6.002 - Fall 2003
Corrections to Homework #1 Solutions
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Exercise 1.1

The solution distributed for Exercise 1.1 reversed the signs conventionally assigned to power. By convention, power is positive when an element is dissipating power, and negative when an element is delivering power. From the notes (Chapter 1.5):

> By our associated variables convention, the product $p = iv$ yields the power supplied *into* the element.

Part A (Section on Power only)

Power supplied into each element:

$$\begin{aligned}
P_V = -iV &= \frac{-V^2}{R_1+R_2+R_3} \\
P_{R_1} = iV_1 &= \frac{VR_1}{R_1+R_2+R_3}\left(\frac{V}{R_1+R_2+R_3}\right) \\
P_{R_2} = iV_2 &= \frac{VR_2}{R_1+R_2+R_3}\left(\frac{V}{R_1+R_2+R_3}\right) \\
P_{R_3} = iV_3 &= \frac{VR_3}{R_1+R_2+R_3}\left(\frac{V}{R_1+R_2+R_3}\right)
\end{aligned}$$

Sum All:

$$P_{total} = \frac{-V^2}{R_1+R_2+R_3} + \frac{V^2R_1}{(R_1+R_2+R_3)^2} + \frac{V^2R_2}{(R_1+R_2+R_3)^2} + \frac{V^2R_3}{(R_1+R_2+R_3)^2} = 0$$

Part B (Section on Power only)

Power supplied into each element:

$$\begin{aligned}
P_I = -IV &= -I^2\left(\frac{R_1R_2R_3}{R_2R_3+R_1R_3+R_1R_2}\right) \\
P_{R_1} = i_1V &= \left[I\left(\frac{R_2R_3}{R_2R_3+R_1R_3+R_1R_2}\right)\right]\left[\frac{IR_1R_2R_3}{R_2R_3+R_1R_3+R_1R_2}\right] \\
P_{R_2} = i_2V &= \left[I\left(\frac{R_1R_3}{R_2R_3+R_1R_3+R_1R_2}\right)\right]\left[\frac{IR_1R_2R_3}{R_2R_3+R_1R_3+R_1R_2}\right] \\
P_{R_3} = i_3V &= \left[I\left(\frac{R_1R_2}{R_2R_3+R_1R_3+R_1R_2}\right)\right]\left[\frac{IR_1R_2R_3}{R_2R_3+R_1R_3+R_1R_2}\right]
\end{aligned}$$

Sum All:

$$P_{total} = \frac{-I^2R_1R_2R_3}{R_2R_3+R_1R_3+R_1R_2} + \left(\frac{I^2(R_1R_2R_3)}{R_2R_3+R_1R_3+R_1R_2}\right)\left(\frac{R_2R_3+R_1R_3+R_1R_2}{R_2R_3+R_1R_3+R_1R_2}\right) = 0$$

Problem 1.1

The solution distributed for Problem 1.1 had an incorrect sign for i_3, the current flowing through resistor R_3. Since the problem defined i_3 as flowing into the negative terminal of R_3, the following solutions are correct:

KVL:

$$\begin{aligned} V &= V_1 + V_2 \\ V_2 &= V_3 \end{aligned}$$

KCL:

$$\begin{aligned} i_1 + i_3 - i_2 &= 0 \\ i_1 &= i_2 - i_3 \end{aligned}$$

Solve for i_3 in terms of i_1:

$$\begin{aligned} i_2 R_2 &= -i_3 R_3 \\ i_2 &= -i_3 \frac{R_3}{R_2} \\ i_1 = -i_3 \frac{R_3}{R_2} - i_3 &= -i_3 \frac{R_2 + R_3}{R_2} \\ i_3 &= -i_1 (\frac{R_2}{R_2 + R_3}) \end{aligned}$$

Solve for i_1 in terms of V:

$$\begin{aligned} V &= i_1 R_1 + i_1 (\frac{R_2 R_3}{R_2 + R_3}) \\ V &= i_1 (\frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2 + R_3}) \\ i_1 &= V (\frac{R_2 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}) \end{aligned}$$

Solve for i_3 in terms of V:

$$i_3 = -i_1 (\frac{R_2}{R_2 + R_3}) = -V (\frac{R_2 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3})(\frac{R_2}{R_2 + R_3}) = \frac{-V R_2}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$