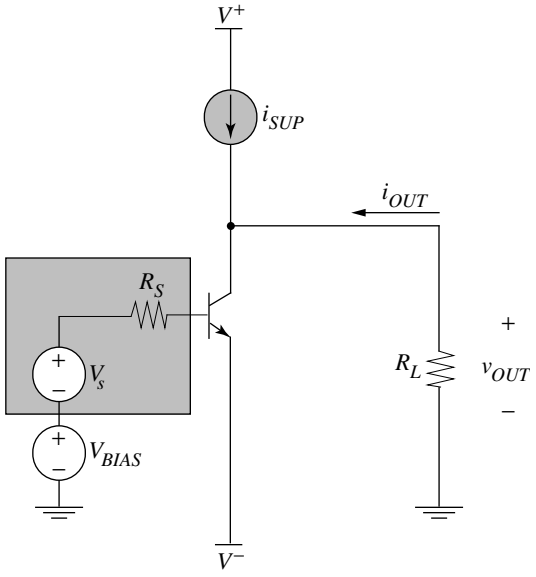
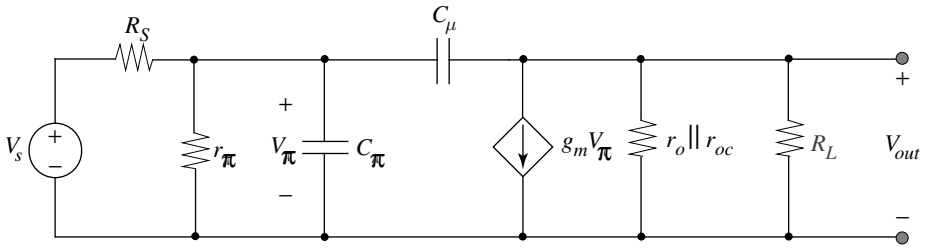
I. Frequency Response of Voltage Amplifiers

A. Common-Emitter Amplifier:



- Operating Point analysis: $V_s = 0$, $R_s = 0$, r_o--->∞, r_{oc}--->∞, R_L--->∞ Find V_{BIAS} such that $I_C = I_{SUP}$ with the BJT in the forward active region
- Frequency Response - Set $V_{BIAS} = 0$. Substitute bipolar small signal model (with capacitors) including R_S, R_L, r_o, r_{oc}, and perform impedance analysis
- **Full Analysis**
- **Miller Approximation**
- **Open Circuit Time Constants**

B. Small-Signal Model



(a)

II. Full Analysis of CE Voltage Amplifier

A. See Pages 639-640 for Analysis

$$\frac{V_{out}}{V_s} = \frac{-g_m R'_{out}\left(\frac{r_\pi}{R_S + r_\pi}\right)\left(1 - \frac{C_\mu}{g_m} j\omega\right)}{1 + j\omega\left(R'_{out}C_\mu + R'_{in}C_\mu\left(1 + g_m R'_{out}\right) + R'_{in}C_\pi\right) - \omega^2 R'_{out}R'_{in}C_\mu C_\pi}$$

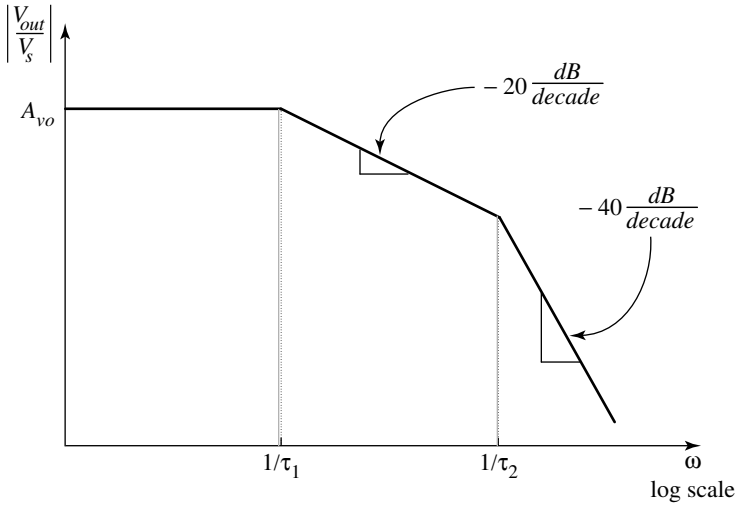
where $R_{in}' = R_S \| r_\pi$ and $R_{out}' = r_o \| r_{oc} \| R_L$

- Zero at g_m/C_μ - Ignore since higher than f_T

B. Examine Denominator of System Transfer Function

$$\frac{V_{out}}{V_s} = \frac{A_{vo}}{\left(1 + j\omega\tau_1\right)\left(1 + j\omega\tau_2\right)} = \frac{A_{vo}}{1 + j\omega\left(\tau_1 + \tau_2\right) - \omega^2\tau_1\tau_2}$$

- A_{vo} is DC gain and τ_1 and τ_2 are the two time constants



$$\tau_1 + \tau_2 = R'_{out}C_\mu + R'_{in}C_\mu\left(1 + g_m R'_{out}\right) + R'_{in}C_\pi$$

$$\tau_1\tau_2 = R'_{out}R'_{in}C_\mu C_\pi$$

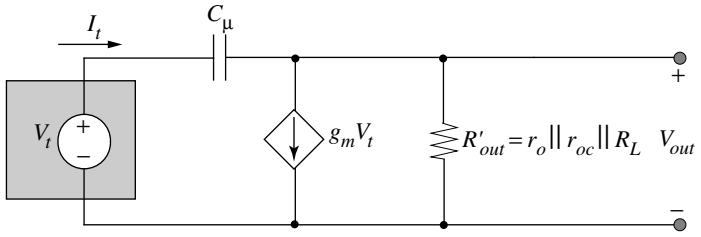
- Could solve for τ_1 and τ_2 but algebraically complex
- Assume that $\tau_1 >> \tau_2$. This implies that $\tau_1 + \tau_2 \approx \tau_1$ This is a conservative estimate since τ_1 is actually smaller

$$\tau_1 \cong R'_{in}\left[C_\pi + \left(1 + g_m R'_{out}\right)C_\mu\right] + R'_{out}C_\mu$$

$$\omega_{3dB} = \frac{1}{\tau_1} = \frac{1}{R'_{in}\left[C_\pi + \left(1 + g_m R'_{out}\right)C_\mu\right] + R'_{out}C_\mu}$$

III. The Miller Approximation

A. Input Impedance Caused by C_μ



- The effect of C_μ on the input can be found by finding the input impedance Z_i which is found from V_t/I_t

$$V_{out} = -g_m V_t R'_{out} + I_t R'_{out}$$

- The Miller Approxiamtion is that the current through C_μ is small compared to that in the transconductance generator

$$I_t \ll \left| g_m V_t \right|$$
$$V_{out} \approx -g_m V_t R'_{out}$$

- We can relate V_t and V_{out} by

$$V_t - V_{out} = \frac{I_t}{j\omega C_\mu}$$

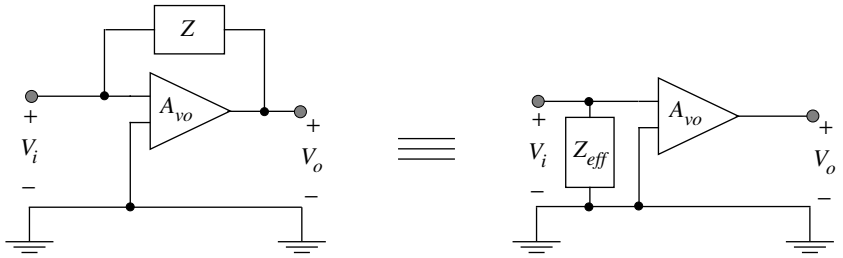
- After Algebra

$$\frac{V_t}{I_t} = Z_{eff} = \frac{1}{j\omega C_\mu \left(1 + g_m R'_{out} \right)} = \frac{1}{j\omega C_\mu \left(1 - A_{vC\mu} \right)}$$

- The effect of C_μ at the input is "Miller multiplied" by $(1\text{-}A_{vC\mu})$

B. Generalized "Miller Effect"

- An impedance connected across an amplifier with voltage gain A_{vo} can be replaced by an impedance to ground ... divided by $(1\text{-}A_{vo})$

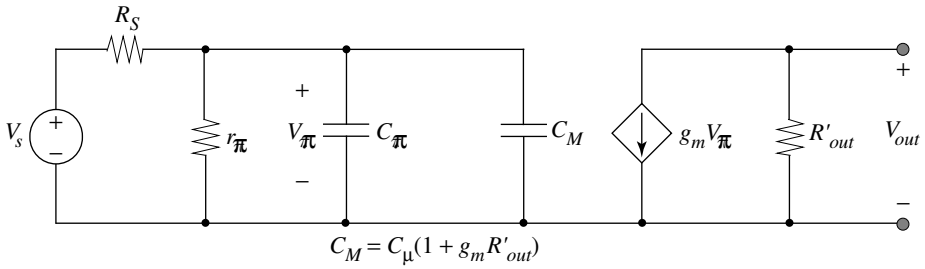


$$Z_{eff} = Z/(1 - A_{vo})$$

- Common-emitter (and common-source): A_{vo} = large and negative
- Capacitance at the input is magnified

C. Frequency for CE Volatage Amplifier Using the Miller Approximation

- The Miller capacitance is lumped together with C_π, which results in a single-pole low-pass *R*C filter at the input



$$\frac{V_{out}}{V_s} = -g_m\left(\frac{r_\pi}{r_\pi + R_S}\right)R'_{out}\left[\frac{1}{1 + j\omega\left(C_\pi + C_M\right)\left(R_S \| r_\pi\right)}\right]$$

- At DC the small signal voltage gain is

$$\frac{v_{out}}{v_s} = -g_m\left(\frac{r_\pi}{r_\pi + R_S}\right)R'_{out}$$

- The frequency at which the magnitude of the voltage gain is reduced by $1/\sqrt{2}$

$$\omega_{3dB} = \frac{1}{\left(R_S \| r_\pi\right)\left(C_\pi + C_M\right)} = \left[\frac{1}{\left(R_S \| r_\pi\right)}\right]\left[\frac{1}{C_\pi + \left(1 + g_m R'_{out}\right)C_\mu}\right]$$

III. Open Circuit Time Constant Analysis

A. Introduction

- No Zeros
- One "dominant" pole ($1/\tau_1 << 1/\tau_2, 1/\tau_3, \ldots, 1/\tau_n$)
- N capacitors

$$\frac{V_{out}}{V_s} = \frac{A_{vo}}{(1 + j\omega\tau_1)\,(1 + j\omega\tau_2)\,(\ldots)\,(1 + j\omega\tau_n)}$$

(the example shows a voltage gain ... it could by I_{out} / V_s or V_{out} / I_s)

- Multiplying out the denominator

$$\frac{V_{out}}{V_s} = \frac{A_{vo}}{1 + b_1 j\omega + b_2 (j\omega)^2 + \ldots + b_n (j\omega)^n} \text{ where } b_1 = \tau_1 + \tau_2 + \tau_3 + \ldots. \tau_n$$

- It can be shown (Gray & Meyer, 3rd ed., pp 502-506 6.301 text) coefficient b_1 can be found exactly by:

$$b_1 = \left(\sum_{i=1}^{N} R_{Ti} C_i \right) = \left(\sum_{i=1}^{N} \tau_{C_{io}} \right), \text{where } N \text{ is the number of capacitors}$$

- τ_{Cio} is the **open-circuit time constant** for capacitor C_i
- C_i is the i^{th} capacitor and R_{Ti} is the Thevenin resistance across the i^{th} capacitor terminals (with all capacitors open-circuited)

B. Estimating the Dominate Pole

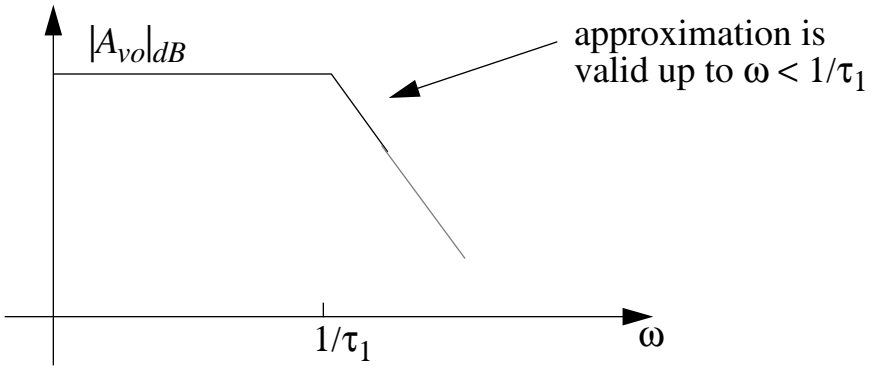
- The dominant pole of the system can be estimated by:
- $b_1 = \tau_1 + \tau_2 + \tau_3 + ... \tau_n \approx \tau_1$

$$b_1 = \underbrace{\left(\sum_i^N R_{Ti} C_i \right)}_{\textit{exact}} \approx \underbrace{\tau_1 = \frac{1}{\omega_1}}_{\textit{approximate}},$$

- ~$R_{Ti}\, C_i$ is the **open-circuit time constant** for capacitor C_i

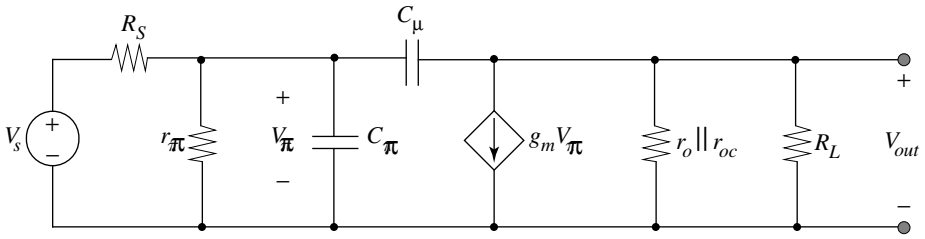
C Power of the Technique

- Estimates the contribution of each capacitor to the dominant pole frequency *separately* .
- Enables the designer to understand what part of a complicated circuit is responsible for limiting the bandwidth of the amplifier.
- The approximate magnitude Bode plot is



IV. Common Emitter Amplifier Analysis Using OCT

A. Small Signal Model



- From the Full Analysis

$$\frac{V_{out}}{V_s} = \frac{-g_m R'_{out}\left(\frac{r_\pi}{R_S + r_\pi}\right)\left(1 - \frac{C_\mu}{g_m} j\omega\right)}{1 + j\omega\left(R'_{out}C_\mu + R'_{in}C_\mu\left(1 + g_m R'_{out}\right) + R'_{in}C_\pi\right) - \omega^2 R'_{out}R'_{in}C_\mu C_\pi}$$

where $R_{in}' = R_S \| r_\pi$ and $R_{out}' = r_o \| r_{oc} \| R_L$

$$b_1 = \left(R'_{out}C_\mu + R'_{in}C_\mu\left(1 + g_m R'_{out}\right) + R'_{in}C_\pi\right)$$

$$\omega_{3dB} \approx \frac{1}{b_1} = \frac{1}{R'_{in}\left[C_\pi + \left(1 + g_m R'_{out}\right)C_\mu\right] + R'_{out}C_\mu}$$

Next Lecture we will show how to calculate b_1

V. Compare the Three Methods for analyzing the CE amp

A. Full Analysis - assumes $\tau_1 + \tau_2 \approx \tau_1$

$$\omega_{3dB} = \frac{1}{\tau_1} = \frac{1}{R'_{in}\left[C_\pi + \left(1 + g_m R'_{out}\right)C_\mu\right] + R'_{out}C_\mu}$$

B. Miller Approximation - does not take into account R'_{out}

$$\omega_{3dB} = \left[\frac{1}{R'_{in}}\right]\left[\frac{1}{C_\pi + \left(1 + g_m R'_{out}\right)C_\mu}\right]$$

C. Open Circuit Time Constant - Assumes dominant pole same as the full analysis

$$\omega_{3dB} = \frac{1}{b_1} = \frac{1}{R'_{in}\left[C_\pi + \left(1 + g_m R'_{out}\right)C_\mu\right] + R'_{out}C_\mu}$$