I. MOSFET Circuit Models

A. Large Signal SPICE Model - NMOS

- **Cutoff**: ($V_{GS} \leq V_{Tn}$)-----> $I_D = 0$
- **Triode**: ($V_{GS} \geq V_{Tn}$ and $V_{DS} \leq V_{GS}$ - V_{Tn})

$$I_D = \left(\frac{W}{L}\right)\mu_n C_{ox}\left[V_{GS} - V_{Tn} - \frac{V_{DS}}{2}\right]V_{DS}$$

- **Saturation**: ($V_{GS} \geq V_{Tn}$ and $V_{DS} \geq V_{GS}$ - V_{Tn})

$$I_D = \left(\frac{W}{2L}\right)\mu_n C_{ox}\left(V_{GS} - V_{Tn}\right)^2\left(1 + \lambda_n V_{DS}\right)$$

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B. Backgate Effect

- The threshold voltage is a function of the bulk-to-source voltage

$$V_{Tn} = V_{TOn} + \gamma_n\left(\sqrt{-V_{BS} - 2\phi_p} - \sqrt{-2\phi_p}\right)$$

- where V_{TOn} is the threshold voltage with $V_{BS} = 0$
- γ_n is the backgate effect parameter

$$\gamma_n = \left(\sqrt{2q\varepsilon_s N_a}\right) / C_{ox}$$

II. MOSFET Small-Signal Model

A. Small Signal Modelling Concepts

- Find an equivalent circuit which relates the incremental changes in i_D, v_{GS}, v_{DS}, etc.
- Since the changes are small, the small-signal equivalent circuit has **linear** elements only (e.g., capacitors, resistors, controlled sources)
- Mathamatically we perform a Taylor expansion around the DC operating point (also called the quiescent point or Q point) defined by the DC voltages $Q(V_{GS}, V_{DS}, V_{BS})$:
- The total drain current in saturation:

$$i_D = (W/2L)\,\mu_n\, C_{ox}\,(v_{GS} - V_{Tn})^2\,(1 + \lambda_n v_{DS}) = i_D(v_{GS,}\, v_{DS}, v_{BS})$$

$$where\ v_{GS} = V_{GS} + v_{gs}\ ,\ i_D = I_D + i_d$$

- We want to find $i_d = (?)\ v_{gs}$

$$i_D = I_D + \left.\frac{\partial i_D}{\partial v_{GS}}\right|_Q (v_{gs}) + \frac{1}{2}\left.\frac{\partial^2 i_D}{\partial v_{GS}^2}\right|_Q (v_{gs})^2 + \ldots$$

If the small-signal voltage is really "small," then we can neglect everything past the linear term --

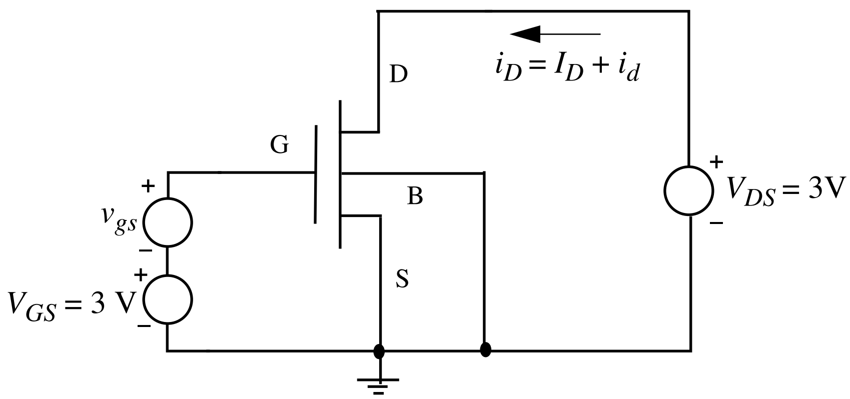
$$i_D = I_D + \left.\frac{\partial i_D}{\partial v_{GS}}\right|_Q (v_{gs}) = I_D + g_m v_{gs}$$

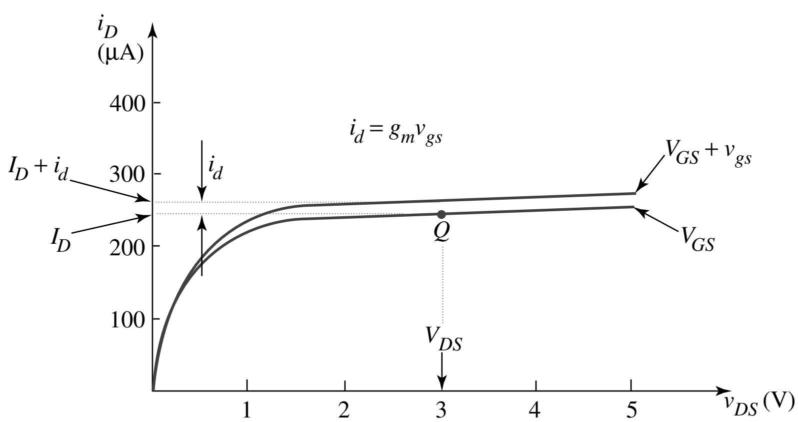
where the partial derivative is defined as the *transconductance*, g_m.

B. Transconductance

- The small-signal drain current due to v_{gs} is therefore given by

$$i_d = g_m\, v_{gs}.$$





C. Quantifying Transconductance

- Evaluating the partial derivative:

$$g_m = \left(\frac{W}{L}\right)\mu_n C_{ox}(V_{GS} - V_{Tn})\ (1 + \lambda_n V_{DS})$$

- We neglect the effect of CLM when calculating the transconductance so that g_m in terms of V_{GS} becomes

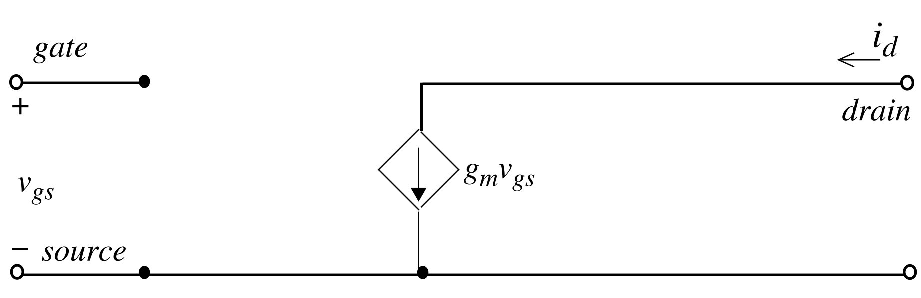
$$\boxed{g_m = \left(\frac{W}{L}\right)\mu_n C_{ox}(V_{GS} - V_{Tn})}$$

- In many circuits we want an expression for g_m in terms of the DC drain current

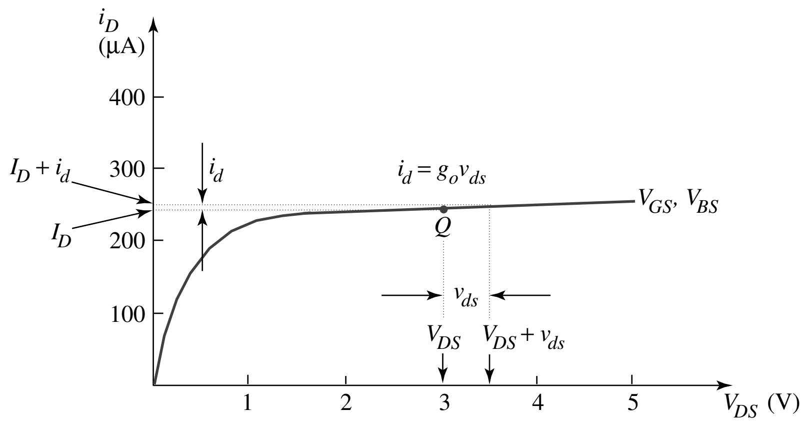
$$\boxed{g_m = \sqrt{2\left(\frac{W}{L}\right)\mu_n C_{ox} I_D}}$$

- For typical values $(W/L) = 10$, $I_D = 100\ \mu A$, and $\mu_n C_{ox} = 50\ \mu AV^{-2}$ we find that $g_m = 320\ \mu AV^{-1} = 0.32$ mS

- The circuit which expresses $i_d = g_m\, v_{gs}$



D. Output Conductance



- The change in drain current due to an incremental change in the drain-source voltage is:

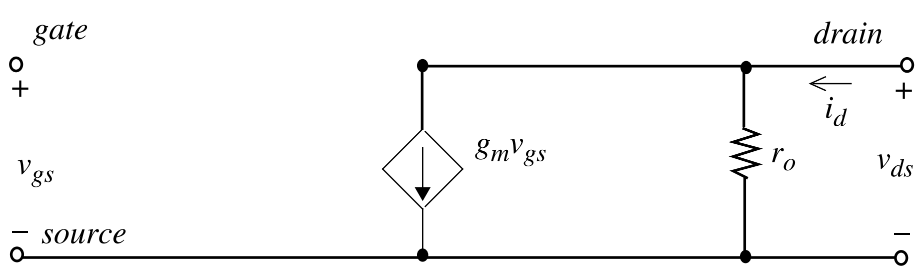
$$g_o = \left.\frac{\partial i_D}{\partial v_{DS}}\right|_Q = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_T)^2 \lambda_n \cong \lambda_n I_D$$

- The output resistance is the inverse of the output conductance

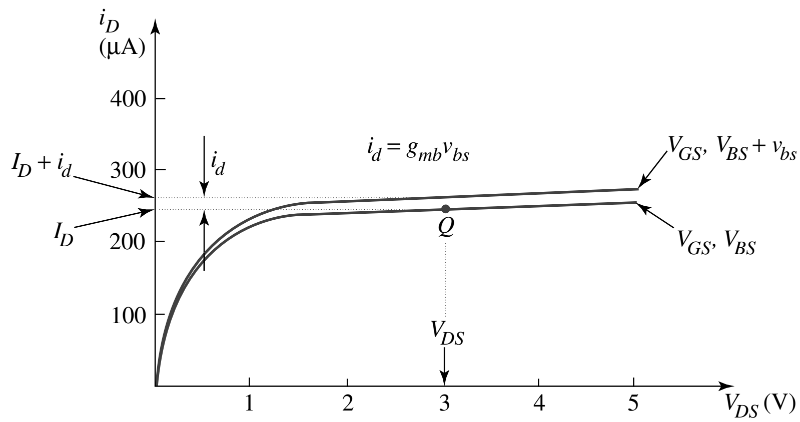
$$\boxed{r_o = \frac{1}{g_o} = \frac{1}{\lambda_n I_D}}$$

- The small-signal circuit model with r_o added looks like:

$$i_d = g_m v_{gs} + (1/r_o) v_{ds}$$



E. Backgate Transconductance



- The change in drain current due to an incremental change in the backgate bias is found using the by the chain rule:

$$g_{mb} = \left.\frac{\partial i_D}{\partial v_{BS}}\right|_Q = \left.\frac{\partial i_D}{\partial V_{Tn}}\right|_Q \left.\frac{\partial V_{Tn}}{\partial v_{BS}}\right|_Q$$

$$\left.\frac{\partial i_D}{\partial V_{Tn}}\right|_Q = -\left(\frac{W}{L}\right)\mu_n C_{ox}(V_{GS} - V_{Tn})(1 + \lambda_n V_{DS}) \approx -g_m$$

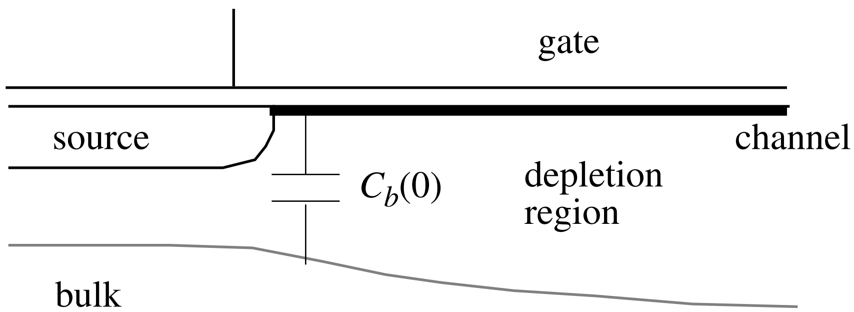
$$\boxed{g_{mb} = (-g_m)\left.\frac{\partial V_{Tn}}{\partial v_{BS}}\right|_Q = (-g_m)\left(\frac{-\gamma_n}{2\sqrt{-2\phi_p - V_{BS}}}\right) = \frac{\gamma_n g_m}{2\sqrt{-2\phi_p - V_{BS}}}}$$

E. Backgate Transconductance (con't)

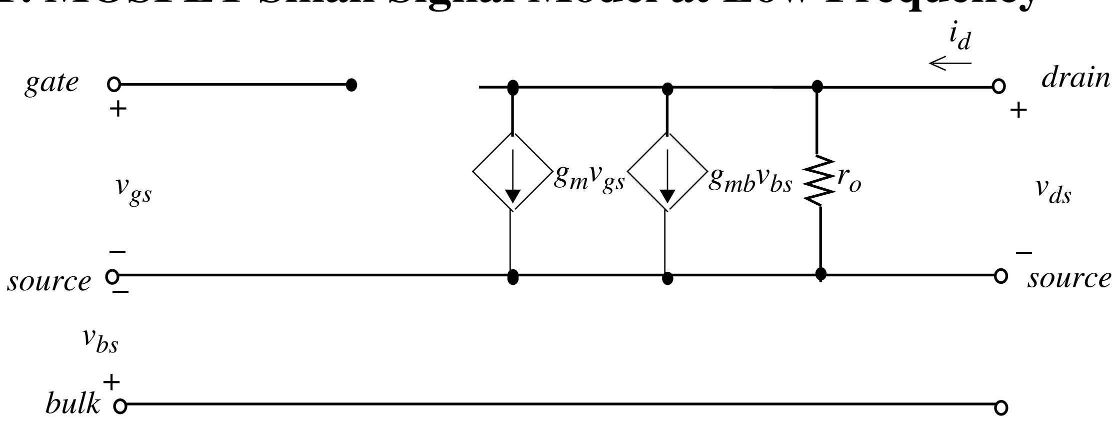
- The ratio of the backgate transconductance g_{mb} to the "front-gate" transconductance g_m to is:

$$\frac{g_{mb}}{g_m} = \frac{\sqrt{2q\varepsilon_s N_a}}{2C_{ox}\sqrt{-2\phi_p - V_{BS}}} = \frac{1}{C_{ox}}\sqrt{\frac{q\varepsilon_s N_a}{2\,(-2\phi_p - V_{BS})}} = \frac{C_b(y{=}0)}{C_{ox}}$$

- where C_b (y=0) is the depletion capacitance --

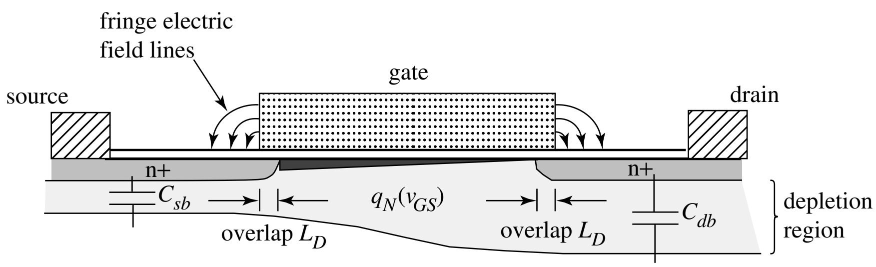


F. MOSFET Small Signal Model at Low Frequency

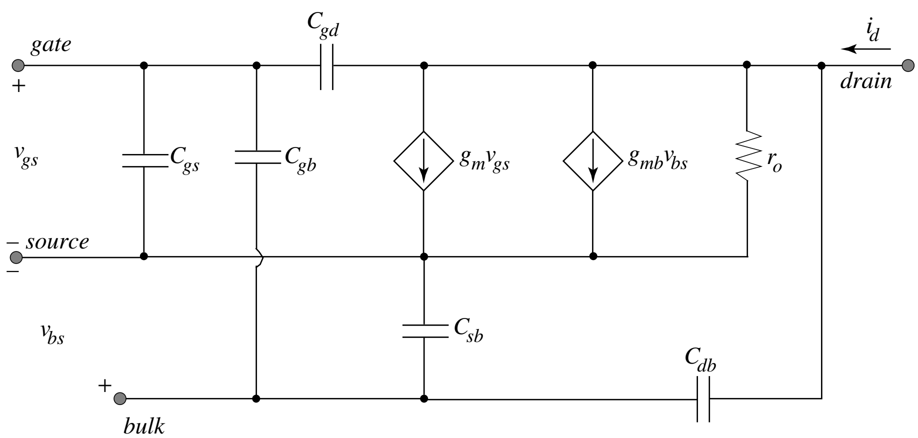


III. MOSFET Small Signal Model at High Frequency

A. Terminal Capacitances



- C_{gs} - Overlap capacitance C_{ov} + **Channel charge**
- C_{gd} - Overlap capacitance C_{ov} only
- C_{gb} - Only parasitic since bulk charge does not change
- C_{db} - Drain depletion charge
- C_{sb} - Source depletion charge



B. Channel Charge

$$q_N(v_{GS}) = -W\int_0^L C_{ox}\left(v_{GS} - V_{Tn} - v_C(y)\right)dy$$

- Recalling that current is related to the electric field

$$dy = \left(\frac{WC_{ox}\mu_n}{i_D}\right)\left(v_{GS} - V_{Tn} - v_C\right)dv_C$$

$$q_N(v_{GS}) = -\left(\frac{W^2\mu_n C_{ox}^2}{i_D}\right)\int_0^{v_{GS} - V_{Tn}}\left(v_{GS} - V_{Tn} - v_C\right)^2 dv_C$$

$$q_N(v_{GS}) = -\frac{2}{3}WLC_{ox}\left(v_{GS} - V_{Tn}\right)$$

- Note bulk Charge is constant with v_{GS} so the channel charge component of C_{gs} is given by

$$q_G(v_{GS}) = -q_N(v_{GS}) - q_{B,max}$$

- In saturation the drain has no control over the channel charge so only C_{gs} has a channel charge component given by

$$\boxed{\left.\frac{dq_G}{dv_{GS}}\right|_{V_{GS}} = \frac{2}{3}WLC_{ox}}$$

C. Parasitic Capacitance from Source & Drain Depletion Regions

- The drain n and p regions have depletion regions whose stored charge changes during the transient.
- Depletion $q_J(v_D)$ is non-linear --> take the worst case and use the zero-bias capacitance C_{jo} as a linear charge-storage element during the transient.
- Perimeter of the drain diffusion is also important and must be included in the calculation as a capacitance/length x perimeter of diffusion:

Area of drain diffusion:

$A_{diff} = W\,L_{diff}$

Perimeter of drain diffusion:

$P = W + 2\,L_{diff}$
(side next to gate isn't counted)