6.002 Quiz #1 Review Packet

This material was compiled from previous 6.002 terms. Thanks to all authors.

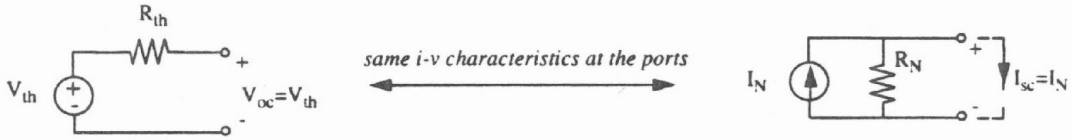
by Jason Kim

1. Primitive Elements

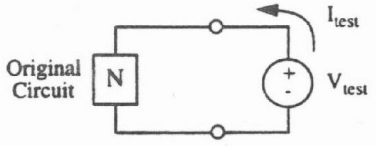
Resistor: time domain: $V = IR$ freq. domain: $Z = R$

- Resistor Network
  - N-unknowns & N-equations (most primitive approach)
  - Node Analysis (KCL, KVL)
    - *KCL: Sum of all currents entering and leaving a node is zero.
    - *KVL: Sum of all voltages around a closed loop path is zero. (be careful with polarities)
    - 1) *Label*(!) all current directions and voltage polarities. (Remember, currents flow from + to -)
    - 2) write KCL & KVL Eq.
    - 3) subst. and solve.
  - Simplification (by inspection)

- Thevenin Equivalent Circuit Model — Norton Equivalent Circuit Model



* Three variables: $V_{th}=V_{oc}$, $R_{th}=R_N$, and $I_N=I_{sc}$. They are related by $V_{oc}=I_{sc}*R_{th}$.
* $V_{th}=V_{oc}$: Leave the port open(thus no current flow at the port) and solve for V_{oc}. For a resistive network, this gives you a point on the V-axis of the i-v plot.
* $I_N=I_{sc}$: Short the port(thus no voltage across the port) and solve for I_{sc} flowing out of + and into -. For a resistive network, this gives you a point on the I-axis of the i-v plot.
* R_{th}: Set all sources to zero, except dependent sources. Solve the resistive network. When setting sources to zero, V source becomes short and I source becomes open. R_{th} may also be found by attaching I_{test} and V_{test} and setting $R_{th} = (V_{test})/(I_{test})$. If dependent sources are present, set only the independent sources to zero and attach I_{test} and V_{test}. Use KCL and KVL to find the expression $(V_{test})/(I_{test})=R_{th}$.



note the direction of I_{test} and polarity of V_{test}

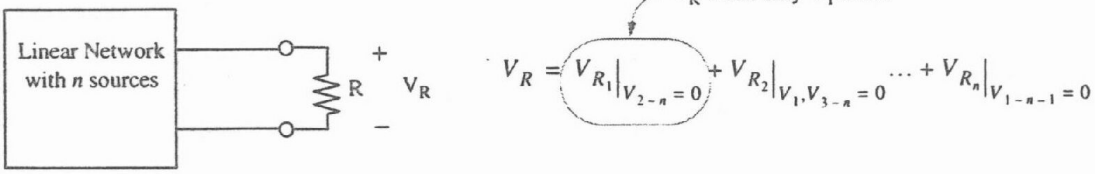
$$R_{th} = \frac{V_{test}}{I_{test}}$$

* For both Thevenin & Norton E.C.M.'s, they have the same power consumption *at the port* as the original circuit, but *not* for its individual components. Power dissipated at R_{th} does *not* equal power dissipated at the resistors of the original circuit. Same holds for the power delivered by the sources in the Thevenin model and the original circuit. Thevenin and Norton E.C.M.'s are for terminal i-v characteristics *only*.

- Power = Real Power = $IV = I^2R = V^2/R$ (energy dissipated as heat)
- $\langle Power \rangle = \frac{1}{T}\int_0^T I(t)V(t)dt$

- Superposition

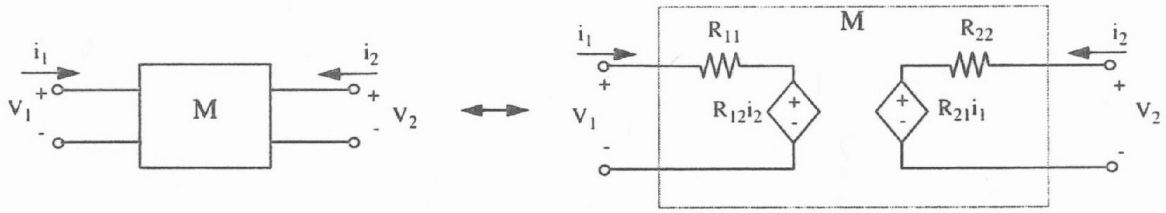
"In a linear network with a number of independent sources, the response can be found by summing the response to each independent sources acting alone, with all other independent sources set to zero."



1) Leave one source on and turn off all other sources.
   (Voltage source "off" = short & Current source "off" = open)
2) Find the effect from the "on" source.
3) Repeat for each sources.
4) Sum the effect from each sources to obtain the total effect.

For cases where a dependent source is present along with multiple sources, DO NOT turn off the dependent source. Leave the dependent source on and carry it in your expressions. Tackle the dependent source term last by solving linear equations. Remember that the variable which the dependent source is depended on is affected by the individual independent sources that you are turning on and off.

- Multiport Network



* By attaching V_1, I_1 and V_2,I_2 at the ports,

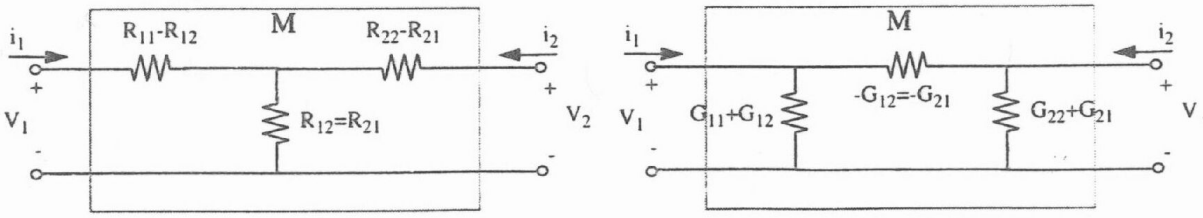
$$V_1 = R_{11}I_1 + R_{12}I_2$$
$$V_2 = R_{21}I_1 + R_{22}I_2$$

The first subscript denote the place(port) of voltage measurement.
The second subscript denote the place(port) of current source.
Thus, R_{12} means the resistance V_1/I_2 when voltage V_1 is measured at open port 1 and I_2 current source is placed at port 2.

* If all the Z-parameters(R_{xx}'s) are identical, then the networks can't be differentiated using only the i-v characteristics at the ports.

* Reciprocity: $R_{12}=R_{21}$.

* Using the definitions from above,

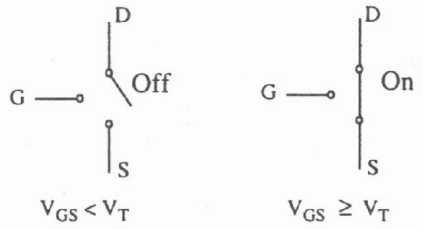


T model Pi model

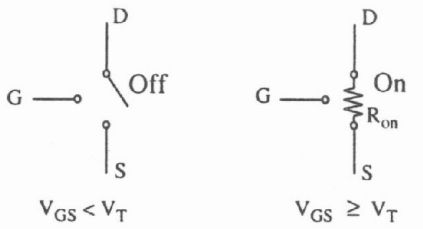
4. MOSFET Transistor

Large Signal Model

S Model:

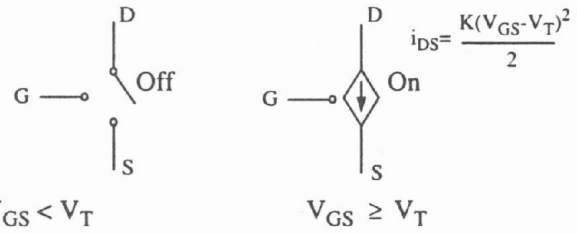


SR Model:



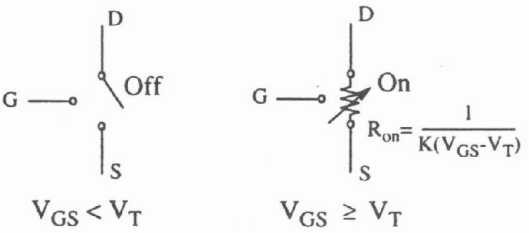
When ($V_{DS} \geq V_{GS} - V_T$),

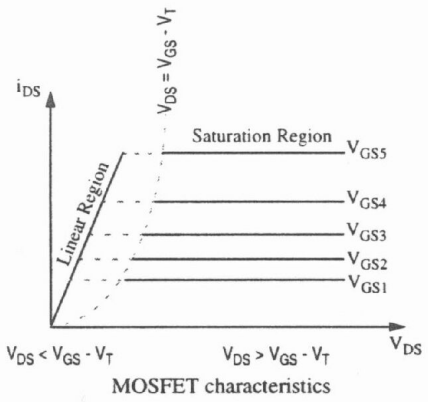
SCS Model *(Saturation)*:



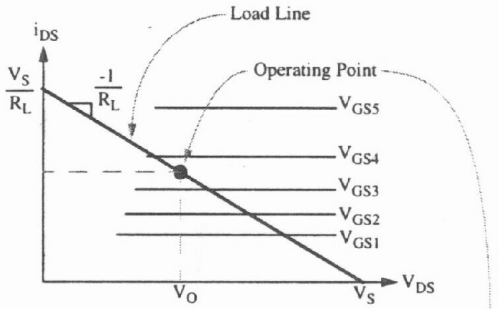
When (VDS < VGS - VT),

SVR Model (Linear):



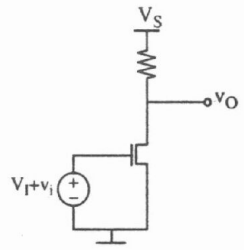


MOSFET characteristics

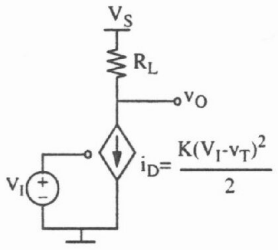


Load Line superimposed on MOS characteristics for operating point analysis of MOSFET Amplifier

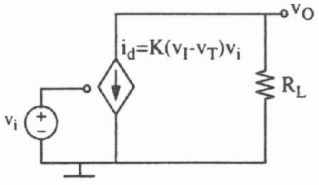
5. MOSFET Amplifier



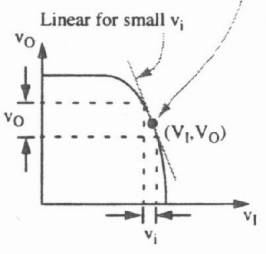
MOSFET Amplifier



Large Signal Model(SCS)



Small Signal Model



Operating Point@V_I

Small Signal gain: $\frac{v_o}{v_i} = -K(V_I - v_T)R_L = -g_m R_L$ *Note that the output is a function of both V_I(DC) and v_i(AC).

This problem studies the use of a mythical MOSFET-like device called a ZFET to construct an amplifier as shown below. The ZFET operates in its saturation region when $v_{GS} \geq 0$ and $v_{DS} > 0$. In this region, the drain-source terminal relation is $i_{DS} = Kv_{GS}^3$, where K is a constant having units of A/V^3. When $v_{DS} = 0$, the ZFET exhibits a short circuit between its drain and source terminals, and is said to operate outside its saturation region. Similarly, the ZFET exhibits an open circuit for $v_{GS} < 0$ as it again operates outside its saturation region. Finally, the gate terminal always exhibits an open circuit. These characteristics are summarized in the figure below, beneath the symbol for the ZFET.

(A) Assuming saturated operation of the ZFET, determine v_{OUT} as a function of v_{IN}.

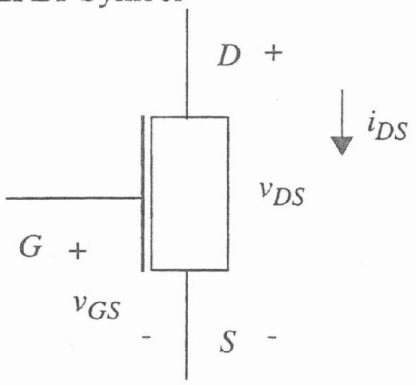
(B) Over what range of v_{IN} will the ZFET operate in its saturation region?

(C) Assume that $V_S = 10$ V, $R_L = 1$ kΩ and $K = 0.001$ A/V^3. Sketch and clearly label v_{OUT} as a function of v_{IN} for -1 V $\leq v_{IN} \leq 3$ V on the axes provided on the answer sheet.

(D) Given the parameters of Part (C), can the amplifier can be used as an inverter that provides a valid output high voltage threshold of $V_{OH} = 7$ V. Why or why not? Assume that $V_{IH} = 6$ V and $V_{IL} = 2$ V.

(E) Assume that the amplifier is biased at an input voltage V_{IN} such that the ZFET exhibits saturated operation; the corresponding bias output voltage is V_{OUT}. For this case, derive the small-signal voltage gain v_{out}/v_{in} of the amplifier.

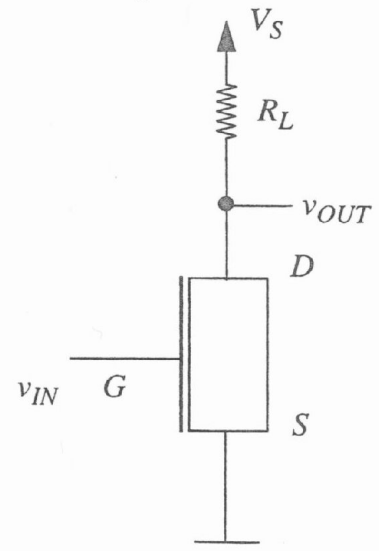
ZFET Symbol



$i_{DS} = Kv_{GS}^3$

For $v_{GS} \geq 0$
$v_{DS} > 0$

ZFET Amplifier



The circuit shown below delivers a nearly constant current to its load despite the fact that the power supply is noisy. The noise is modeled by the small signal v_s superimposed on the constant supply voltage V_S. Thus, V_S and v_s are the large-signal and small-signal components of the total power supply voltage v_S, respectively. I_L and i_l are the large-signal and small-signal components of the load current i_L, respectively. The noise v_s in the power supply voltage satisfies $v_s \ll V_S$, and is responsible for the presence of i_l in i_L.

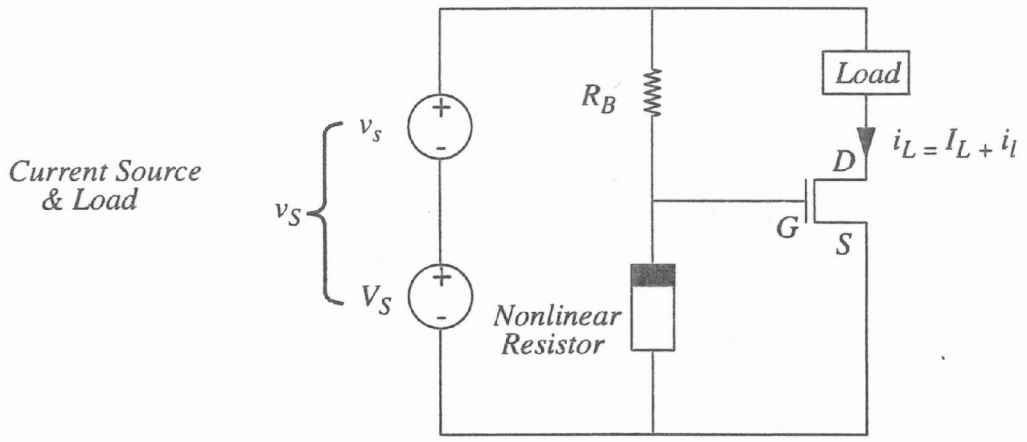
The current source contains a MOSFET which operates in its saturation region such that $i_{DS} = \frac{K}{2}(v_{GS} - v_T)^2$. The current source also contains a nonlinear resistor whose terminal characteristics are described graphically below. Assume that $V_S > V_N > v_T$.

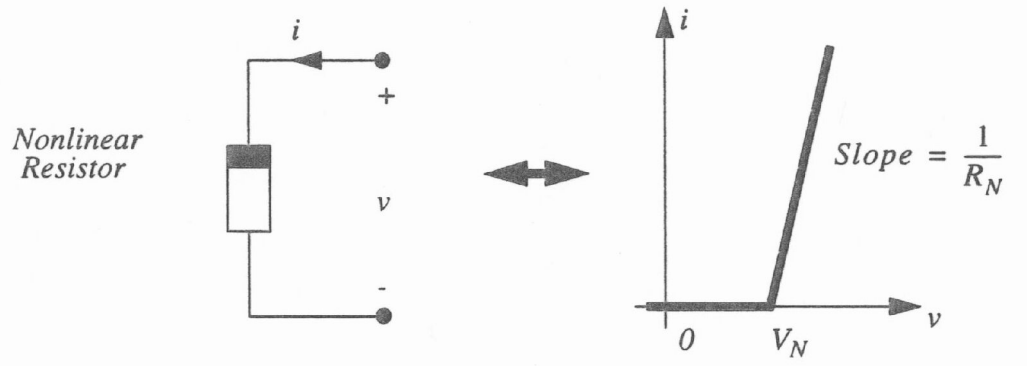
(A) Assume $v_s = 0$. Determine V_{GS}, the large-signal component of v_{GS}, in terms of R_B, R_N, V_N and V_S.

(B) Following the result of Part (A), determine I_L in terms of R_B, R_N, V_N, V_S, K and v_T.

(C) Now assume that $v_s \neq 0$. Draw a small-signal circuit model for the combined circuit comprising the power supply, current source and load, with which i_l can be found from v_s. Clearly label the value of each component in the circuit model.

(D) Using the small-signal model from Part (C), determine the ratio i_l/v_s.





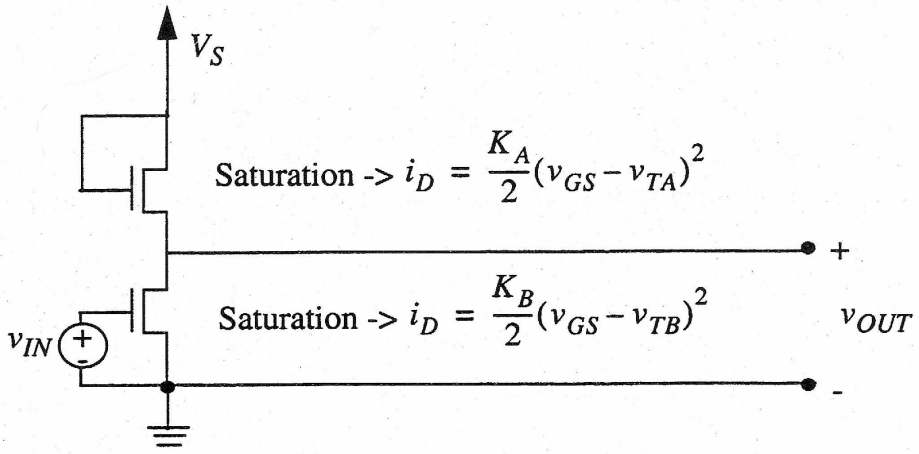
A "linear" MOSFET amplifier may be constructed using two MOSFETs as shown below. Note that the transconductances K_A and K_B and the threshold voltages V_{TA} and V_{TB} of the two MOSFETs are different.

(A) Assuming that both MOSFETs operate in their saturation regions, determine v_{OUT} as a function of v_{IN}.

(B) Determine the range of v_{IN} over which both MOSFETs operate in their saturated regions.

(C) Determine the range of v_{OUT} over which both MOSFETs operate in their saturated regions.

(D) What is the small-signal gain of the amplifier over the range found in Parts (B) and (C)?

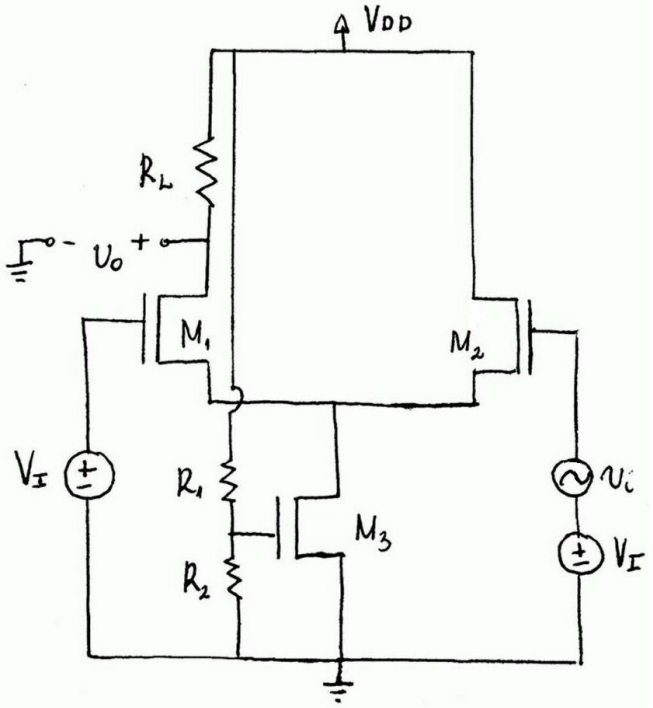


All transistors have the same Vt and K.

a) Assume that K, Vt, R_L, R2, and V_I are all given. Find the range for R1 so that all transistors are biased in saturation region. Also, assume that v_i =0.

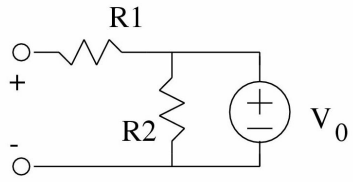
What is the corresponding range of the output voltage?
What function M1, M2 and M3 transistors have in the circuit?

b) Find the small signal gain in terms of K, Vt, R_L, R1, R2 and V_I.

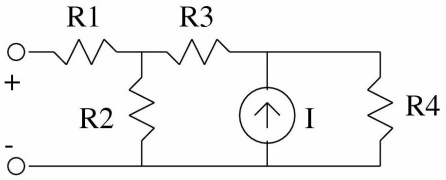


Question: Find Thevenin and Norton equivalents for each of the following circuits.

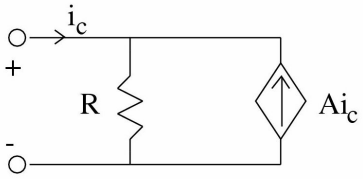
a.)



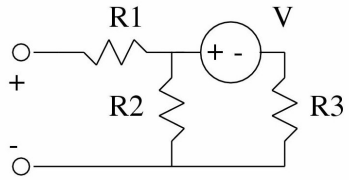
b.)



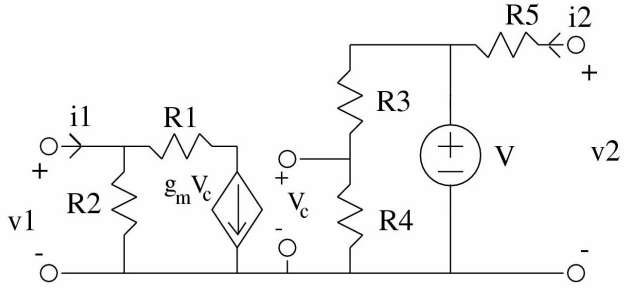
c.)

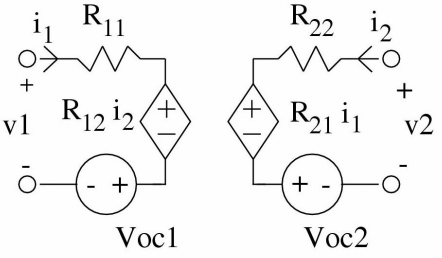


d.)

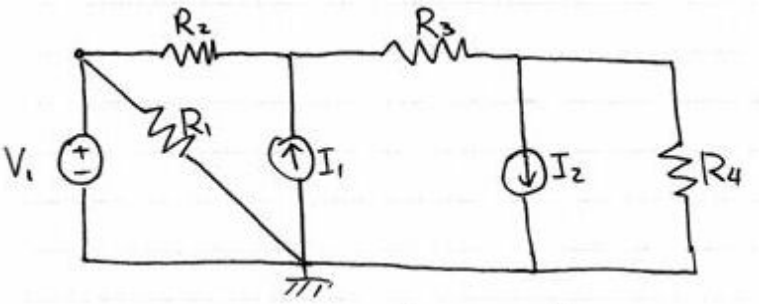
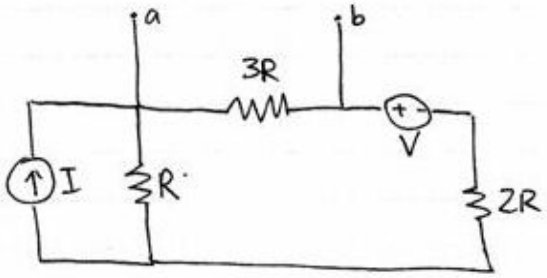


e.) Find the general 2 port equivalent model for the following circuit.





a) How many node-to-reference voltages are unknown?
b) Write node equations to solve for those nodes. (Do not solve.)



Find the Thevenin equivalent at terminals ab.