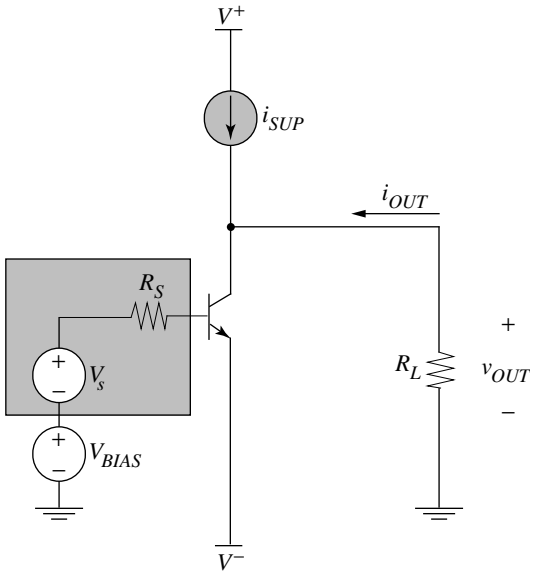
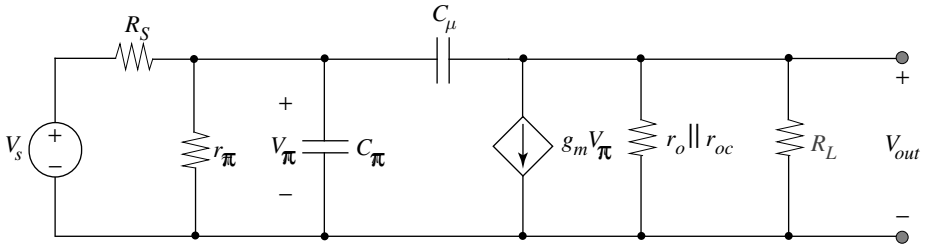
# I. Finding the Open Circuit Time Constants $\tau_j$'s

## A. Example: CE Amplifier




- Small signal model

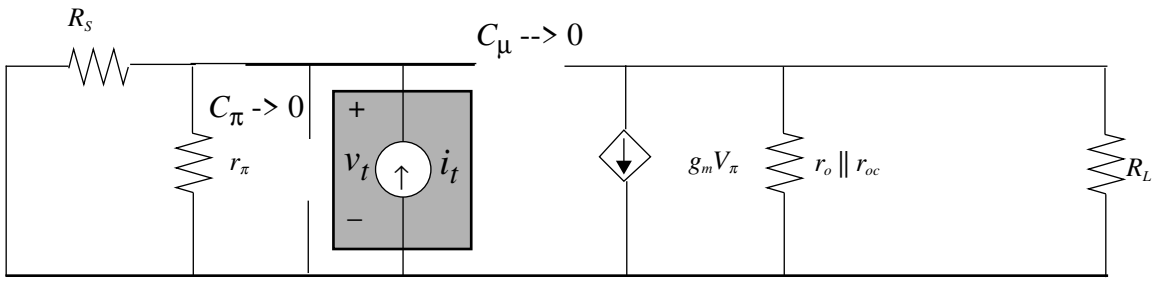



(a)

## B. Procedure

- Eliminate all independent sources (e.g., $V_s$ --> 0)
- Open-circuit all capacitors
- Find Thevenin resistance by applying $i_t$ and measuring $v_t$.

## C. Time Constant for $C_\pi$



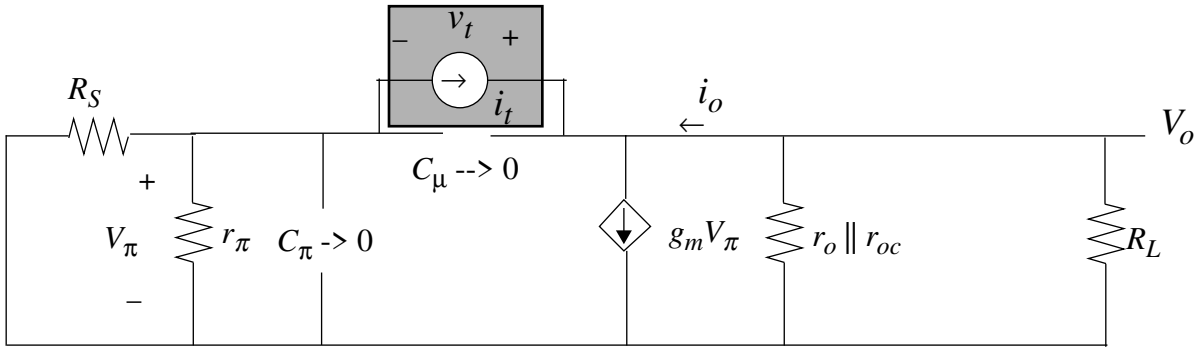

- Result: (by inspection)

$$R_{T\pi} = R_s \| r_\pi$$

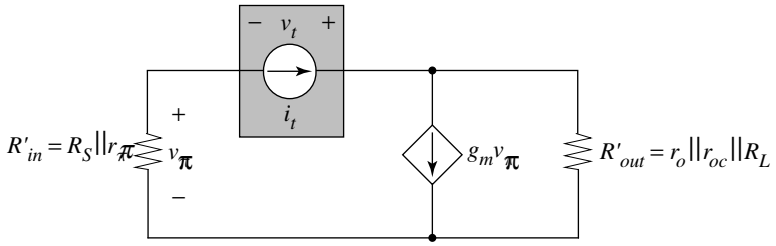
$$\tau_{C_{\pi o}} = R_{T\pi} C_\pi$$

# D. Time Constant for $C_\mu$

- Same procedure:




- Let $R'_{in} = R_s \parallel r_\pi$ and $R'_{out} = r_o \parallel r_{oc} \parallel R_L$




$$i_t = \frac{v_t + v_\pi}{R'_{out}} + g_m v_\pi \qquad \frac{v_\pi}{R'_{in}} = -i_t \text{ eliminate } v_\pi$$

$$\frac{v_t}{i_t} = R_{T\mu} = R'_{out} + R'_{in}\left(1 + g_m R'_{out}\right)$$

$$\tau_{C_{\mu o}} = R_{T\mu} C_\mu = \left[R'_{out} + R'_{in}\left(1 + g_m R'_{out}\right)\right] C_\mu$$

## E. Dominant Pole for CE Amplifier

- Sum Individual time constants

$$b_1 = (R_{T\pi}C_\pi + R_{T\mu}C_\mu)$$

$$b_1 = R'_{in}C_\pi + R'_{in}\left(1 + g_m R'_{out}\right)C_\mu + R'_{out}C_\mu$$

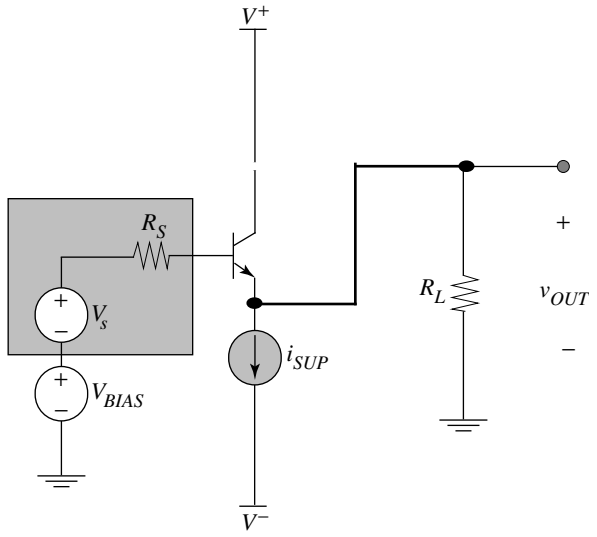
- Assume $\tau_1 >> \tau_2$

$$b_1 = \tau_1 + \tau_2 \approx \tau_1$$

$$\omega_{3dB} \approx \frac{1}{b_1} = \frac{1}{R'_{in}C_\pi + R'_{in}\left(1 + g_m R'_{out}\right)C_\mu + R'_{out}C_\mu}$$

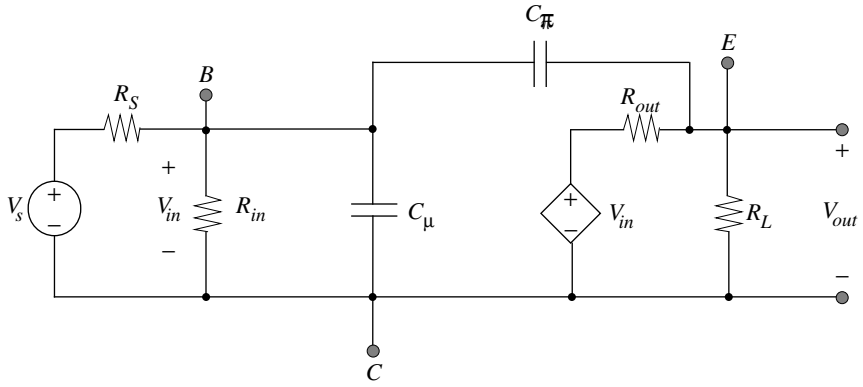
- Very similar result to the Miller effect calculation
- Addtional term $R'_{out}C_\mu$ is taken into account

# II. Common Collector Frequency Response




## A. Small Signal Model

- Add $C_\mu$ and $C_\pi$ to the two-port model from Chapter 8



## B. Low Frequency Voltage Gain

$$\frac{v_{out}}{v_s} = \left(\frac{R_{in}}{R_S + R_{in}}\right)(1)\left(\frac{R_L}{R_L + R_{out}}\right)$$

- Substituting values for input and output resistance

$$\frac{v_{out}}{v_s} = \left(\frac{r_\pi + \beta_o R_L}{R_S + r_\pi + B_o R_L}\right)(1)\left(\frac{R_L}{R_L + \left(1/g_m\right) + \left(R_S/\beta_o\right)}\right)$$

## C. Use Miller Approximation to Find Dominant Pole

- Voltage gain from B to E across $C_\pi$

$$A_{vC\pi} = \frac{R_L}{R_{out} + R_L} = \frac{R_L}{1/g_m + R_L}$$

- Total Capacitance seen at the input $C_T = (1\text{-}A_{vC\pi})C_\pi + C_\mu$

$$C_T = C_\pi\left(\frac{1/g_m}{\left(1/g_m\right) + R_L}\right) + C_\mu$$

- Thevenin resistance seen by $C_T$

$$R_T = R_S \parallel R_{in}$$

## D. Common Collector Dominant Pole

- The dominant time constant for a CC amplifier is

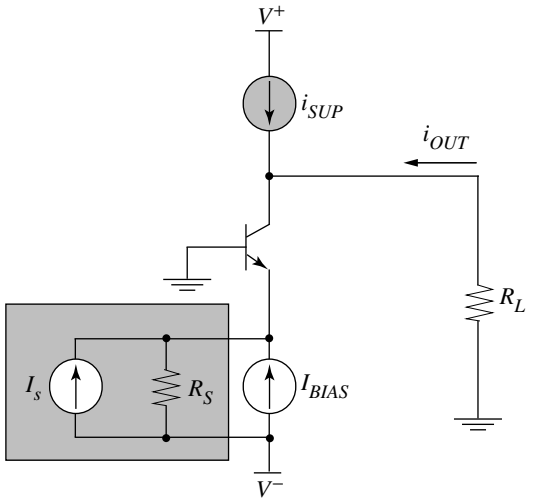
$$\tau = \left( R_S \| R_{in} \right) \left[ \frac{C_\pi}{1 + g_m R_L} + C_\mu \right]$$

- Substitute for $R_{in}$ and look at $\omega_{3dB}$

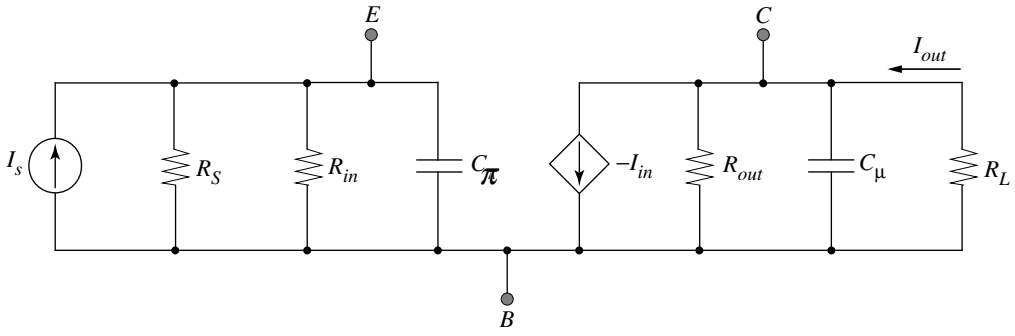
$$\omega_{3dB} = \frac{1}{\left[ \left( R_S \| \left( r_\pi + \beta_o R_L \right) \right) \left( \frac{C_\pi}{1 + g_m R_L} + C_\mu \right) \right]}$$

- Effect of $C_\pi$ is small
- In general $R_S < R_{in}$---> frequency response is dominated by $R_S C_\mu$

# III. Common Base Amplifier Frequency Response




## A. Small Signal Model




- DC Gain

$$\frac{i_{out}}{i_s} = \left(\frac{R_S}{R_{in}+R_S}\right)(1)\left(\frac{R_{out}}{R_L+R_{out}}\right)$$

## B. Use Method of Open Circuit Time Constants

- Thevenin resistance across $C_\pi$

$$R_{T\pi} = R_S \| R_{in} = R_S \| \left( 1/g_m \right) \approx 1/g_m$$

- Thevenin resistance across $C_\mu$

$$R_{T\mu} = R_{out} \| R_L \approx \beta_o r_o \| R_L \approx R_L$$

- Summing the open circuit time constants and taking reciprocal

$$\omega_{3dB} \approx \frac{1}{\left( C_\pi / g_m \right) + R_L C_\mu}$$

# IV. Summary of Frequency Response of Single-Stages

CE/CS: with voltage output - suffers from Miller effect

CE/CS: with current output - "wideband"

CB/CG: "wideband"

CC/CD: "wideband"

- "Wideband" means that the stage can operate near the frequency limit of the device ... $f_T$
- Frequency limitation is set by external circuit $R_S$ and $R_L$