Lecture 10

MOSFET (III)

MOSFET Equivalent Circuit Models

Outline

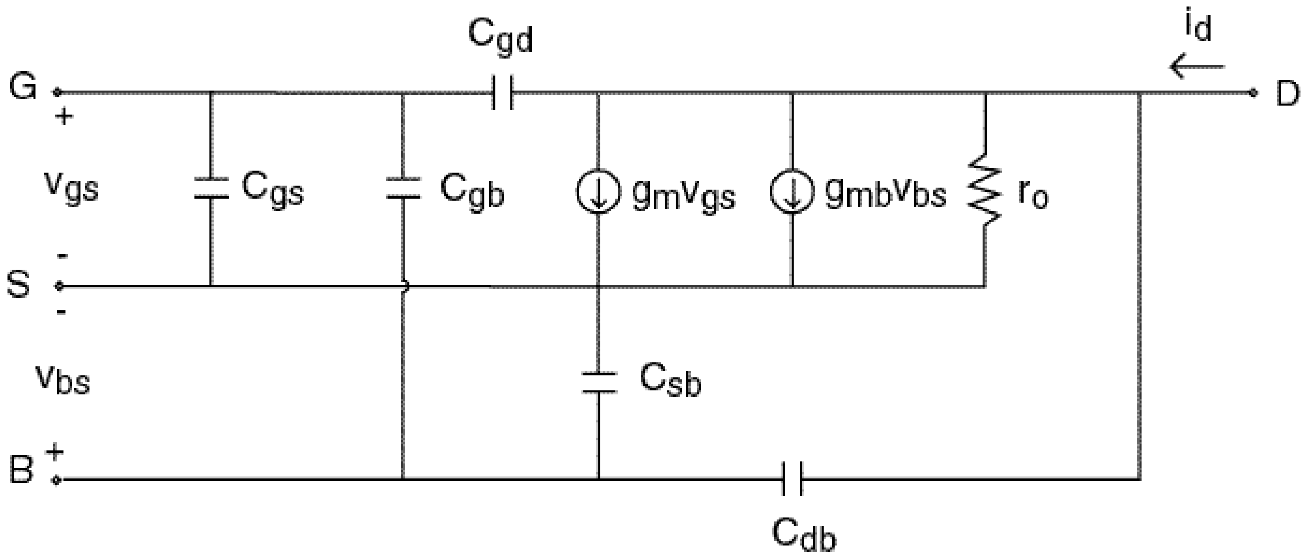
- Low-frequency small-signal equivalent circuit model
- High-frequency small-signal equivalent circuit model

Reading Assignment:

Howe and Sodini; Chapter 4, Sections 4.5-4.6

Summary of Key Concepts

High-frequency small-signal equivalent circuit model of MOSFET



In saturation:

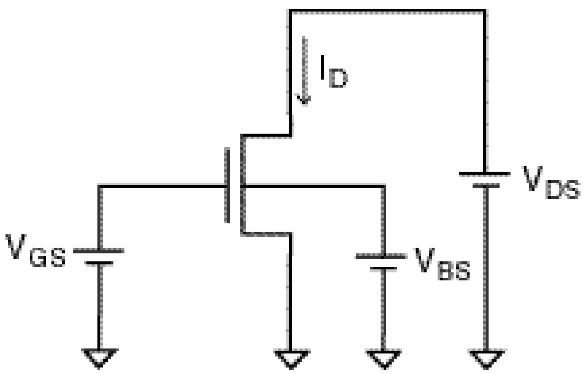
$$g_m \propto \sqrt{\frac{W}{L} I_D}$$

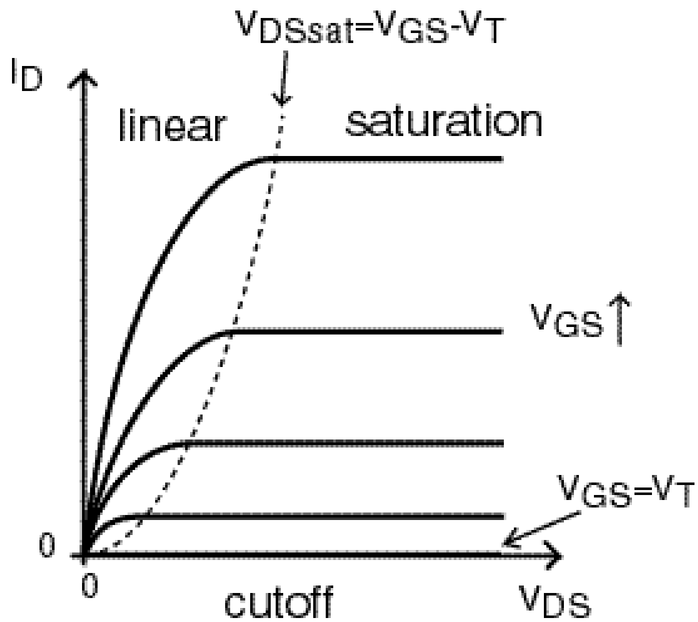
$$r_o \propto \frac{L}{I_D}$$

$$C_{gs} \propto WLC_{ox}$$

1. Low-frequency small-signal equivalent circuit model

Regimes of operation of MOSFET:





- Cut-off

$$\mathbf{I_D} = 0$$

- Linear / Triode:

$$\mathbf{I_D} = \frac{\mathbf{W}}{\mathbf{L}} \mu_{\mathbf{n}} \mathbf{C_{ox}} \left[\mathbf{V_{GS}} - \frac{\mathbf{V_{DS}}}{2} - \mathbf{V_T} \right] \bullet \mathbf{V_{DS}}$$

- Saturation

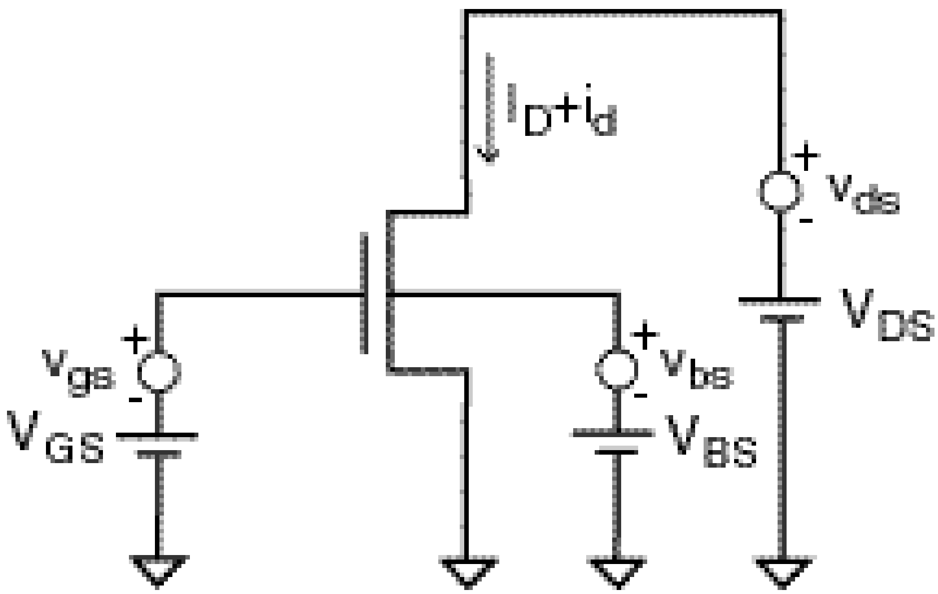
$$\mathbf{I_D} = \mathbf{I_{Dsat}} = \frac{\mathbf{W}}{2\mathbf{L}} \mu_{\mathbf{n}} \mathbf{C_{ox}} [\mathbf{V_{GS}} - \mathbf{V_T}]^2 \bullet [1 + \lambda (\mathbf{V_{DS}} - \mathbf{V_{DSsat}})]$$

Effect of back bias

$$\mathbf{V_T}(\mathbf{V_{BS}}) = \mathbf{V_{To}} + \gamma \left[\sqrt{-2\phi_{\mathbf{p}} - \mathbf{V_{BS}}} - \sqrt{-2\phi_{\mathbf{p}}} \right]$$

Small-signal device modeling

In many applications, we are only interested in the response of device to ***small-signal*** applied on top of a bias.



Key Points:

- Small-signal is ***small***
  - ⇒ response of non-linear components becomes linear
- Since response is linear, ***superposition*** can be used
  - ⇒ effects of different small signals are independent from each other.

Mathematically:

$$\mathbf{i_D(V_{GS}, V_{DS}, V_{BS}; v_{gs}, v_{ds}, v_{bs}) \approx}$$

$$\mathbf{I_D\left(V_{DS}, V_{DS}, V_{BS}\right) + i_d(v_{gs}, v_{ds}, v_{bs})}$$

With i_d linear on small-signal drives:

$$\mathbf{i_d \approx g_m v_{gs} + g_o v_{ds} + g_{mb} v_{bs}}$$

Define:

$g_m \equiv$ ***transconductance*** [S]
$g_o \equiv$ ***output or drain conductance*** [S]
$g_{mb} \equiv$ ***backgate transconductance*** [S]

Approach to computing g_m, g_o, and g_{mb}.

$$\mathbf{g_m \approx \left.\frac{\P i_D}{\P v_{GS}}\right|_Q}$$

$$\mathbf{g_o \approx \left.\frac{\P i_D}{\P v_{DS}}\right|_Q}$$

$$\mathbf{g_{mb} \approx \left.\frac{\P i_D}{\P v_{BS}}\right|_Q}$$

$$\mathbf{Q \equiv [v_{GS} = V_{GS}, v_{DS} = V_{DS}, v_{BS} = V_{BS}]}$$

Transconductance

In saturation regime:

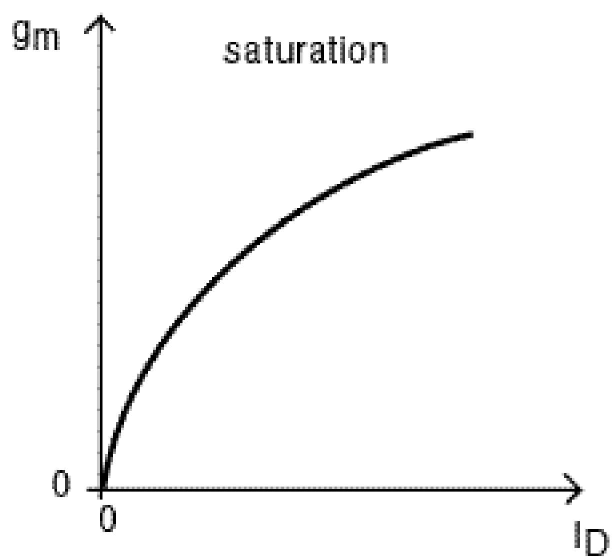
$$\mathbf{i_D} = \frac{\mathbf{W}}{2\mathbf{L}} \mu_\mathbf{n} \mathbf{C_{ox}} \left[\mathbf{v_{GS}} - \mathbf{V_T}\right]^2 \bullet \left[1 + \lambda\left(\mathbf{v_{DS}} - \mathbf{V_{DSsat}}\right)\right]$$

Then (neglecting channel length modulation) the transconductance is:

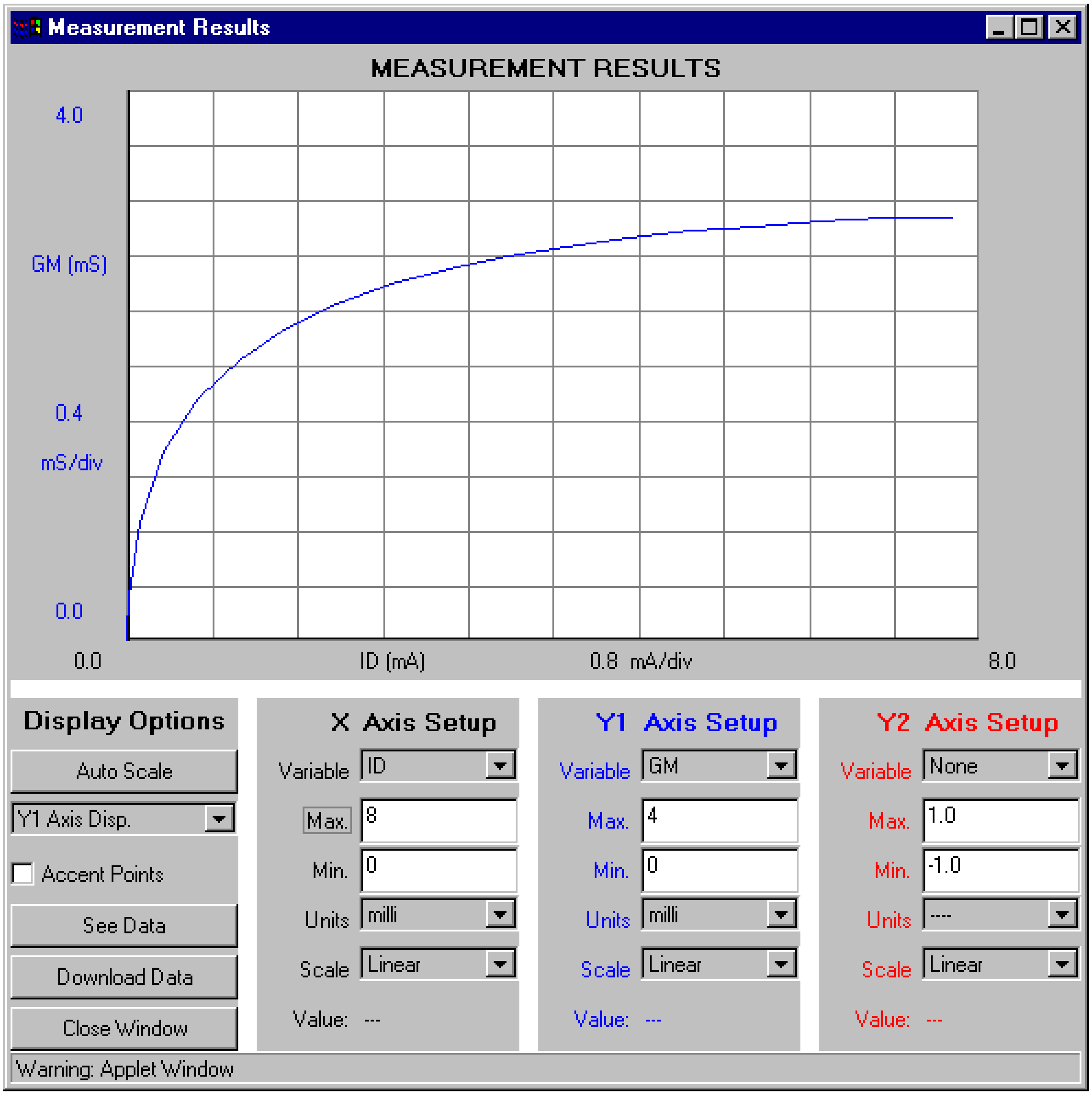
$$\mathbf{g_m} = \left.\frac{\partial \mathbf{i_D}}{\partial \mathbf{v_{GS}}}\right|_\mathbf{Q} \approx \frac{\mathbf{W}}{\mathbf{L}} \mu_\mathbf{n} \mathbf{C_{ox}} \left(\mathbf{V_{GS}} - \mathbf{V_T}\right)$$

Rewrite in terms of I_D:

$$\mathbf{g_m} = \sqrt{2 \frac{\mathbf{W}}{\mathbf{L}} \mu_\mathbf{n} \mathbf{C_{ox}} \mathbf{I_D}}$$

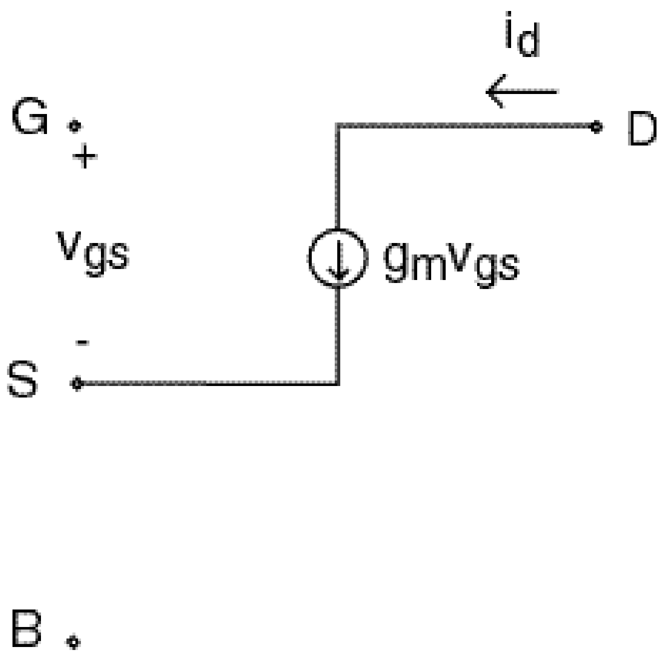


Transconductance (contd.)



Transconductance (contd.)

Equivalent circuit model representation of g_m:



Output conductance

In saturation regime:

$$\mathbf{i_D} = \frac{\mathbf{W}}{2\mathbf{L}} \mu_{\mathbf{n}} \mathbf{C_{ox}} \left[\mathbf{v_{GS}} - \mathbf{V_T}\right]^2 \bullet \left[1 + \lambda\left(\mathbf{v_{DS}} - \mathbf{V_{DSsat}}\right)\right]$$

Then:

$$\mathbf{g_o} = \left.\frac{\partial \mathbf{i_D}}{\partial \mathbf{v_{DS}}}\right|_{\mathbf{Q}} = \frac{\mathbf{W}}{2\mathbf{L}} \mu_{\mathbf{n}} \mathbf{C_{ox}} \left(\mathbf{V_{GS}} - \mathbf{V_T}\right)^2 \bullet \lambda \approx \lambda \mathbf{I_D}$$

Output resistance is the inverse of output conductance:

$$\mathbf{r_o} = \frac{1}{\mathbf{g_o}} = \frac{1}{\lambda \mathbf{I_D}}$$

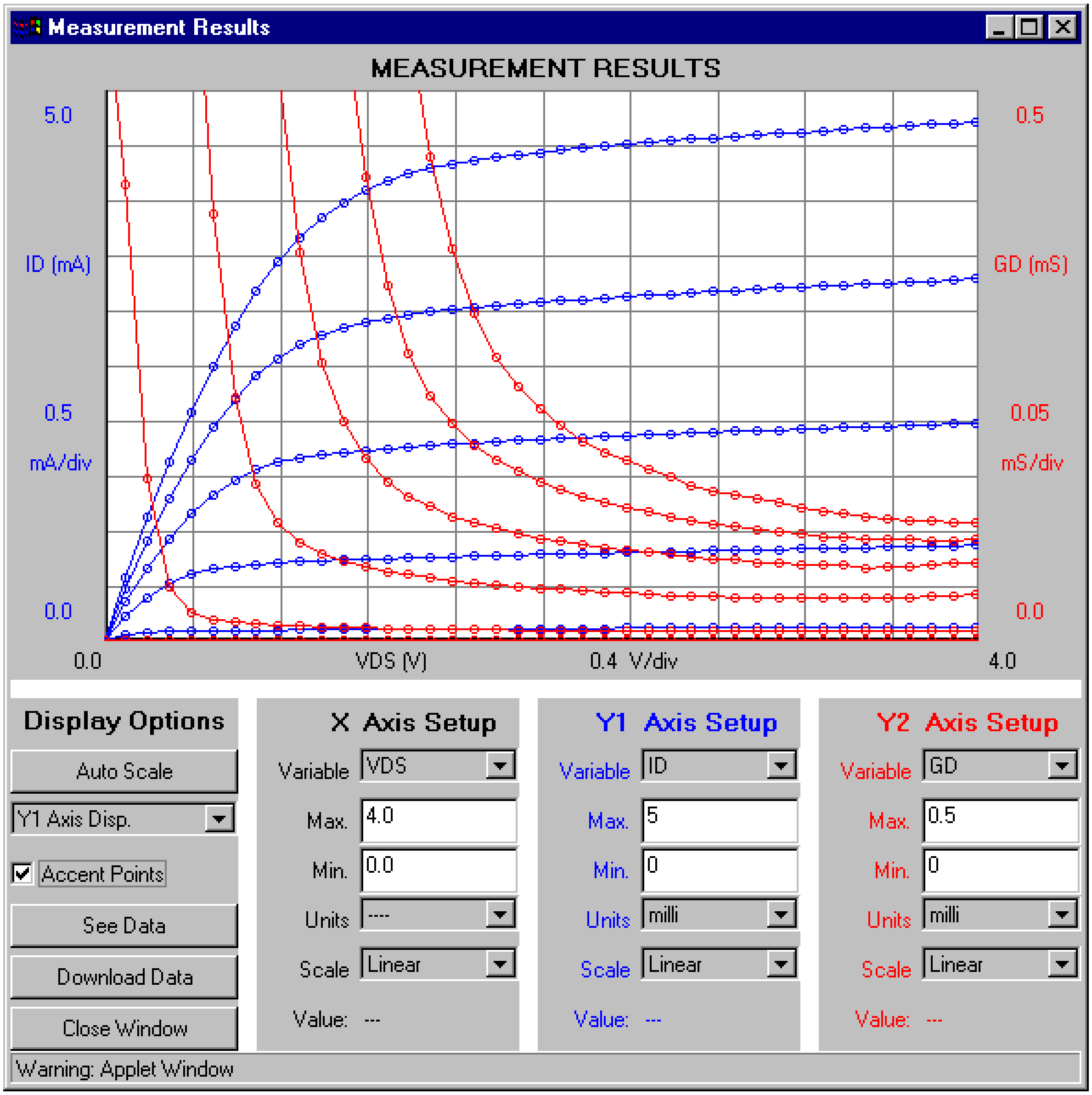
Remember also:

$$\lambda \propto \frac{1}{\mathbf{L}}$$

Hence:

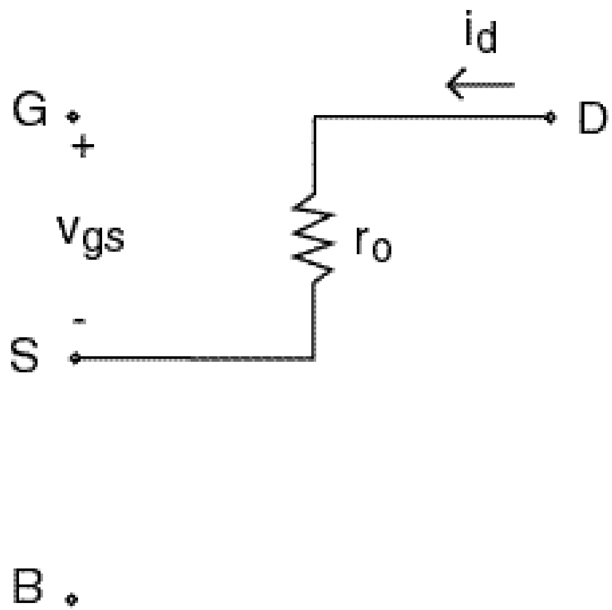
$$\mathbf{r_o} \propto \mathbf{L}$$

Output conductance (contd.)



Output conductance (contd.)

Equivalent circuit model representation of g_o:



Backgate transconductance

In saturation regime (neglect channel length modulation):

$$\mathbf{i_D} \approx \frac{\mathbf{W}}{2\mathbf{L}}\mu_{\mathbf{n}}\mathbf{C_{ox}}\left[\mathbf{v_{GS}} - \mathbf{V_T}\right]^2$$

Then:

$$\mathbf{g_{mb}} = \left.\frac{\partial \mathbf{i_D}}{\partial \mathbf{v_{BS}}}\right|_{\mathbf{Q}} = -\frac{\mathbf{W}}{\mathbf{L}}\mu_{\mathbf{n}}\mathbf{C_{ox}}\left(\mathbf{V_{GS}} - \mathbf{V_T}\right)\bullet\left(\left.\frac{\partial \mathbf{V_T}}{\partial \mathbf{v_{BS}}}\right|_{\mathbf{Q}}\right)$$

Since:

$$\mathbf{V_T}(\mathbf{v_{BS}}) = \mathbf{V_{To}} + \gamma\left[\sqrt{-2\phi_{\mathbf{p}} - \mathbf{v_{BS}}} - \sqrt{-2\phi_{\mathbf{p}}}\right]$$

Then :

$$\left.\frac{\partial \mathbf{V_T}}{\partial \mathbf{v_{BS}}}\right|_{\mathbf{Q}} = \frac{-\gamma}{2\sqrt{-2\phi_{\mathbf{p}} - \mathbf{V_{BS}}}}$$

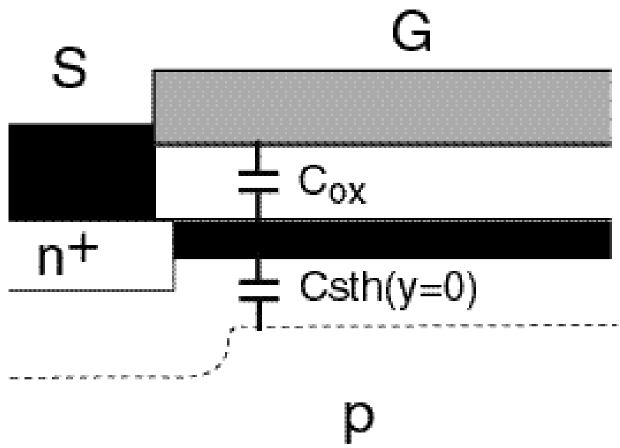
Hence:

$$\mathbf{g_{mb}} = \frac{\gamma\,\mathbf{g_m}}{2\sqrt{-2\phi_{\mathbf{p}} - \mathbf{V_{BS}}}}$$

Backgate transconductance (contd.)

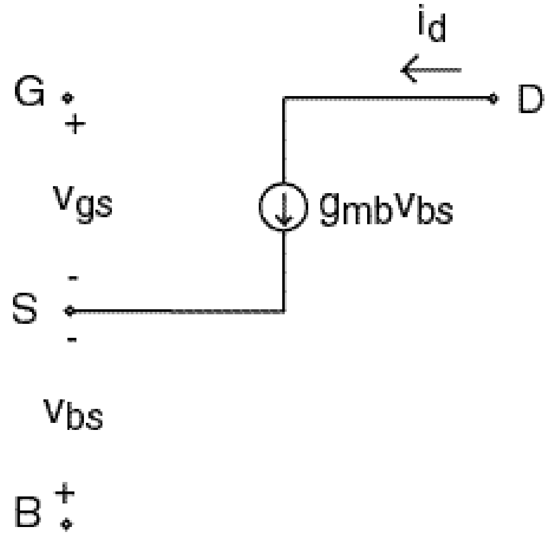
Another way to write this (see book):

$$\frac{g_{mb}}{g_m} = \frac{C_b(y=0)}{C_{ox}}$$

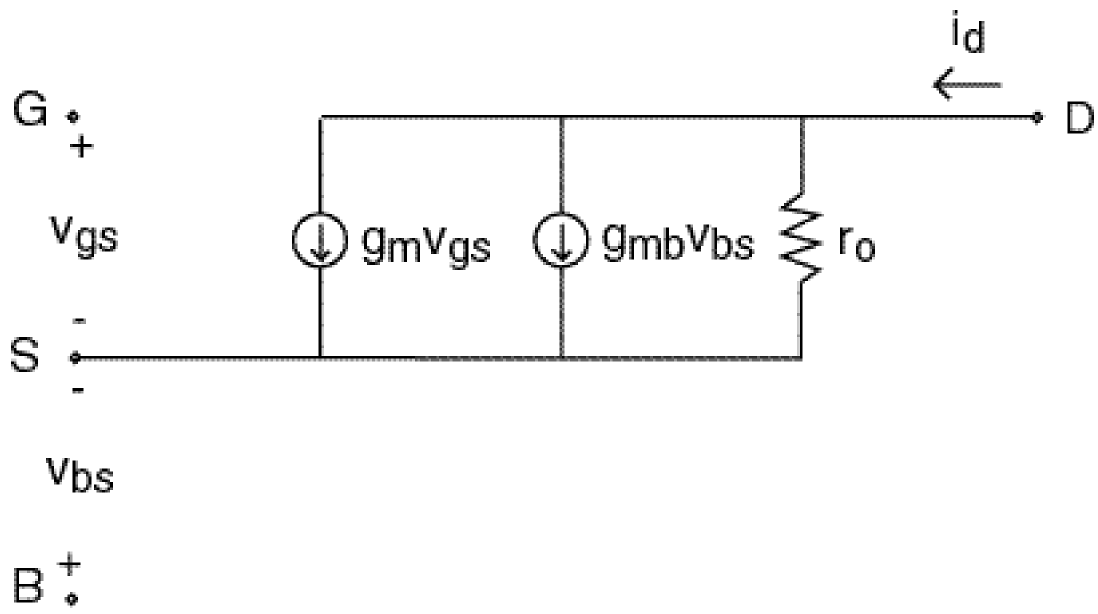


Ratio of backgate transconductance to transconductance is equal to the ratio of depletion capacitance at source to oxide capacitance.

Equivalent circuit representation of g_{mb}:

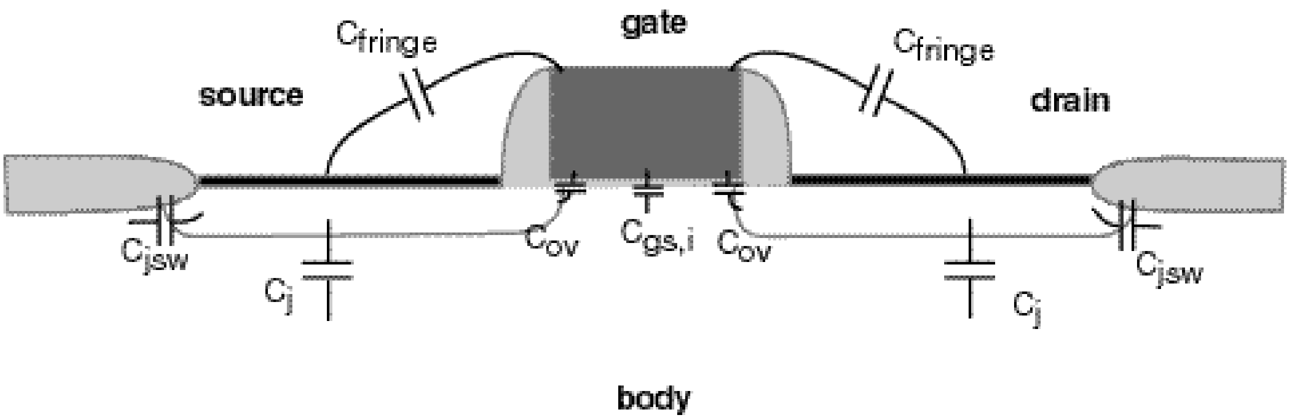


Complete MOSFET small-signal equivalent circuit model for low frequency:



2. High-frequency small-signal equivalent circuit model

Need to add capacitances. In saturation:



$C_{gs} \equiv$ channel charge + overlap capaciatance, C_{ov}

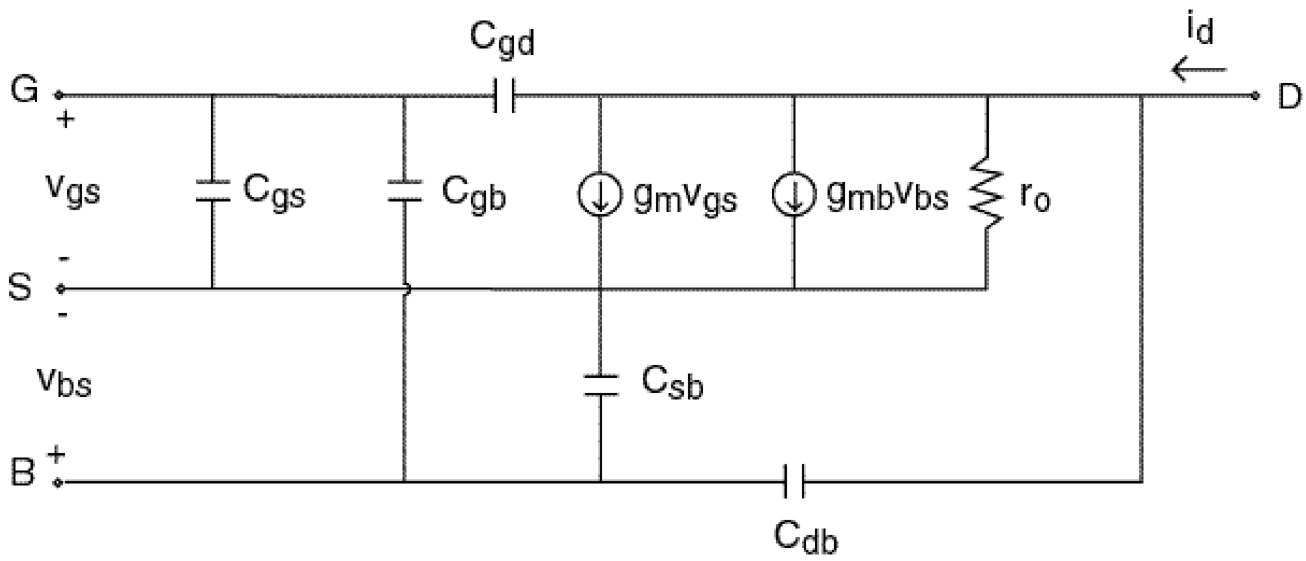
$C_{gd} \equiv$ overlap capacitance, C_{ov}

$C_{gb} \equiv$ only parasitic capacitance

$C_{sb} \equiv$ source junction depletion capacitance (+sidewall)

$C_{db} \equiv$ source junction depletion capacitance (+sidewall)

Complete MOSFET high-frequency small-signal equivalent circuit model:



Inversion layer charge in saturation

$$q_N(v_{GS}) = W\int_0^L Q_n(y)\,dy = W\int_0^{v_{GS}-V_T} Q_n(y)\bullet\frac{dy}{dv_C}\bullet dv_C$$

Note that q_N is total inversion charge in the channel & $v_C(y)$ is the channel voltage. But:

$$\frac{dv_C}{dy} = -\frac{i_D}{W\mu_n Q_n(y)}$$

Then:

$$q_N(v_{GS}) = -\frac{W^2 L\mu_n}{i_D}\bullet\int_0^{V_{GS}-V_T}[Q_n(y)]^2\bullet dv_C$$

Remember:

$$Q_n(v_C) = -C_{ox}[v_{GS} - v_C(y) - V_T]$$

Then:

$$q_N(v_{GS}) = -\frac{W^2 L\mu_n}{i_D}\bullet\int_0^{v_{GS}-V_T}[v_{GS} - v_C(y) - V_T]^2\bullet dv_C$$

Inversion layer charge in saturation (contd.)

Do integral, substitute i_D in saturation and get:

$$q_N(v_{GS}) = -\frac{2}{3} WLC_{ox}(v_{GS} - V_T)$$

Gate charge:

$$q_G(v_{GS}) = -q_N(v_{GS}) - Q_{B,max}$$

Intrinsic gate-to-source capacitance:

$$C_{gs,i} = \frac{dq_G}{dv_{GS}} = \frac{2}{3} WLC_{ox}$$

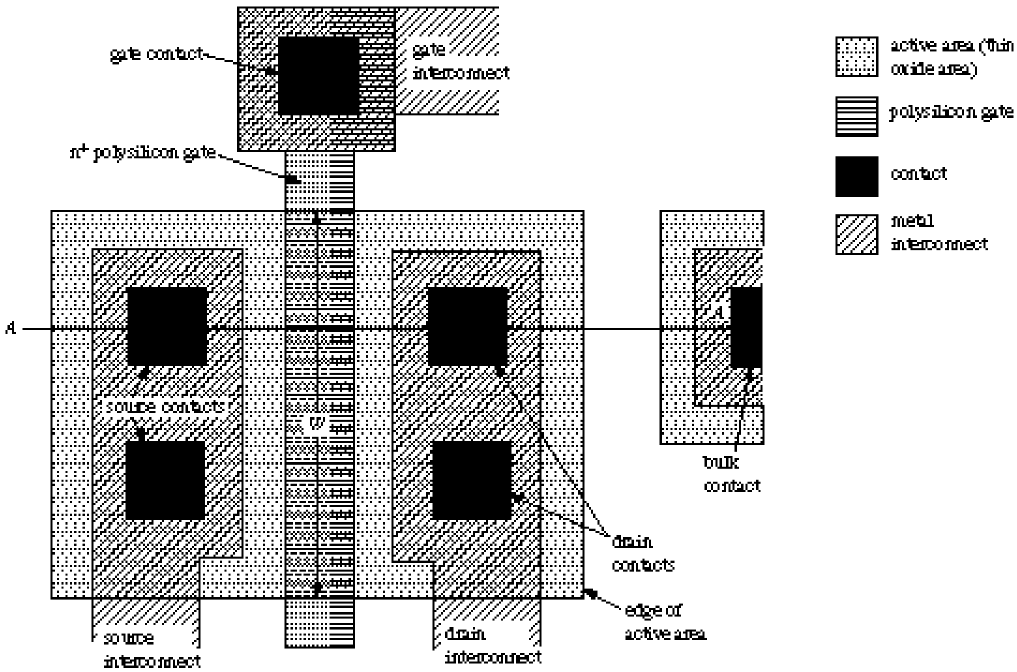
Must add overlap capaciatnce:

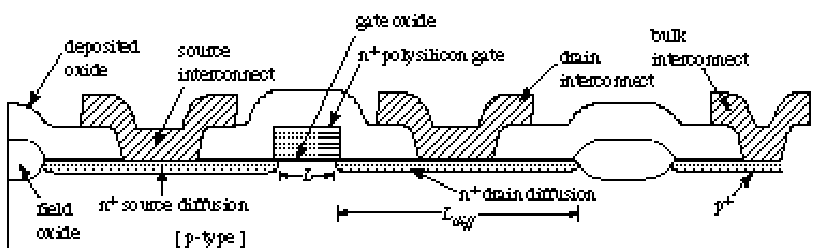
$$C_{gs} = \frac{2}{3} WLC_{ox} + WC_{ov}$$

Gate-to-drain capacitance — only overlap capacitance:

$$C_{gd} = WC_{ov}$$

Other capacitances





Body-to-source capacitance = source junction capacitance:

$$C_{bs} = C_j + C_{jsw} = WL_{diff}\sqrt{\frac{q\epsilon_s N_a}{2(\phi_B - V_{BS})}} + (2L_{diff} + W)C_{JSW}$$

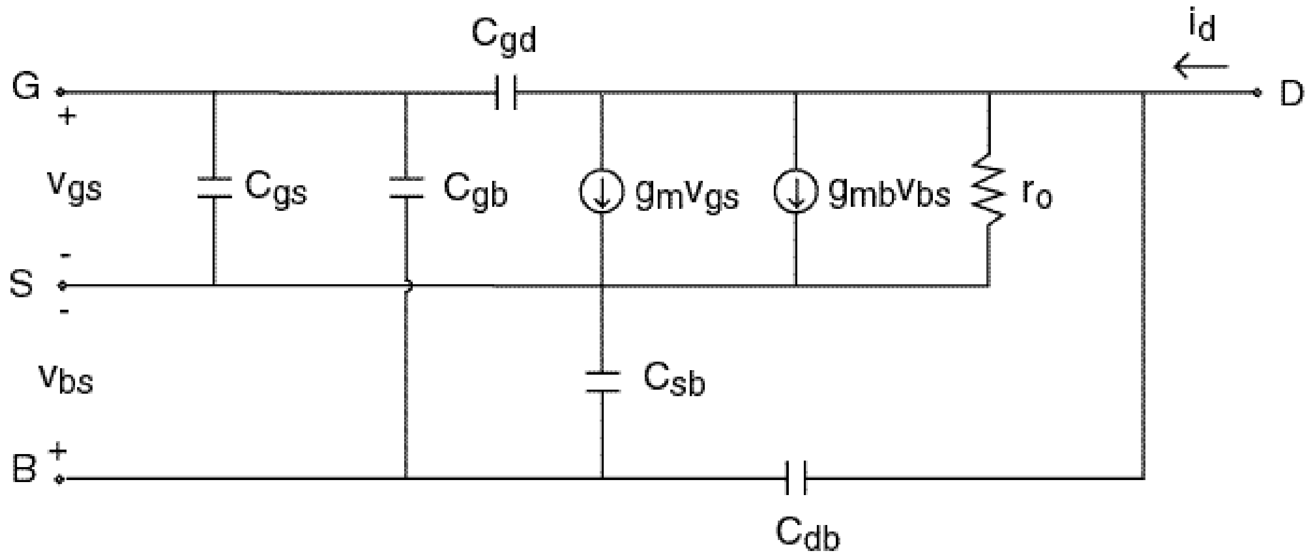
Body-to-drain capacitance = drain junction capacitance:

$$C_{bd} = C_j + C_{jsw} = WL_{diff}\sqrt{\frac{q\epsilon_s N_a}{2(\phi_B - V_{BS})}} + (2L_{diff} + W)C_{JSW}$$

What did we learn today?

Summary of Key Concepts

High-frequency small-signal equivalent circuit model of MOSFET



In saturation:

$$g_m \propto \sqrt{\frac{W}{L} I_D}$$

$$r_o \propto \frac{L}{I_D}$$

$$C_{gs} \propto WLC_{ox}$$