Lecture 22

Frequency Response of Amplifiers (II)

VOLTAGE AMPLIFIERS

Outline

1. Full Analysis
2. Miller Approximation
3. Open Circuit Time Constant

Reading Assignment:
Howe and Sodini, Chapter 10, Sections 10.1-10.4

Summary of Key Concepts

- Full Analysis
  - Assumes that $\tau_1 + \tau_2 \approx \tau_1$

$$\omega_{3dB} \approx \frac{1}{\tau_1} = \frac{1}{R'_{out}C_\mu + R'_{in}C_\mu\left(1+g_m R'_{out}\right) + R'_{in}C_\pi}$$

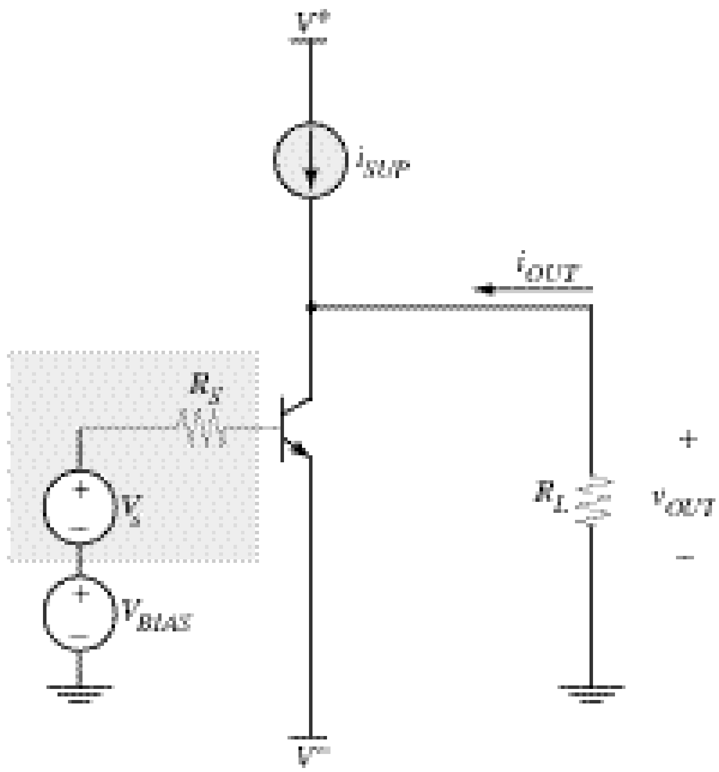
- Miller Approximation
  - Does not take into account R'_{out}

$$\omega_3 \quad = \left[\frac{1}{R'}\right]\left[\frac{1}{C_\pi + \left(1+g \quad '_{out}\right)C_\mu}\right]$$

- Open Circuit Time Constant (OCT)
  - Assumes a dominant pole as full analysis

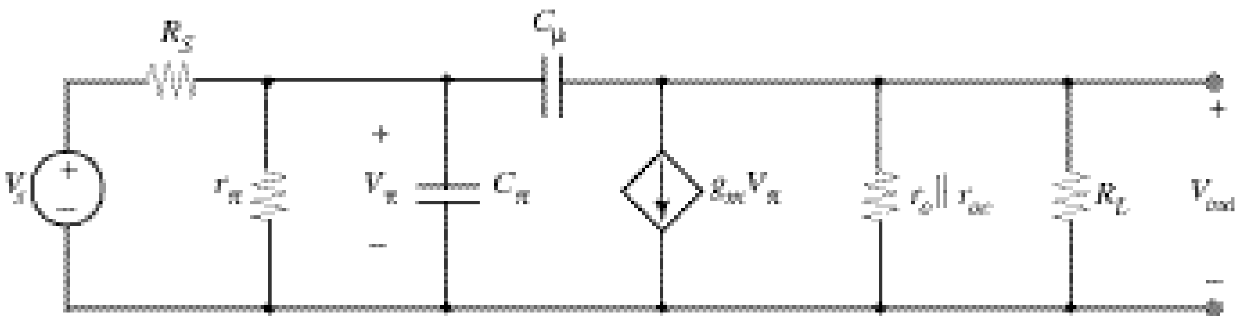
$$\omega_{3dB} \approx \frac{1}{b_1} = \frac{1}{R'_{out}C_\mu + R'_{in}C_\mu\left(1+g_m R'_{out}\right) + R'_{in}C_\pi}$$

Common Emitter Amplifier



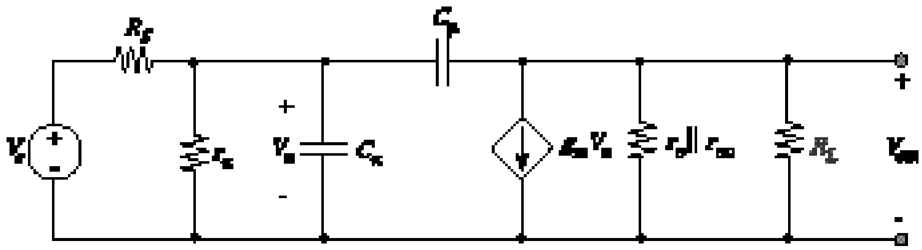
- Operating Point Analysis
  - v_s=0, R_S = 0, $r_o \to \infty$, $r_{oc} \to \infty$, $R_L \to \infty$
  - Find V_{BIAS} such that I_C=I_{SUP} with the BJT in the forward active region

Frequency Response Analysis of the Common Emitter Amplifier

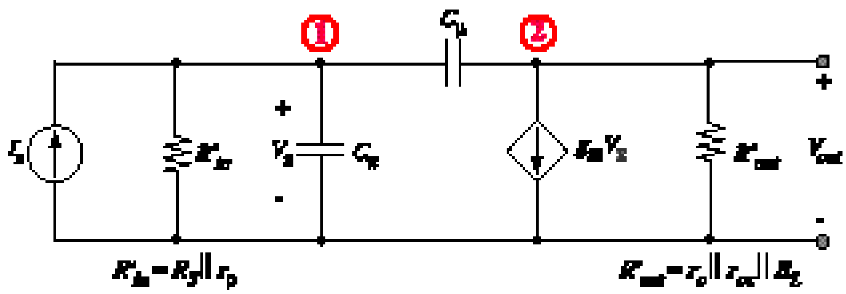


- Frequency Response
  - Set $V_{BIAS} = 0$.
  - Substitute BJT small signal model (with capacitors) including R_S, R_L, r_o, r_{oc}
  - Perform impedance analysis

1. Full Analysis of CE Voltage Amplifier



Replace source voltage and resistance with source current and conductance



Node 1:

$$\mathbf{I_s = \frac{V_\pi}{R'_{in}} + j\omega C_\pi V_\pi + j\omega C_\mu \left(V_\pi - V_{out}\right)}$$

Node 2:

$$\mathbf{g_m V_\pi + \frac{V_{out}}{R'_{out}} = j\omega C_\mu \left(V_\pi - V_{out}\right)}$$

Full Frequency Response Analysis (contd.)

- Re-arranging **2** and obtain an expression for V_π
- Substituting it into **1** and with some manipulation, we can obtain an expression for V_{out} / I_s:

$$\frac{\mathbf{V_{out}}}{\mathbf{I_s}} = \frac{-\mathbf{R'_{in} R'_{out}} (\mathbf{g_m} - \mathbf{j\omega C_\mu})}{1 + \mathbf{j\omega}(\mathbf{R'_{out} C_\mu} + \mathbf{R'_{in} C_\mu} + \mathbf{R'_{in} C_\pi} + \mathbf{g_m R'_{out} R'_{in} C_\mu}) - \omega^2 \mathbf{R'_{out} R'_{in} C_\mu C_\pi}}$$

Changing input current source back to a voltage source:

$$\frac{\mathbf{V_{out}}}{\mathbf{V_s}} = \frac{-\mathbf{g_m R'_{out}} \left(\frac{\mathbf{r_\pi}}{\mathbf{R_S} + \mathbf{r_\pi}}\right)\left(1 - \mathbf{j\omega} \frac{\mathbf{C_\mu}}{\mathbf{g_m}}\right)}{1 + \mathbf{j\omega}(\mathbf{R'_{out} C_\mu} + \mathbf{R'_{in} C_\mu}(1 + + \mathbf{g_m R'_{out}}) + \mathbf{R'_{in} C_\pi}) - \omega^2 \mathbf{R'_{out} R'_{in} C_\mu C_\pi}}$$

where $R'_{in} = \mathbf{R_S} \,||\, \mathbf{r_\pi}$ and $R'_{out} = \mathbf{r_o} \,||\, \mathbf{r_{oc}} \,||\, \mathbf{R_L}$

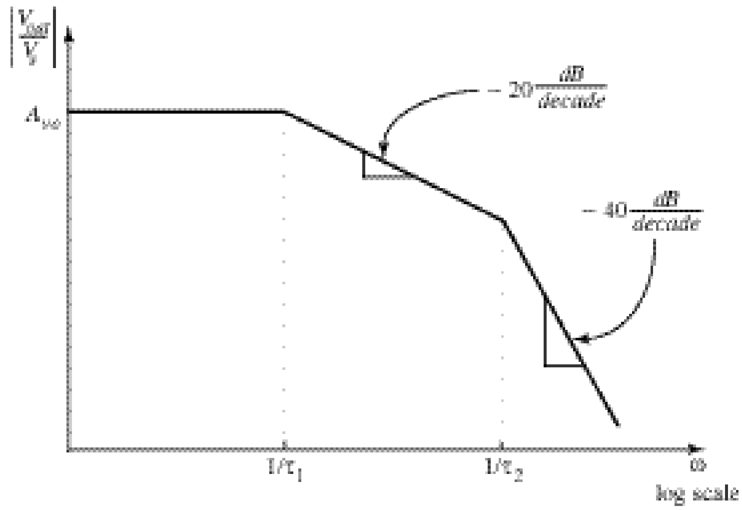
We can ignore zero at g_m/C_μ because it is higher than f_T.

The gain can be expressed as:

$$\frac{\mathbf{V_{out}}}{\mathbf{V_s}} = \frac{\mathbf{A_v}}{(1 + \mathbf{j\omega\tau_1})(1 + \mathbf{j\omega\tau_2})} = \frac{\mathbf{A_v}}{(1 - \omega^2 \tau_1 \tau_2) - \mathbf{j\omega}(\tau_1 + \tau_2)}$$

where A_v is the DC gain and τ_1 and τ_2 are the two time constants associated with the capacitors

Denominator of the System Transfer Function



$$\tau_1 + \tau_2 = \mathbf{R'_{out}C_\mu} + \mathbf{R'_{in}C_\mu}\left(1 + +\mathbf{g_m R'_{out}}\right) + \mathbf{R'_{in}C_\pi}$$

$$\tau_1 \bullet \tau_2 = \mathbf{R'_{out}R'_{in}C_\mu C_\pi}$$

We could solve for τ_1 and τ_2 but is algebraically complex.

- However, if we assume that $\tau_1 >> \tau_2$,$\Rightarrow$ $\tau_1 + \tau_2 \approx \tau_1$.
- This is a conservative estimate since τ_1 is actually smaller.

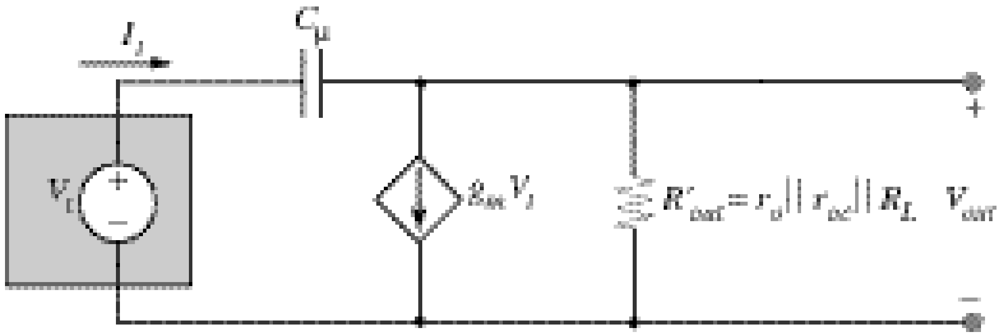
$$\tau_1 \approx \mathbf{R'_{in}}\left[\mathbf{C_\pi} + \mathbf{C_\mu}\left(1 + \mathbf{g_m R'_{out}}\right)\right] + \mathbf{R'_{out}C_\mu}$$

Then:

$$\omega_{3dB} = \frac{1}{\tau_1} = \frac{1}{\mathbf{R'_{in}}\left[\mathbf{C_\pi} + \mathbf{C_\mu}\left(1 + \mathbf{g_m R'_{out}}\right)\right] + \mathbf{R'_{out}C_\mu}}$$

2. The Miller Approximation

Effect of C_μ on the Input Impedance:



The input impedance Z_i is determined by applying a test voltage V_t to the input and measuring I_t:

$$\mathbf{V_{out} = -g_m V_t R'_{out} + I_t R'_{out}}$$

The Miller Approximation assumes that current through C_μ is small compared to the transconductance generator

$$\mathbf{I_t << |g_m V_t|}$$

$$\mathbf{V_{out} \approx -g_m V_t R'_{out}}$$

We can relate V_t and V_{out} by

$$\mathbf{V_t - V_{out} = \frac{I_t}{j\omega C_\mu}}$$

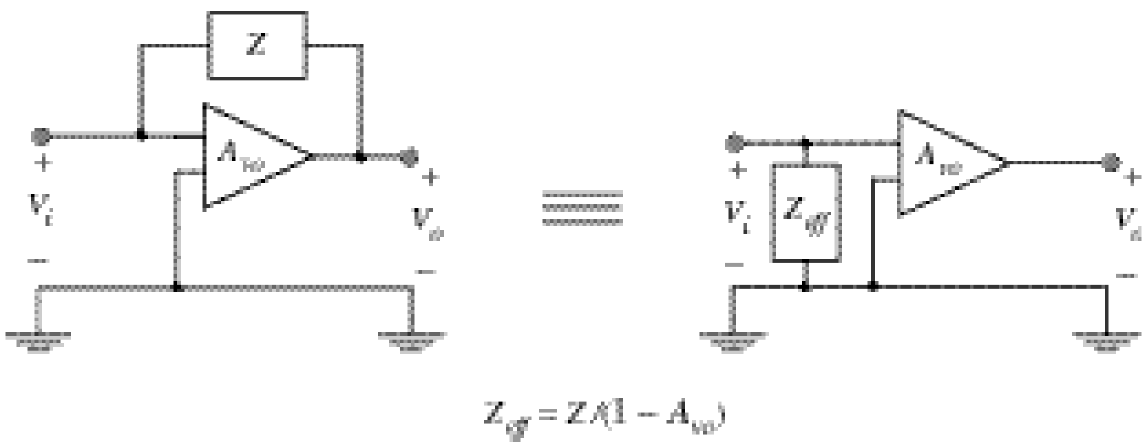
The Miller Approximation (contd.)

After some Algebra:

$$\frac{\mathbf{V_t}}{\mathbf{I_t}} = \mathbf{Z_{i,eff}} = \frac{1}{\mathbf{j\omega C_\mu}(1+\mathbf{g_m R'_{out}})} = \frac{1}{\mathbf{j\omega C_\mu}(1-\mathbf{A_{vC_\mu}})}$$

The effect of C_μ at input is that C_μ is "Miller multiplied" by $(1-A_{vC\mu})$

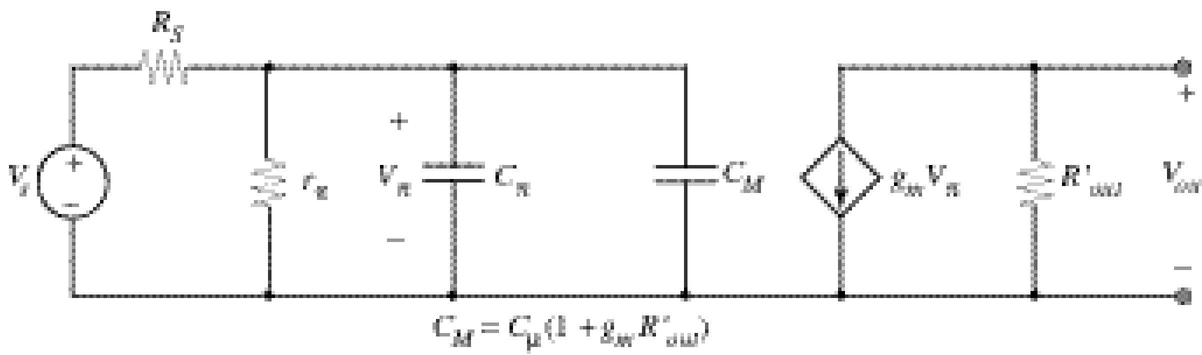
Generalized "Miller Effect"



- An impedance connected across an amplifier with voltage gain A_{vo} can be replaced by an an impedance to ground … divided by $(1-A_{vo})$
- A_{vo} is large and negative for common-emitter and common-source amplifiers
- Capacitance at input is magnified.

$$\mathbf{Z_{eff}} = \frac{\mathbf{Z}}{(1-\mathbf{A_{vo}})}$$

Frequency Response of the CE Voltage Amplifier Using Miller Approximation



- The Miller capacitance is lumped together with C_π, which results in a single pole low pass filter at the input

$$\frac{\mathbf{V_{out}}}{\mathbf{V_s}} = -\mathbf{g_m}\left(\frac{\mathbf{r_\pi}}{\mathbf{r_\pi} + \mathbf{R_S}}\right)\mathbf{R'_{out}}\left[\frac{1}{1 + \mathbf{j}\omega(\mathbf{C_\pi} + \mathbf{C_M})(\mathbf{R_S} \parallel \mathbf{r_\pi})}\right]$$

- At DC the small signal voltage gain is

$$\frac{\mathbf{V_{out}}}{\mathbf{V_s}} = -\mathbf{g_m}\left(\frac{\mathbf{r_\pi}}{\mathbf{r_\pi} + \mathbf{R_S}}\right)\mathbf{R'_{out}}$$

- The frequency at which the magnitude of the voltage gain is reduced by $1/\sqrt{2}$ is

$$\omega_{3dB} = \frac{1}{(\mathbf{R_s} \parallel \mathbf{r_\pi})(\mathbf{C_\pi} + \mathbf{C_M})} = \left[\frac{1}{(\mathbf{R_s} \parallel \mathbf{r_\pi})}\right]\left[\frac{1}{\mathbf{C_\pi} + (1 + \mathbf{g_m}\mathbf{R'_{out}})\mathbf{C_\mu}}\right]$$

3. Open Circuit Time Constant Analysis

Assumptions:

- No zeros
- One "dominant" pole ($1/\tau_1 << 1/\tau_2$, $1/\tau_3$ … $1/\tau_n$)
- N capacitors

$$\frac{\mathbf{V_{out}}}{\mathbf{V_s}} = \frac{\mathbf{A_v}}{(1+\mathbf{j}\omega\tau_1)(1+\mathbf{j}\omega\tau_2)(\ldots)(1+\mathbf{j}\omega\tau_\mathbf{n})}$$

The example shows a voltage gain; however, it could be I_{out}/V_s or V_{out}/I_s.

Multiplying out the denominator:

$$\frac{\mathbf{V_{out}}}{\mathbf{V_s}} = \frac{\mathbf{A_v}}{1+\mathbf{b}_1(\mathbf{j}\omega)+\mathbf{b}_2(\mathbf{j}\omega)^2+\ldots+\mathbf{b_n}(\mathbf{j}\omega)^\mathbf{n}}$$

where $b_1 = \tau_1 + \tau_2 + \tau_3 + \ldots + \tau_n$

It can be shown that the coefficient b_1 can be found exactly [see Gray & Meyer, 3rd Edition, pp. 502-506]

$$\mathbf{b}_1 = \left(\sum_{\mathbf{i}=1}^{\mathbf{N}} \mathbf{R_{Ti}C_i}\right) = \left(\sum_{\mathbf{i}}^{\mathbf{N}} \tau_{\mathbf{C_{io}}}\right)$$

- τ_{Cio} is the open-circuit time constant for capacitor C_i
- C_i is the i^{th} capacitor and R_{Ti} is the Thevenin resistance across the i^{th} capacitor terminals (with all capacitors open-circuited)

Open Circuit Time Constant Analysis

Estimating the Dominant Pole

The dominant pole of the system can be estimated by:

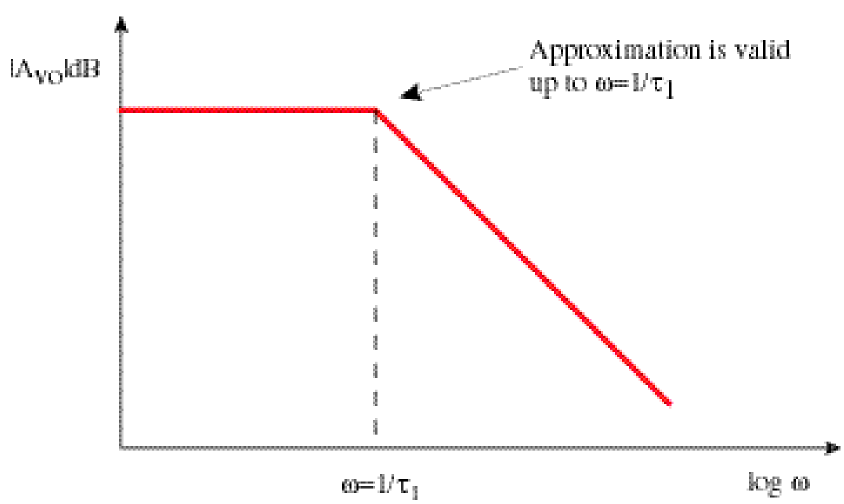
$b_1 = \tau_1 + \tau_2 + \tau_3 + \ldots + \tau_n$

$$\mathbf{b}_1 = \left(\sum_{\mathbf{i}=1}^{\mathbf{N}} \mathbf{R}_{\mathbf{Ti}} \mathbf{C}_{\mathbf{i}} \right) \approx \tau_1 = \frac{1}{\omega_1}$$

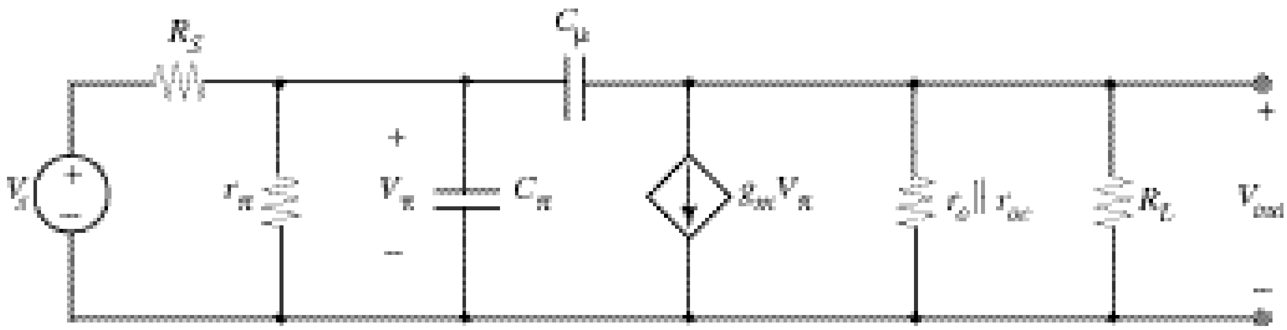
$R_{Ti}C_i$ is the **open-circuit time constant** for capacitor C_i

Power of the Technique:

- Estimates the contribution of each capacitor to the dominant pole frequency separately
- Enables the designer to understand what part of a complicated circuit is responsible for limiting the bandwidth of amplifier
- The approximate magnitude of the Bode Plot is



Common Emitter Amplifier Analysis Using OCT



From the Full Analysis

$$\frac{\mathbf{V_{out}}}{\mathbf{V_s}} = \frac{-\mathbf{g_m R'_{out}}\left(\frac{\mathbf{r_\pi}}{\mathbf{R_S + r_\pi}}\right)\left(1 - \mathbf{j}\omega\frac{\mathbf{C_\mu}}{\mathbf{g_m}}\right)}{1 + \mathbf{j}\omega\left(\mathbf{R'_{out}C_\mu + R'_{in}C_\mu}(1 + +\mathbf{g_m R'_{out}}) + \mathbf{R'_{in}C_\pi}\right) - \omega^2\mathbf{R'_{out}R'_{in}C_\mu C_\pi}}$$

where $R'_{in} = \mathbf{R_S} \| \mathbf{r_\pi}$ and $R'_{out} = \mathbf{r_o} \| \mathbf{r_{oc}} \| \mathbf{R_L}$

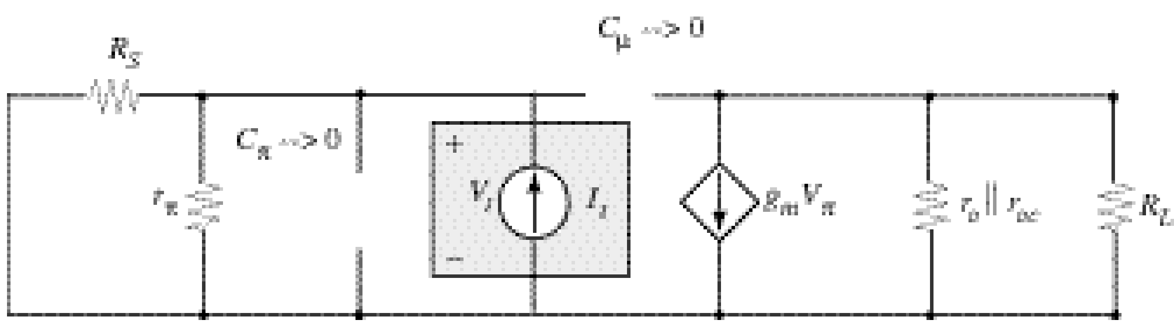
$$\mathbf{b}_1 = \mathbf{R'_{out}C_\mu + R'_{in}C_\mu}(1 + +\mathbf{g_m R'_{out}}) + \mathbf{R'_{in}C_\pi}$$

$$\omega_{3dB} \approx \frac{1}{\mathbf{b}_1} = \frac{1}{\mathbf{R'_{out}C_\mu + R'_{in}C_\mu}(1 + +\mathbf{g_m R'_{out}}) + \mathbf{R'_{in}C_\pi}}$$

Common Emitter Amplifier Analysis Using OCT—Procedure

1. Eliminate all independent sources [e.g. $V_s \rightarrow 0$]
2. Open-circuit all capacitors
3. Find the Thevenin resistance by applying i_t and measuring v_t.

Time Constant for C_π



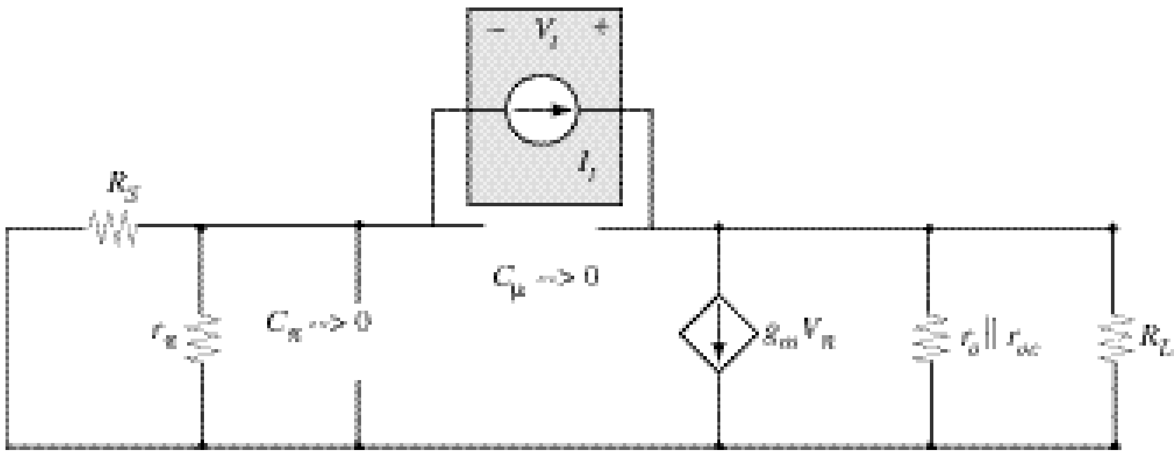
Result obtained by inspection

$$\mathbf{R_{T\pi} = R_S \| r_\pi}$$

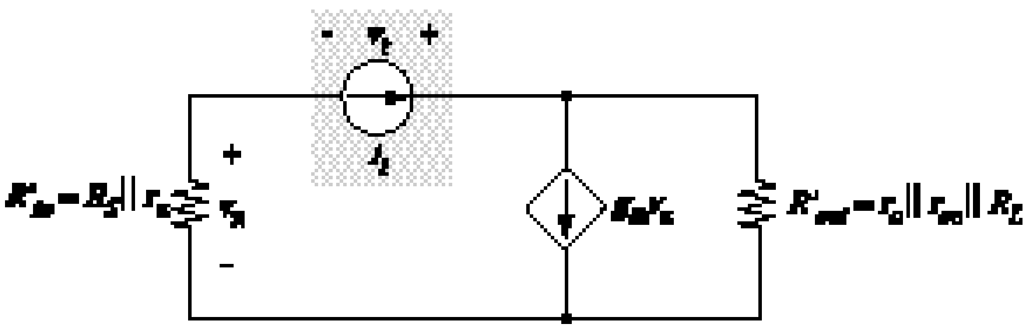
$$\boldsymbol{\tau_{C_{\pi o}} = R_{T\pi} C_\pi}$$

Common Emitter Amplifier Analysis Using OCT—Time Constant for C_μ

Using the same procedure



Let $R'_{in} = \mathbf{R_S} \| \mathbf{r_\pi}$ and $R'_{out} = \mathbf{r_o} \| \mathbf{r_{oc}} \| \mathbf{R_L}$



$$-\mathbf{i_t} = \frac{\mathbf{v_\pi}}{\mathbf{R'_{in}}} \qquad \mathbf{i_t} = \frac{\mathbf{v_t} + \mathbf{v_\pi}}{\mathbf{R'_{out}}} + \mathbf{g_m v_\pi}$$

Eliminate v_π:

$$\frac{\mathbf{v_t}}{\mathbf{i_t}} = \mathbf{R_{T\mu}} = \mathbf{R'_{out}} + \mathbf{R'_{in}}(1 + \mathbf{g_m R'_{out}})$$

$$\tau_{C_{\mu o}} = \mathbf{R_{T\mu} C_\mu} = [\mathbf{R'_{out}} + \mathbf{R'_{in}}(1 + \mathbf{g_m R'_{out}})]\mathbf{C_\mu}$$

Common Emitter Amplifier Analysis Using OCT—Dominant Pole

Summing individual time constants

$$\mathbf{b}_1 = \mathbf{R}_{\mathbf{T\pi}}\mathbf{C}_{\pi} + \mathbf{R}_{\mathbf{T\mu}}\mathbf{C}_{\mu}$$

$$\mathbf{b}_1 = \mathbf{R}'_{\mathbf{out}}\mathbf{C}_{\mu} + \mathbf{R}'_{\mathbf{in}}\mathbf{C}_{\mu}\left(1 + \mathbf{g}_{\mathbf{m}}\mathbf{R}'_{\mathbf{out}}\right) + \mathbf{R}'_{\mathbf{in}}\mathbf{C}_{\pi}$$

Assume $\tau_1 >> \tau_2$

$$\mathbf{b}_1 = \tau_1 + \tau_2 \approx \tau_1$$

$$\mathbf{b}_1 = \mathbf{R}'_{\mathbf{out}}\mathbf{C}_{\mu} + \mathbf{R}'_{\mathbf{in}}\mathbf{C}_{\mu}\left(1 + \mathbf{g}_{\mathbf{m}}\mathbf{R}'_{\mathbf{out}}\right) + \mathbf{R}'_{\mathbf{in}}\mathbf{C}_{\pi}$$

$$\omega_{\mathbf{3dB}} \approx \frac{1}{\mathbf{b}_1} = \frac{1}{\mathbf{R}'_{\mathbf{out}}\mathbf{C}_{\mu} + \mathbf{R}'_{\mathbf{in}}\mathbf{C}_{\mu}\left(1 + \mathbf{g}_{\mathbf{m}}\mathbf{R}'_{\mathbf{out}}\right) + \mathbf{R}'_{\mathbf{in}}\mathbf{C}_{\pi}}$$

This result is very similar to the Miller Effect calculation
Additional term $R'_{out}C_{\mu}$ taken into account

Compare the Three Methods of Analyzing the Frequency Response of CE Amplifier

Full Analysis—assumes $\tau_1 + \tau_2 \approx \tau_1$

$$\omega_{3dB} \approx \frac{1}{\tau_1} = \frac{1}{R'_{out}C_\mu + R'_{in}C_\mu\left(1+g_m R'_{out}\right) + R'_{in}C_\pi}$$

Miller Approximation—does not take into account R_{out}

$$\omega_{3dB} = \left[\frac{1}{R'_{in}}\right]\left[\frac{1}{C_\pi + \left(1+g_m R'_{out}\right)C_\mu}\right]$$

Open Circuit Time Constant—assumes dominant pole is same as full analysis

$$\omega_{3dB} \approx \frac{1}{b_1} = \frac{1}{R'_{out}C_\mu + R'_{in}C_\mu\left(1+g_m R'_{out}\right) + R'_{in}C_\pi}$$

What did we learn today?

Summary of Key Concepts

- Full Analysis
  - Assumes that $\tau_1 + \tau_2 \approx \tau_1$

$$\omega_{3dB} \approx \frac{1}{\tau_1} = \frac{1}{\mathbf{R'_{out}C_\mu + R'_{in}C_\mu(1+g_m R'_{out}) + R'_{in}C_\pi}}$$

- Miller Approximation
  - Does not take into account R'_{out}

$$\omega_{3dB} = \left[\frac{1}{\mathbf{R'_{in}}}\right]\left[\frac{1}{\mathbf{C_\pi + (1+g_m R'_{out})C_\mu}}\right]$$

- Open Circuit Time Constant (OCT)
  - Assumes a dominant pole as full analysis

$$\omega_{3dB} \approx \frac{1}{\mathbf{b}_1} = \frac{1}{\mathbf{R'_{out}C_\mu + R'_{in}C_\mu(1+g_m R'_{out}) + R'_{in}C_\pi}}$$