factor of $\sqrt{2}$; this frequency difference should be $2\alpha_T$; Q is then given by $Q = \omega_{SP}/2\alpha_T$.

It is important to note that $\phi_S(\omega_S)$ is the phase shift of v_{OUT} relative to v_{IN}. The most convenient manner in which to measure ϕ_S is to measure the time delay from zero crossings of v_{IN} to zero crossings of v_{OUT}. This time delay may then be converted to a phase shift. Unfortunately, v_{IN} is internal to the signal generator, and hence not measurable. However, the SYNC output of the generator and v_{IN} have the same zero crossings, which is why the SYNC output is used in this exercise as a reference for measuring phase shift.

Replace the 220 Ω resistor with the measured 1000 Ω resistor and repeat the exercise. Replace the 1000 Ω resistor with the measured 220 Ω resistor for the next exercise.

(3-5) In this exercise you will use the network from Figure 1 to filter a music signal.

Obtain a CD player, a powered speaker, and a 0.1-1.1 μF decade capacitor box from the instrument desk. Connect the 0.1-1.1 μF decade capacitor box in place of the 0.0047 μF capacitor in your circuit from the previous exercise. Use the knobs on the capacitor box to set the capacitance to 0.5 μF. Connect the speaker across the resistor so that you can listen to v_{OUT}. Turn on the CD player and powered speaker and adjust the volume so that you can hear the music.

Listen to the music and observe the signal v_{OUT} on the oscilloscope as you adjust the knobs on the capacitor box from 0.1 to 1.1 μF. What happens to the music when you vary the capacitance? Why?

Post-lab Exercises

Complete these exercises in your lab notebook using the responses which you calculated during the pre-lab exercises, and the data which you measured during the in-lab exercises. The primary goal of the post-lab exercises is to explain any discrepancies which may exist between the responses you calculated during the pre-lab exercises and the responses you measured during the in-lab exercises.

(3-1) Plot the recorded transient response data on the corresponding graph prepared during the pre-lab exercises. How well does the measured data match the theoretical graph? That is, how do they differ?

(3-2) Consider again the transient response. In chart form for the two cases of R, compare your calculated and measured voltage V_{TP}, frequency ω_T, and decay rate α_T. How do the measured and calculated parameters compare? Next, use the measured values of the network elements, including the parasitic resistance of the inductor, to re-calculate the parameters. Include the new parameters in the chart. Do the new parameters compare more favorably with the measured parameters? Do the new parameters explain the differences observed in Post-Lab Exercise 3-1?

(3-3) First, convert the amplitudes of v_{OUT} measured in sinusoidal steady state to measurements of $|H_S(\omega_S)|$ by dividing the measured amplitudes by the 10 V amplitude of v_{IN}. Second, convert the zero-crossing time differences measured in sinusoidal steady state to measurements of $\phi_S(\omega_S)$ by dividing the time differences by the corresponding waveform period, and then multiplying by 360 degrees. Finally, plot the frequency response data on the corresponding graph prepared during the pre-lab exercises. How well does the measured data match the theoretical graph? That is, how do they differ?

(3-4) Consider again the frequency response. In chart form for the two cases of R, compare your calculated and measured peak voltage ratio H_{SP}, frequency ω_{SP}, and Q. How do the measured and calculated parameters compare? Next, use the measured values of the network elements, including the parasitic resistance of the inductor, to re-calculate the parameters. Include the new parameters in the chart. Do the new parameters compare more favorably with the measured parameters? Do the new parameters explain the differences observed in Post-Lab Exercise 3-3?

Using MATLAB for Lab 3

You are encouraged, although not required, to use Matlab to plot the graphs in Pre-Lab Exercises 3-2 and 3-5. *Note: This document is provided specifically for these exercises. There are a number of resources for general help with Matlab on Athena. See http://web.mit.edu/olh/Matlab/.* To use Matlab, you must first type "add matlab" at the Athena prompt, and then invoke Matlab by typing the command "matlab" at the Athena prompt.

Pre-Lab Exercise (3-2)

At the Matlab prompt, enter the values for L, C, R, R_{IN}, and V_{TI}.

```
L = 47e-3;
C = 0.0047e-6;
R = 220;
RIN = 50;
VTI = 10;
```

Your ultimate goal is to plot the transient response of $v_{OUT}(t)$ for $0 \leq t \leq 0.3$ ms. In order to do this, you must generate a time vector `t`, and a voltage vector `vOUT`. Start by using the "`linspace`" command to generate a time vector `t` of evenly spaced time values between 0 and 0.3 ms.

```
t = linspace(0,0.3e-3,1000);
```

Type "`help linspace`" at the Matlab prompt for details on the "`linspace`" command. In Pre-Lab Exercise 3-1, you came up with an expression of the form $v_{OUT}(t) = V_{TO}e^{-\alpha_T t}\sin(\omega_T t + \phi_T)$. You will use this expression to generate the voltage vector `vOUT`. Enter the parameters V_{TO}, α_T, ω_T, and ϕ_T into Matlab in terms of `L, C, R, RIN` and `VTI`. After entering these parameters, you are ready to generate the voltage vector `vOUT`.

```
vOUT = VTO * exp(-1 * alphaT * t) .* sin(omegaT * t + phiT);
```

You should now have two vectors, `t` and `vOUT`, that you can use to plot the transient response. Use the "`plot`" command to generate a plot.

```
plot(t,vOUT);
```

You should use further commands to better format your graph. Try the commands "`grid`", "`title`", "`xlabel`", "`ylabel`", and "`axis`". For help with any Matlab command, type "`help` *command*" at the Matlab prompt.

Now repeat for `R = 1000`.

Pre-Lab Exercise (3-5)

This exercise asks you to create a log-log graph of H_S and a linear-log graph of ϕ_S. You will find this to be very easy with the help of Matlab. Start by creating two row vectors listing the numerator and denominator coefficients in descending powers of s. For example, consider the transfer function $H_S(s) = \frac{sR}{s^2 L + s(R+R_{IN}) + \frac{1}{C}}$. The following command creates a row vector `num` listing the numerator coefficients in descending powers of s.

```
num = [R 0];
```

Note the zero coefficient corresponding to s^0. Now create a row vector `den` for the denominator.

```
den = [L (R+RIN) (1/C)];
```

Now you can use the Matlab command "`tf`" to create a transfer function from the `num` and `den` row vectors.

```
HS = tf(num,den)
```

Next you want to generate a vector of frequencies for the x-axis of your magnitude and phase plots. Pre-Lab Exercise 3-5 asks you to create graphs over the range $2\pi\times 1\ \text{kHz} \le \omega_S \le 2\pi\times 100\ \text{kHz}$. Create a frequency vector of logarithmically spaced frequencies in this range.

```
omegaS = logspace(log10(2*pi*1e3), log10(2*pi*100e3), 1000);
```

For more information, type "`help logspace`" at the Matlab prompt. Now you can use the "`bode`" command to calculate the magnitude and phase at each of the frequencies specified in the `omegaS` vector.

```
[MAG,PHASE] = bode(HS, omegaS);
```

Pre-Lab Exercise 3-5 asks you to graph the magnitude on a log-log plot. You can do this in Matlab using the "`loglog`" command.

```
loglog(omegaS/(2*pi*10e3),MAG(:));
```

Now you have a magnitude graph. Note that the x-axis is normalized to 10 kHz as specified in Pre-Lab Exercise 3-5. As with the previous exercise, you should format your graph with the commands "`grid`", "`title`", "`xlabel`", "`ylabel`", etc. The pre-lab asks for a linear-log graph of the phase. Use the "`semilogx`" command to generate a linear-log graph.

```
semilogx(omegaS/(2*pi*10e3),PHASE(:));
```

Now you are finished. After formatting your graph, repeat the exercise for `R = 1000`.