Lecture 15

The pn Junction Diode (II)

Equivalent Circuit Model

Outline

- I-V characteristics (contd.)
- Small-signal equivalent circuit model
- Carrier charge storage
    - –Diffusion capacitance

Reading Assignment:

Howe and Sodini; Chapter 6, Sections 6.4, 6.5, 6.9

Summary of Key Concepts

Small-signal behavior of diode:

- ***Conductance***: associated with current-voltage characteristics
  - – g_d I in forward bias,
  - – g_d negligible in reverse bias

- ***Junction capacitance***: associated with charge modulation in depletion region

$$C_j \quad \frac{1}{\sqrt{\varphi_B - V}}$$

- ***Diffusion capacitance***: associated with charge storage in QNRs to maintain quasi-neutrality.

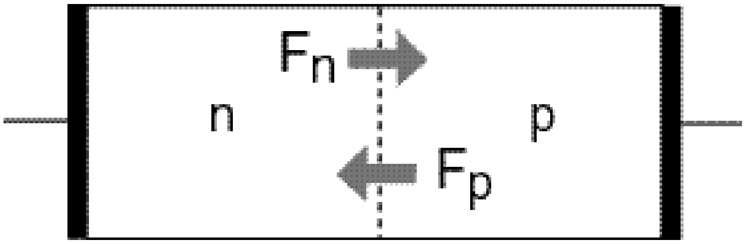
$$C_d \quad \exp\left[\frac{qV}{kT}\right]$$

1. I-V Characteristics (contd.)

Diode Current equation:

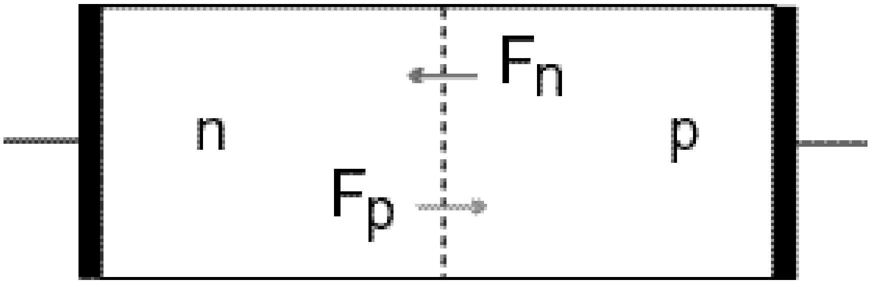
$$I = I_o\left[\exp\left(\frac{qV}{kT}\right) - 1\right]$$

Physics of forward bias:



- Potential drop across SCR reduced by V
  - – minority carrier injection in QNRs
- Minority carrier diffusion through QNRs
- Minority carrier recombination at contacts to the QNRs (and surfaces)
- Large supply of carriers injected into the QNRs

– $I \quad \exp\left[\frac{qV}{kT}\right]$



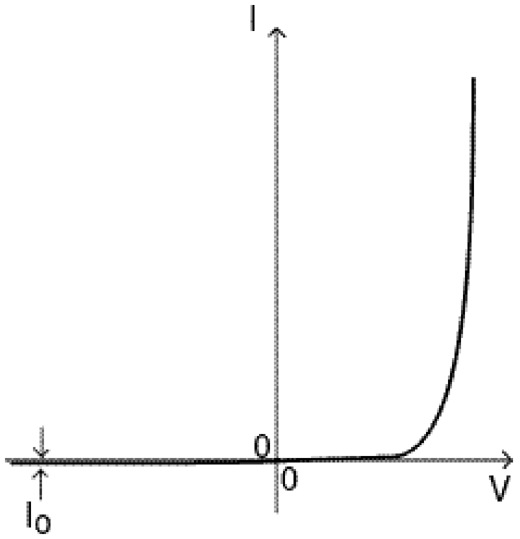
Physics of reverse bias:

- Potential drop across SCR increased by V
  - minority carrier extraction from QNRs
- Minority carrier diffusion through QNRs
- Minority carrier generation at surfaces & contacts of QNRs
- Very small supply of carriers available for extraction
  - I saturates to small value

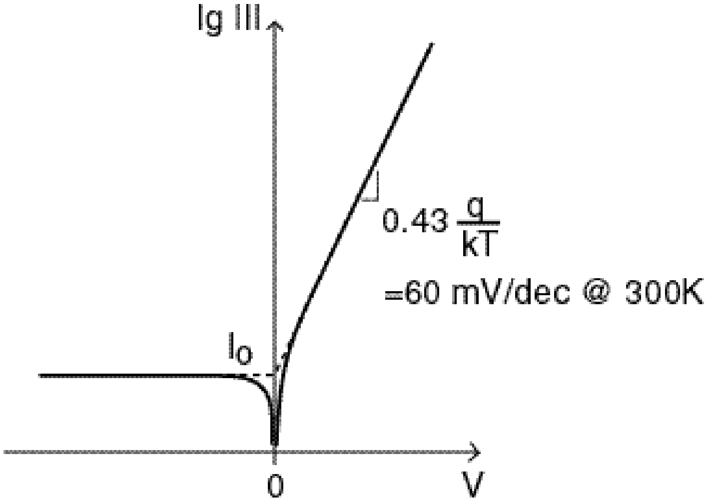
I-V Characteristics (contd)

Diode Current equation:

$$I = I_o\left[\exp\left(\frac{qV}{kT}\right) - 1\right]$$

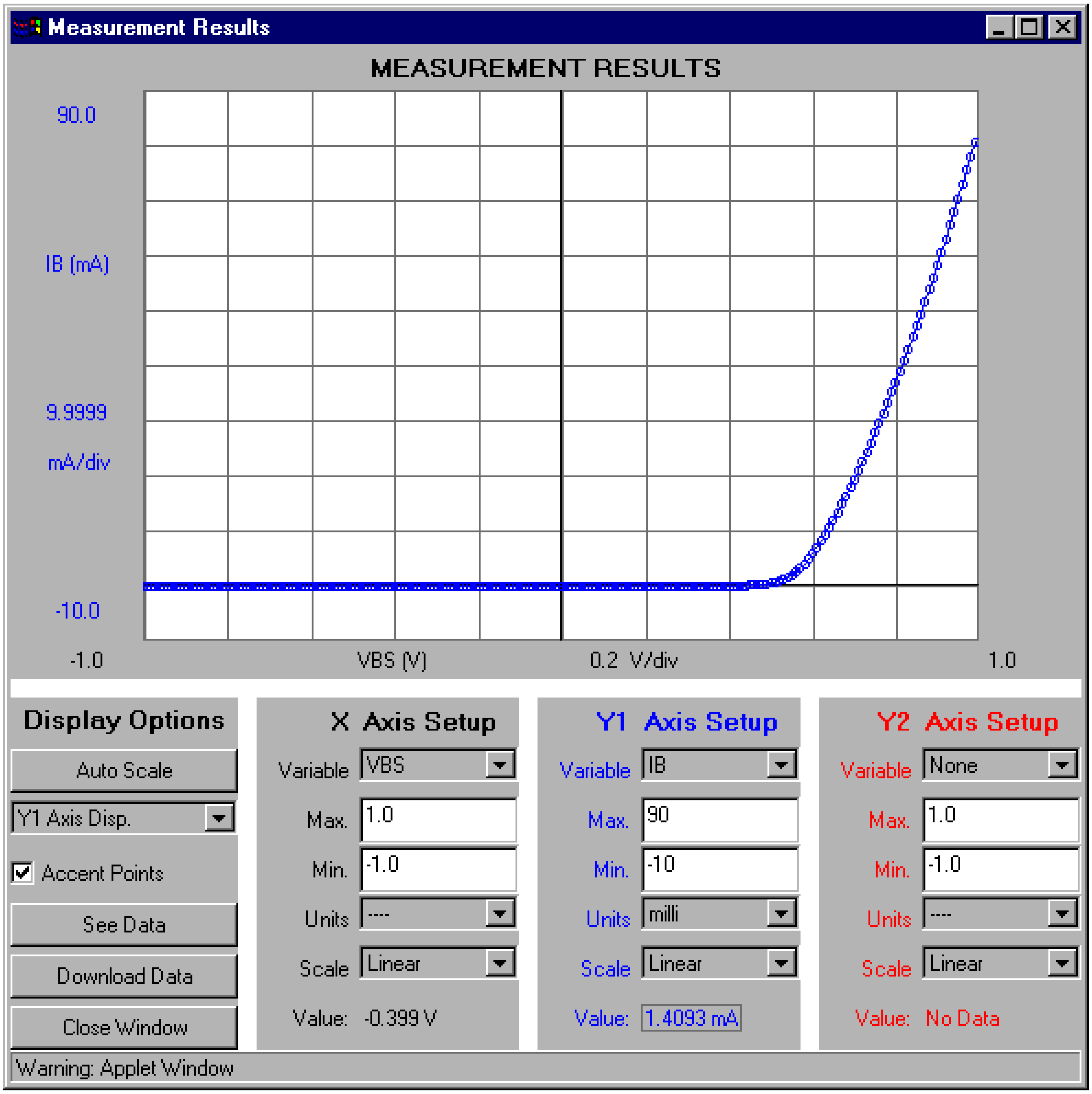


linear scale



semilogarithmic scale

Body diode of NMOSFET (linear scale)



Display Options

X Axis Setup

Variable: VBS
Max.: 1.0
Min.: -1.0
Units: ----
Scale: Linear
Value: -0.399 V

Y1 Axis Setup

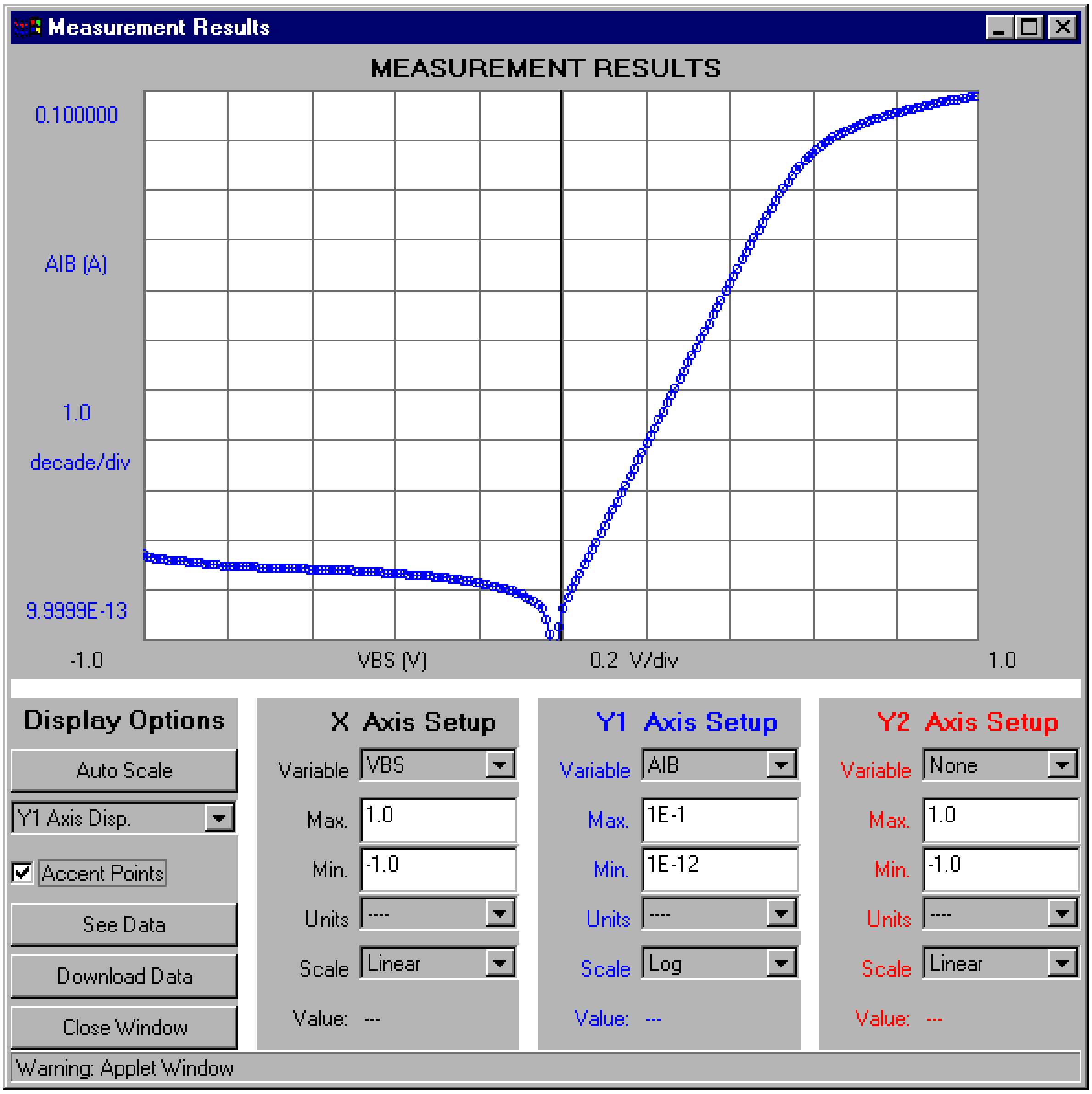
Variable: IB
Max.: 90
Min.: -10
Units: milli
Scale: Linear
Value: 1.4093 mA

Y2 Axis Setup

Variable: None
Max.: 1.0
Min.: -1.0
Units: ----
Scale: Linear
Value: No Data

Warning: Applet Window

Body diode of NMOSFET (logarithmic scale)



	X Axis Setup	Y1 Axis Setup	Y2 Axis Setup
Variable	VBS	AIB	None
Max.	1.0	1E-1	1.0
Min.	-1.0	1E-12	-1.0
Units	----	----	----
Scale	Linear	Log	Linear
Value:	---	---	---

AIB is $|I_B|$, absolute of the body current

Key dependencies of diode current:

$$I = qAn_i^2\left(\frac{1}{N_a}\cdot\frac{D_n}{W_p - x_p} + \frac{1}{N_d}\cdot\frac{D_p}{W_n - x_n}\right)\cdot\left[\exp\left(\frac{qV}{kT}\right) - 1\right]$$

- $I \quad \frac{n_i^2}{N}\left[\exp\left(\frac{qV}{kT}\right) - 1\right] \equiv$ ***excess*** minority carrier concentration at the edge of SCR
  - In forward bias: $I \quad \frac{n_i^2}{N}\exp\left(\frac{qV}{kT}\right)$: the more carriers are injected, the more current flows
  - In reverse bias: $I \quad -\frac{n_i^2}{N}$: when all minority carriers are removed, the current saturates

- $I \quad D$: faster diffusion   more current

- $I \quad \frac{1}{W_{QNR}}$: shorter region to diffuse through   more current

- $I \quad A$: bigger diode   more current

2. Small-signal equivalent circuit model

Examine effect of small signal overlapping bias:

$$I + i = I_o\left[\exp\left(\frac{q(V+v)}{kT}\right) - 1\right]$$

If v small enough, linearize exponential characteristics:

$$I + i \approx I_o\left[\exp\left(\frac{qV}{kT}\right)\exp\left(\frac{qv}{kT}\right) - 1\right] \approx I_o\left[\exp\left(\frac{qV}{kT}\right)\left(1 + \frac{qv}{kT}\right) - 1\right]$$

$$= I_o\left[\exp\left(\frac{qV}{kT}\right) - 1\right] + I_o \exp\left(\frac{qV}{kT}\right)\frac{qv}{kT}$$

Then:

$$i = \frac{q(I + I_o)}{kT} \cdot v$$

From a small signal point of view. Diode behaves as ***conductance*** of value:

$$g_d = \frac{q(I + I_o)}{kT}$$

Small-signal equivalent circuit model, so far:

g_d depends on bias. In forward bias:

$$g_d \approx \frac{qI}{kT}$$

g_d is linear in diode current.

Must add capacitance associated with depletion region:

Depletion or junction capacitance:

$$C_j = A\sqrt{\frac{q\varepsilon_s N_a N_d}{2(N_a + N_d)(\varphi_B - V)}}$$

can rewrite as:

$$C_j = A\sqrt{\frac{q\varepsilon_s N_a N_d}{2(N_a + N_d)\varphi_B}} \cdot \sqrt{\frac{\varphi_B}{(\varphi_B - V)}}$$

or,

$$C_j = \frac{C_{jo}}{\sqrt{1 - \frac{V}{\varphi_B}}}$$

where:

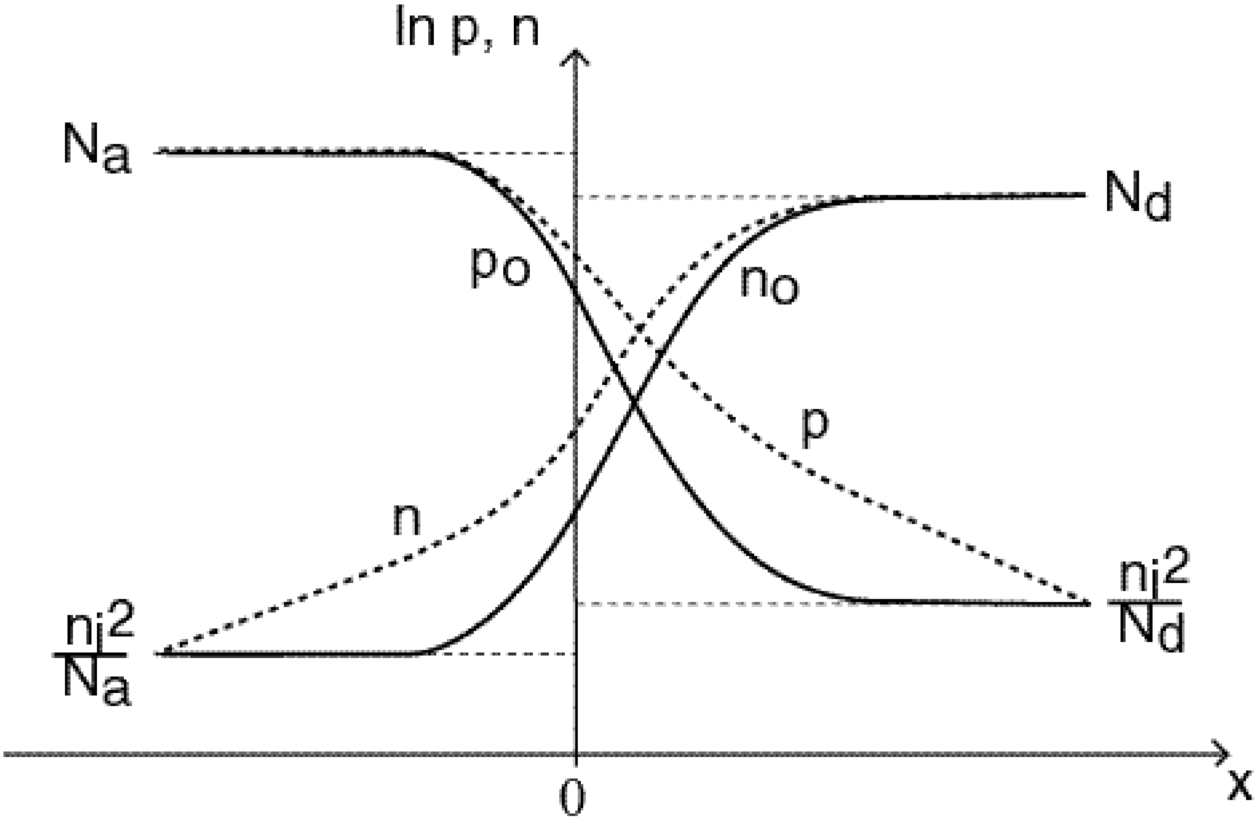
$C_{jo} \equiv$ ***zero-voltage junction capacitance***

3. Carrier charge storage:

diffusion capacitance

What happens to majority carriers?

Carrier picture thus far:

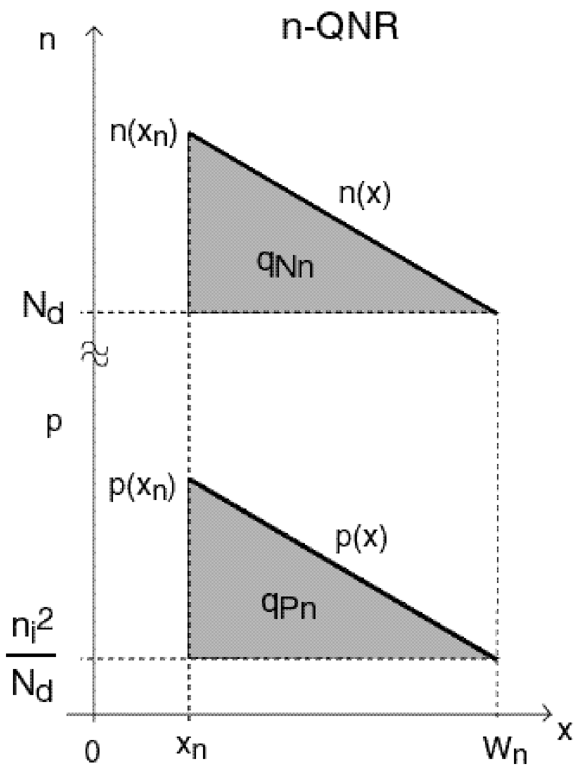


If QNR minority carrier concentration ↑ but majority carrier concentration unchanged quasi-neutrality is violated.

Quasi-neutrality demands that at every point in QNR:

excess minority carrier concentration

= excess majority carrier concentration



Mathematically:

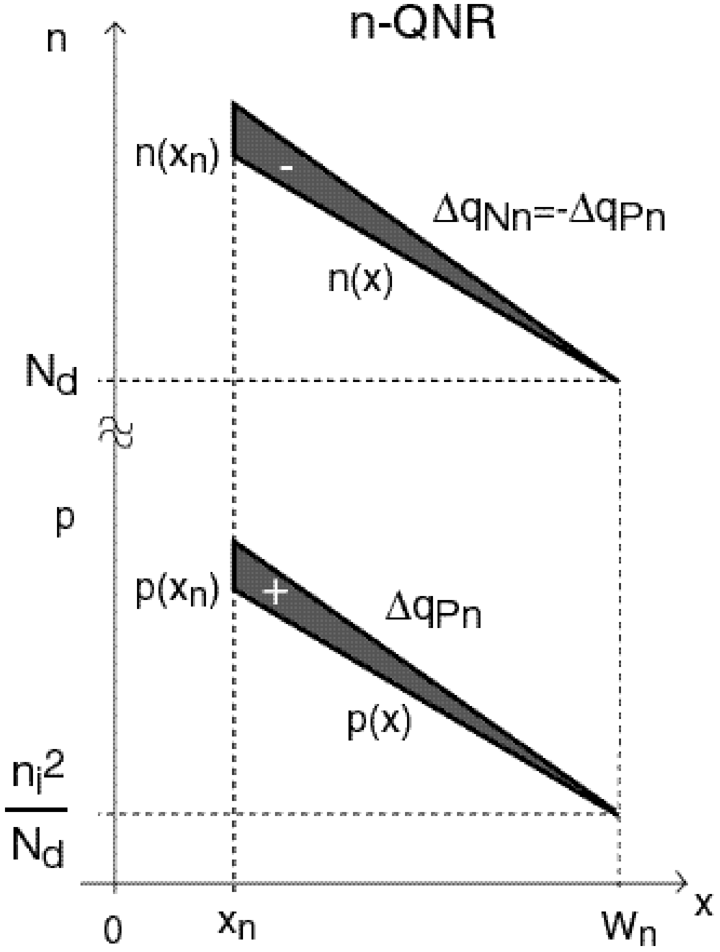
$$p'(x) = p(x) - p_o \approx n'(x) = n(x) - n_o$$

Define integrated carrier charge:

$$q_{Pn} = qA\frac{1}{2}p'(x_n)\cdot\left(W_n - x_n\right) =$$

$$= qA\frac{W_n - x_n}{2}\frac{n_i^2}{N_d}\exp\left[\frac{qV}{kT} - 1\right] = -q_{Nn}$$

Now examine small increase in V:



Small increase in V small increase in q_{Pn} small increase in $|q_{Nn}|$

Behaves as capacitor of capacitance:

$$C_{dn} = \frac{dq_{Pn}}{dV} = qA\frac{W_n - x_n}{2}\frac{n_i^2}{N_d}\frac{q}{kT}\exp\left[\frac{qV}{kT}\right]$$

Can write in terms of I_p (portion of diode current due to holes in n-QNR):

$$C_{dn} = \frac{q}{kT}\frac{(W_n - x_n)^2}{2D_p} qA \frac{n_i^2}{N_d}\frac{D_p}{W_n - x_n}\exp\left[\frac{qV}{kT}\right]$$

$$\approx \frac{q}{kT}\frac{(W_n - x_n)^2}{2\,D_p} I_p$$

Define ***transit time*** of holes through n-QNR:

$$\tau_{Tp} = \frac{(W_n - x_n)^2}{2D_p}$$

Transit time is the ***average time for a hole to diffuse through n-QNR*** [will discuss in more detail in BJT]

Then:

$$C_{dn} \approx \frac{q}{kT} \cdot \tau_{Tp} \cdot I_p$$

Similarly for p-QNR:

$$C_{dp} \approx \frac{q}{kT} \cdot \tau_{Tn} \cdot I_n$$

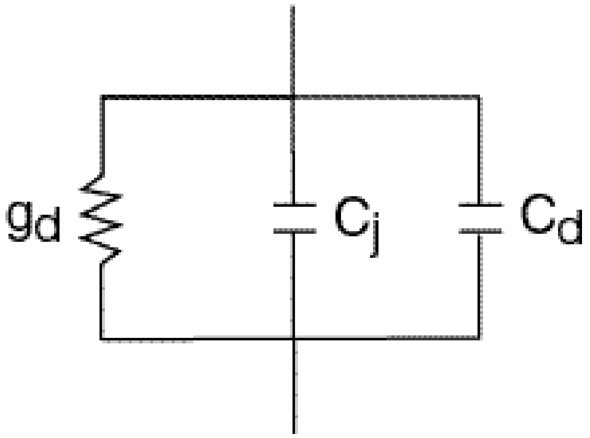
where τ_{Tn} is ***transit time*** of electrons through p-QNR:

$$\tau_{Tn} = \frac{\left(W_p - x_p\right)^2}{2D_n}$$

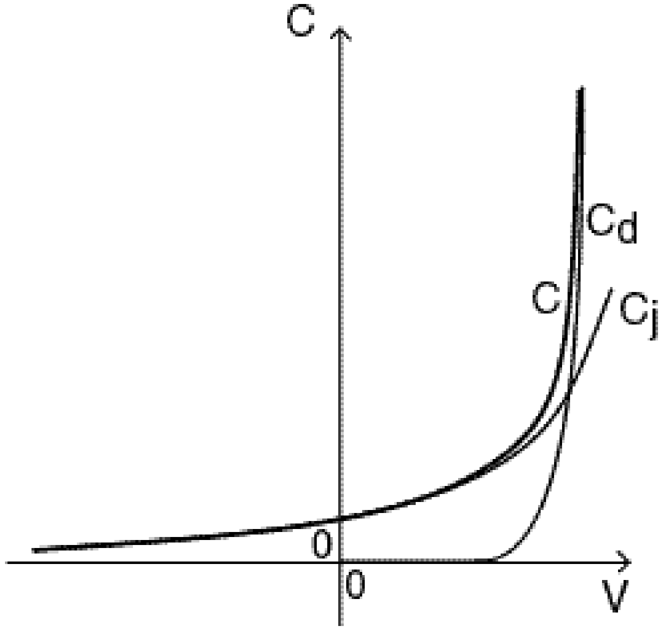
Both capacitors sit in ***parallel*** total diffusion capacitance:

$$C_d = C_{dn} + C_{dp} = \frac{q}{kT}\left(\tau_{Tn} I_n + \tau_{Tp} I_p\right)$$

Complete small-signal equivalent circuit model for diode:



Bias dependence of C_j and C_d:



- C_j dominates in reverse bias and small forward bias

$$\frac{1}{\sqrt{\varphi_B - V}}$$

- C_d dominates in strong forward bias

$$\exp\left[\frac{qV}{kT}\right]$$

What did we learn today?

Summary of Key Concepts

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