Lecture 4

PN Junction and MOS Electrostatics(I)

Semiconductor Electrostatics in Thermal Equilibrium

Outline

- Non-uniformly doped semiconductor in thermal equilibrium
- Relationships between potential, $\varphi(x)$ and equilibrium carrier concentrations, $p_o(x)$, $n_o(x)$
  - –Boltzmann relations & "60 mV Rule"
- Quasi-neutral situation

Reading Assignment:
Howe and Sodini; Chapter 3, Sections 3.1-3.2

Summary of Key Concepts

- It is possible to have an electric field inside a semiconductor in thermal equilibrium
  - ***Non-uniform doping distribution***
- In thermal equilibrium, there is a fundamental relationship between potential, $\varphi(x)$ and the equilibrium carrier concentrations $p_o(x)$ & $n_o(x)$
  - **Boltzmann relations (or "60 mV Rule")**
- In a slowly varying doping profile, majority carrier concentration tracks well the doping concentration.

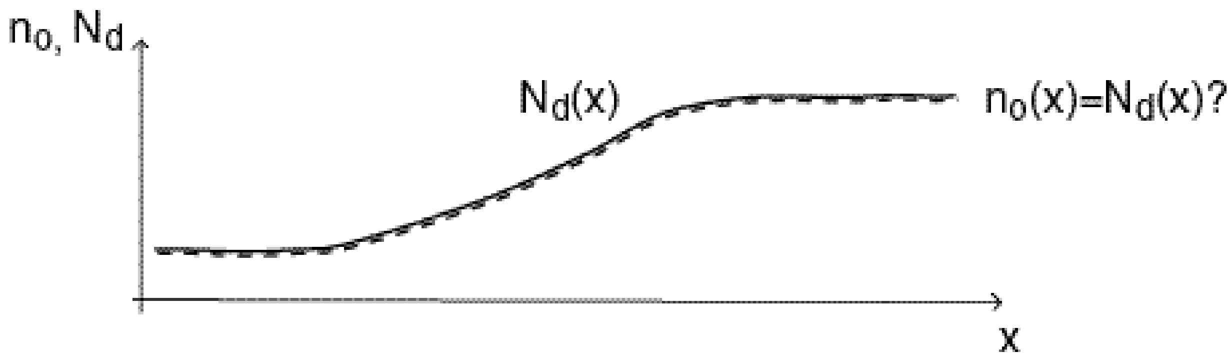
1. Non-uniformly doped semiconductor in thermal equilibrium

Consider a piece of n-type Si in thermal equilibrium with non-uniform dopant distribution:

What is the resulting electron concentration in thermal equilibrium?

OPTION 1: Every donor gives out one electron

$$n_o(x) = N_d(x)$$



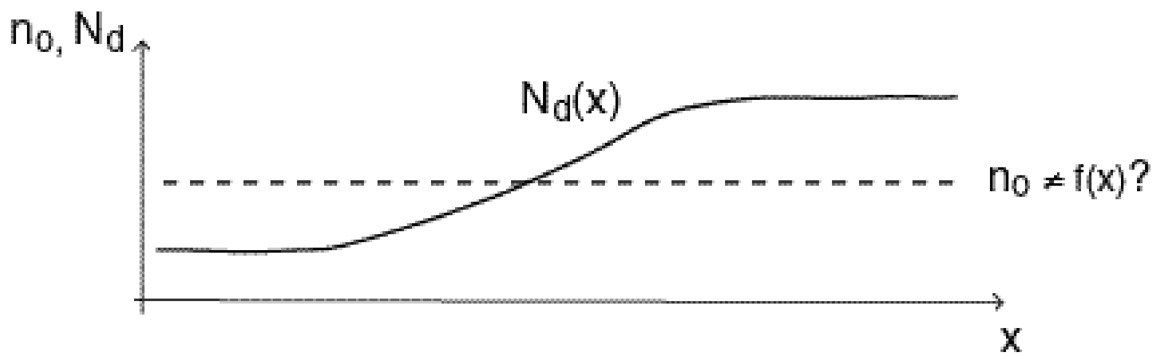
Gradient of electron concentration

net electron diffusion

not in thermal equilibrium!

OPTION 2: electron concentration uniform in space

$$n_o(x) = n_{ave} \neq f(x)$$



Think about space charge density:

$$\rho(x) \approx q\left[N_d(x) - n_o(x)\right]$$

If $N_d(x) \neq n_o(x)$

$\rho(x) \neq 0$

electric field

<u>**net electron drift**</u>

not in thermal equilibrium!

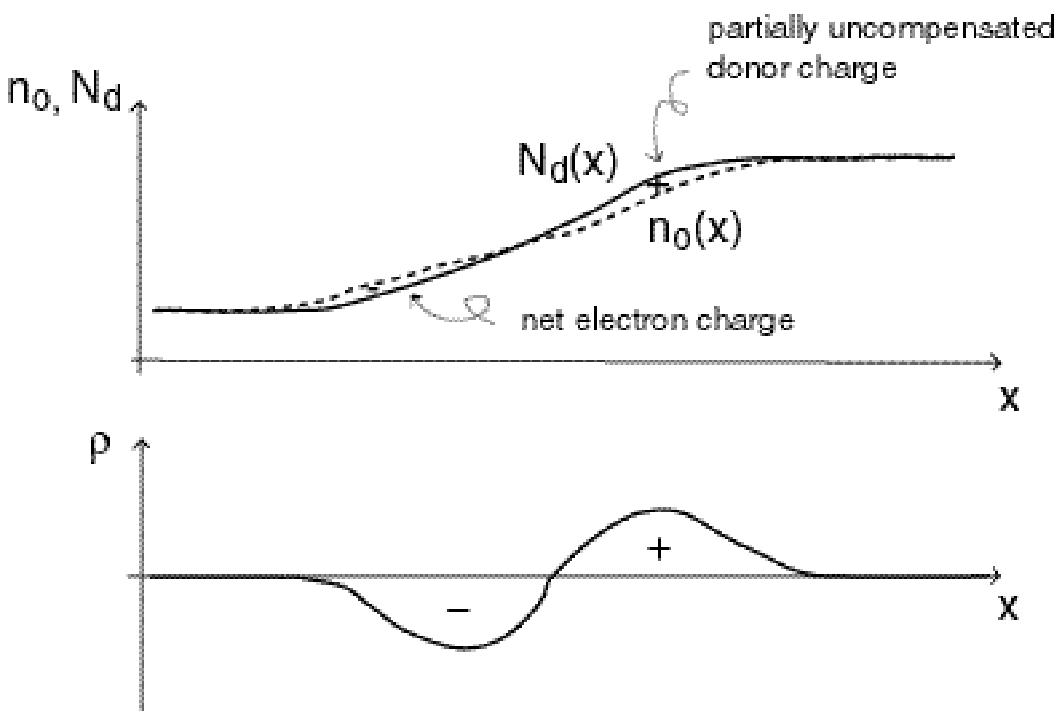
OPTION 3: Demand that $J_n = 0$ in thermal equilibrium at every x ($J_p = 0$ too)

Diffusion precisely balances Drift

Let us examine the electrostatics implications of $n_o(x) \neq N_d(x)$

Space charge density

$$\rho(x) = q\left[N_d(x) - n_o(x)\right]$$



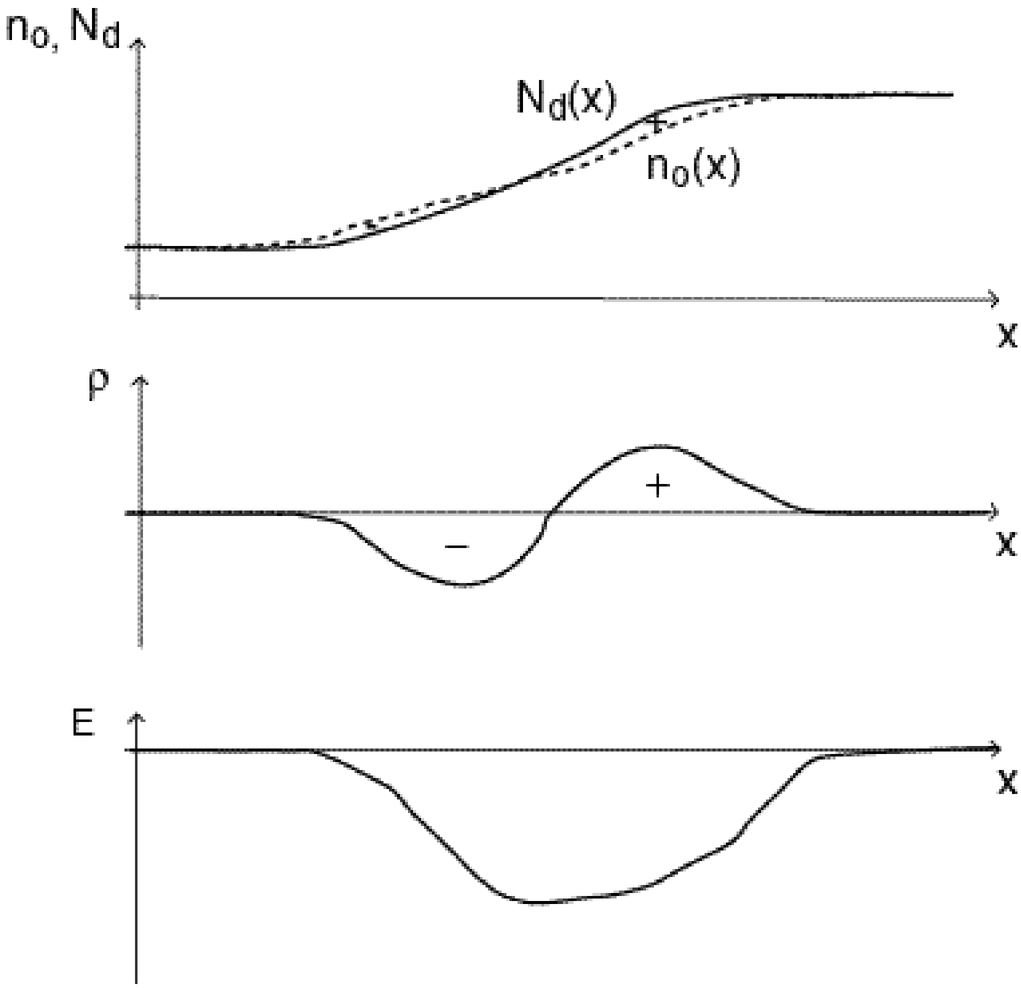
Electric Field

Gauss' equation:

$$\frac{dE}{dx} = \frac{\rho(x)}{\varepsilon_s}$$

Integrate from x = 0:

$$E(x) - E(0) = \frac{1}{\varepsilon_s}\int_0^x \rho(x')\,dx'$$



Electrostatic Potential

$$\frac{d\varphi}{dx} = -E(x)$$

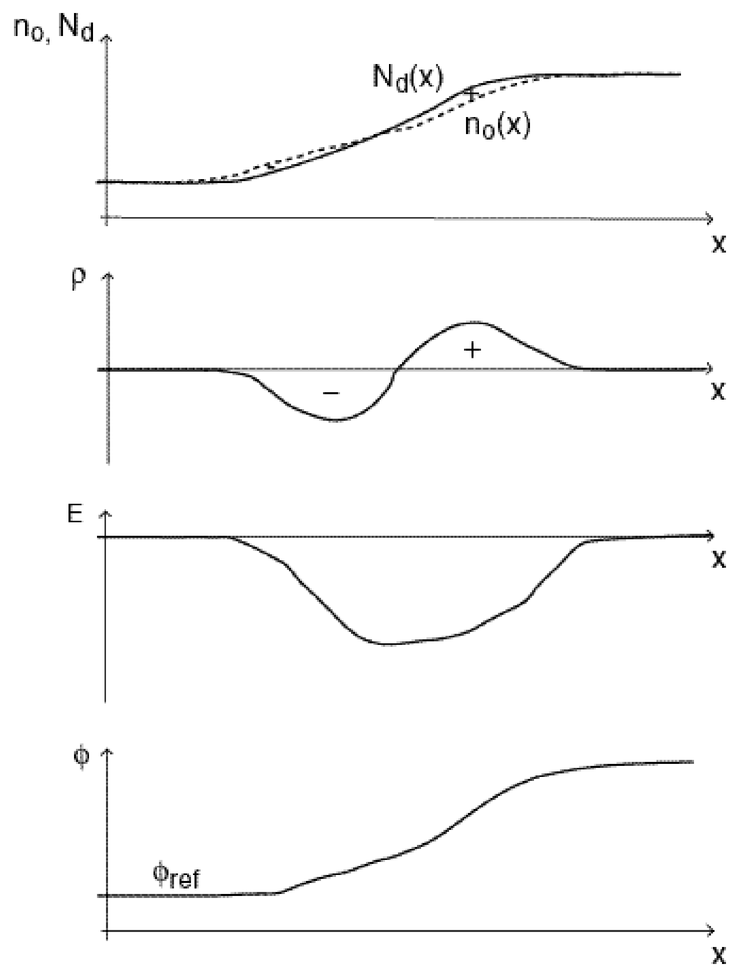
Integrate from x=0:

$$\varphi(x) - \varphi(0) = -\int_0^x E(x')\,dx'$$

Need to select reference

(**physics is in the potential difference, not in absolute value!**);

Select $\varphi(x = 0) = \varphi_{ref}$



Given $N_d(x)$, want to know $n_o(x)$, $\rho(x)$, $E(x)$ and $\varphi(x)$

Equations that describe the problem:

$$J_n = q\mu_n n_o E + qD_n \frac{dn_o}{dx} = 0$$

$$\frac{dE}{dx} = \frac{q}{\varepsilon_s}\left(N_d - n_o\right)$$

Express them in terms of φ:

$$-q\mu_n n_o \frac{d\varphi}{dx} + qD_n \frac{dn_o}{dx} = 0 \qquad (1)$$

$$-\frac{d^2\varphi}{dx^2} = \frac{q}{\varepsilon_s}\left(N_d - n_o\right) \qquad (2)$$

Plug (1) into (2):

$$\frac{d^2\left(\ln n_o\right)}{dx^2} = \frac{q^2}{\varepsilon_s kT}\left(n_o - N_d\right)$$

One equation with one unknown. Given $N_d(x)$, we can solve for $n_o(x)$ and all the rest, but…

No analytical solutions for most situations!

2. Relationships between potential, $\varphi(x)$ and equilibrium carrier concentrations, $p_o(x)$, $n_o(x)$ (Boltzmann relations)

From [1]:

$$\frac{\mu_n}{D_n} \cdot \frac{d\varphi}{dx} = \frac{1}{n_o} \cdot \frac{dn_o}{dx}$$

Using Einstein relation:

$$\frac{q}{kT} \cdot \frac{d\varphi}{dx} = \frac{d(\ln n_o)}{dx}$$

Integrate:

$$\frac{q}{kT}(\varphi - \varphi_{ref}) = \ln n_o - \ln n_{o,ref} = \ln \frac{n_o}{n_{o,ref}}$$

Then:

$$\mathbf{n_o = n_{o,ref}} \exp\left[\frac{\mathbf{q(\varphi - \varphi_{ref})}}{\mathbf{kT}}\right]$$

Any reference is good

In 6.012, $\varphi_{ref} = 0$ at $n_{o,ref} = n_i$

Then:

$$n_o = n_i e^{q\varphi/kT}$$

If we do same with holes (starting with $J_h = 0$ in thermal equilibrium, or simply using $n_0 p_o = n_i^2$);

$$p_o = n_i e^{-q\varphi/kT}$$

We can re-write as:

$$\varphi = \frac{kT}{q} \cdot \ln \frac{n_o}{n_i}$$

and

$$\varphi = -\frac{kT}{q} \cdot \ln \frac{p_o}{n_i}$$

“60 mV” Rule

At room temperature for Si:

$$\varphi = (25\,\mathrm{mV}) \cdot \ln\frac{n_o}{n_i} = (25\,\mathrm{mV}) \cdot \ln(10) \cdot \log\frac{n_o}{n_i}$$

Or

$$\varphi \approx (60\,\mathrm{mV}) \cdot \log\frac{n_o}{n_i}$$

EXAMPLE 1:

$n_o = 10^{18}\,\mathrm{cm}^{-3}$ $\qquad$ $\varphi = (60\,\mathrm{mV}) \times 8 = 480\,\mathrm{mV}$

"60 mV" Rule: contd.

With holes:

$$\varphi = -(25\,\mathrm{mV}) \cdot \ln \frac{p_o}{n_i} = -(25\,\mathrm{mV}) \cdot \ln(10) \cdot \log \frac{p_o}{n_i}$$

Or

$$\varphi \approx -(60\,\mathrm{mV}) \cdot \log \frac{p_o}{n_i}$$

EXAMPLE 2:

$$n_o = 10^{18}\,\mathrm{cm}^{-3} \qquad p_o = 10^{2}\,\mathrm{cm}^{-3}$$

$$\varphi = -(60\,\mathrm{mV}) \times -8 = 480\,\mathrm{mV}$$

Relationship between φ, n_o and p_o:

Note: φ cannot exceed 550 mV or be smaller than -550 mV.
(Beyond this point different physics come into play.)

Example 3: Compute potential difference in thermal equilibrium between region where $n_o = 10^{17}$ cm^{-3} and $p_o = 10^{15}$ cm^{-3}.

$$\phi(n_o = 10^{17}\,\text{cm}^{-3}) = 60 \times 7 = 420\,\text{mV}$$

$$\phi(p_o = 10^{15}\,\text{cm}^{-3}) = -60 \times 5 = -300\,\text{mV}$$

$$\phi(n_o = 10^{17}\,\text{cm}^{-3}) - \phi(p_o = 10^{15}\,\text{cm}^{-3}) = 720\,\text{mV}$$

Example 4: Compute potential difference in thermal equilibrium between region where $n_o = 10^{20}$ cm^{-3} and $p_o = 10^{16}$ cm^{-3}.

$$\phi(n_o = 10^{20}\,\text{cm}^{-3}) = \phi_{max} = 550\,\text{mV}$$

$$\phi(p_o = 10^{16}\,\text{cm}^{-3}) = -60 \times 6 = -360\,\text{mV}$$

$$\phi(n_o = 10^{20}\,\text{cm}^{-3}) - \phi(p_o = 10^{16}\,\text{cm}^{-3}) = 910\,\text{mV}$$

3. Quasi-neutral situation

Back to equation that gives $n_o(x)$ in terms of $N_d(x)$:

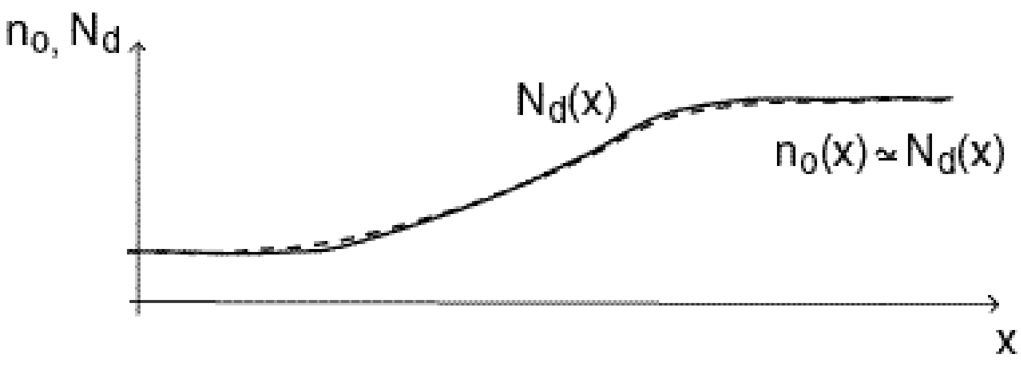
$$\frac{d^2(\ln n_o)}{dx^2} = \frac{q^2}{\varepsilon_s kT}(n_o - N_d)$$

If $N_d(x)$ changes slowly with x $n_o(x)$ also changes slowly $\frac{d^2(\ln n_o)}{dx^2}$ small

Then:

$$n_o(x) \approx N_d(x)$$

$n_o(x)$ tracks $N_d(x)$ well   minimum space charge   semiconductor is quasi-neutral



What did we learn today?

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