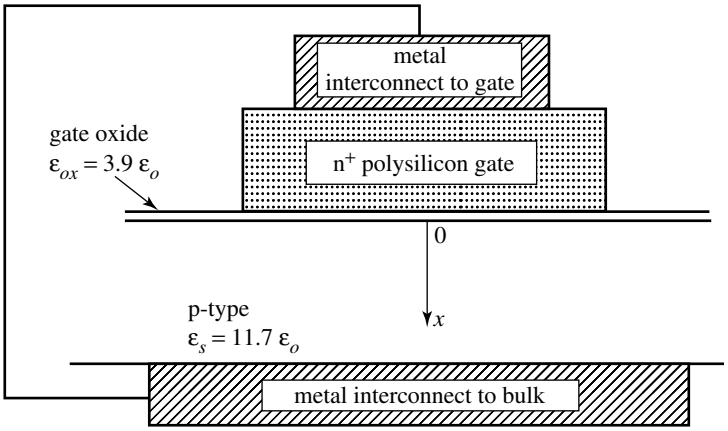
I. The MOS Capacitor in Thermal Equilibrium

A. Introduction

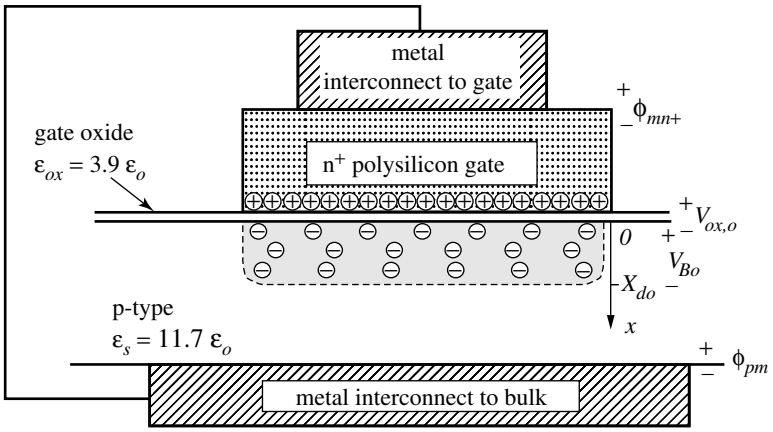


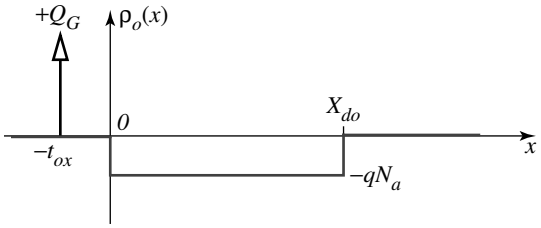
- Oxide = SiO_2 ... a near-perfect insulator.
- Assume zero charge in the oxide, electric field is constant and potential is linear in the oxide.
- n^+ polysilicon has a potential which is the maximum possible in silicon:

$$\phi_{n+} = 550 \text{ mV}$$

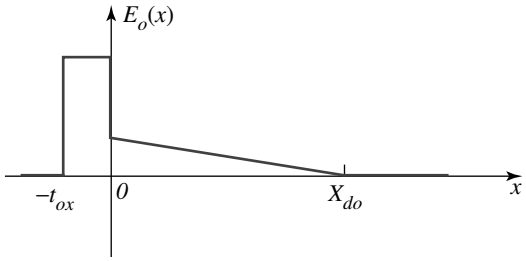
- Example parameters t_{ox}=15nm=150A $N_a = 10^{17}$ cm-3
- ϕ_p = -420 mV
- The *surface potential* ϕ_s is the potential at the S_i/S_iO_2 interfaceg
- Strategy: same as pn junction electrostatics: first thermal equilibrium, then with an applied bias.

B. Thermal Equilibrium MOS Electrostatics - Qualitative

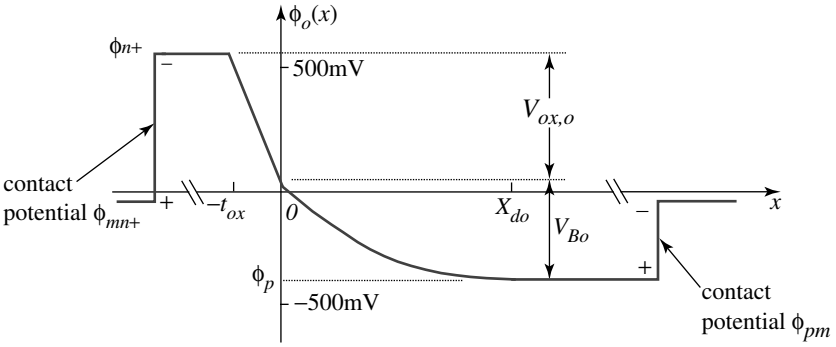




(a)



(b)



C. Quantitative Electrostatics

- E Field and Potential in OXIDE

$$E(0^+) = \frac{-Q_B}{\varepsilon_s} = \frac{qN_aX_{d_o}}{\varepsilon_s}$$

$$E(0^-) = \frac{\varepsilon_s}{\varepsilon_{ox}}\left(\frac{qN_aX_{d_o}}{\varepsilon_s}\right) = \frac{qN_aX_{d_o}}{\varepsilon_{ox}} = E_{ox}$$

$$\left(\phi_{n+} - \phi_p\right) = \left(\phi(0) - \phi_p\right) + \left(\frac{qN_aX_{d_o}t_{ox}}{\varepsilon_{ox}}\right)$$

- E Field and Potential in Silicon

$$E(x) = \frac{qN_a\left(X_{d_o} - x\right)}{\varepsilon_s}$$

$$\phi_{n+} - \phi_p = \frac{qN_aX_{d_o}}{C_{ox}} + \frac{qN_aX_{d_o}^2}{2\varepsilon_s}$$

- Result

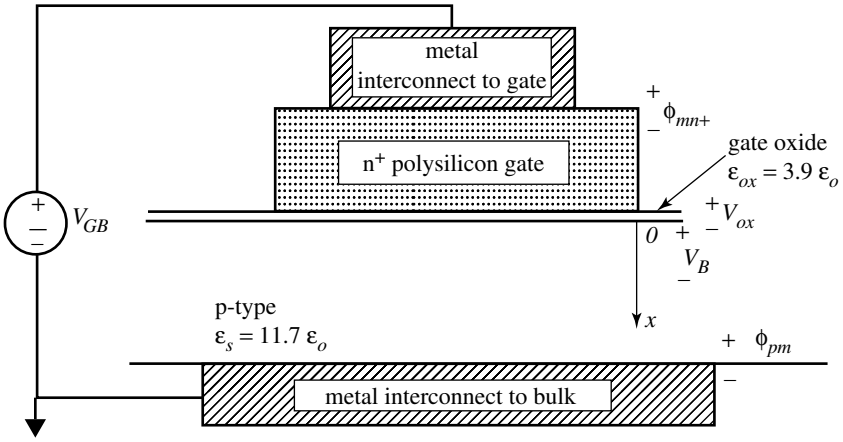
$$X_{d_o} = t_{ox}\left(\frac{\varepsilon_s}{\varepsilon_{ox}}\right)\left(\sqrt{1 + \frac{2C_{ox}^2\left(\phi_{n+} - \phi_p\right)}{q\varepsilon_s N_a}} - 1\right)$$

$$\phi_s = \phi(x=0) = \left(\phi_{n+} - \phi_p\right) - \frac{qN_aX_{d_o}}{C_{ox}} + \phi_p$$

II. MOS Capacitor under Bias

A. Introduction

- No steady-state current between the n^+ poly gate and the substrate.
- $J_n = 0$ and $J_p = 0$
- Absence of current implies that we can relate potential to carrier concentration in the silicon substrate.



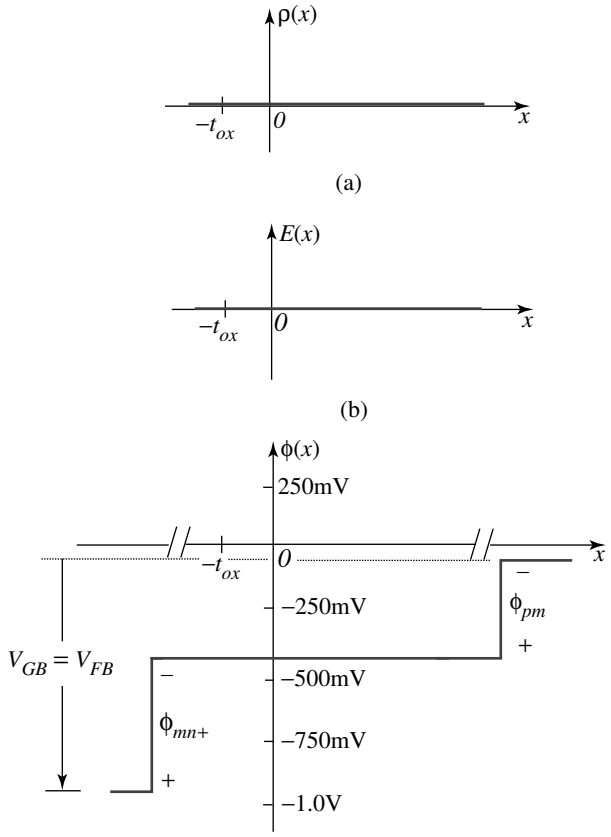
- Flatband condition: cancel built-in drop by applying

$$V_{GB} = -(\phi_{n+} - \phi_p) = V_{FB}$$

$$V_{FB} = \text{-970 mV for } N_a = 10^{17}\ \text{cm}^{-3}$$

- For $V_{GB} = V_{FB}$, the internal potential is constant through the structure since both the gate and the p-type substrate have the same potential.

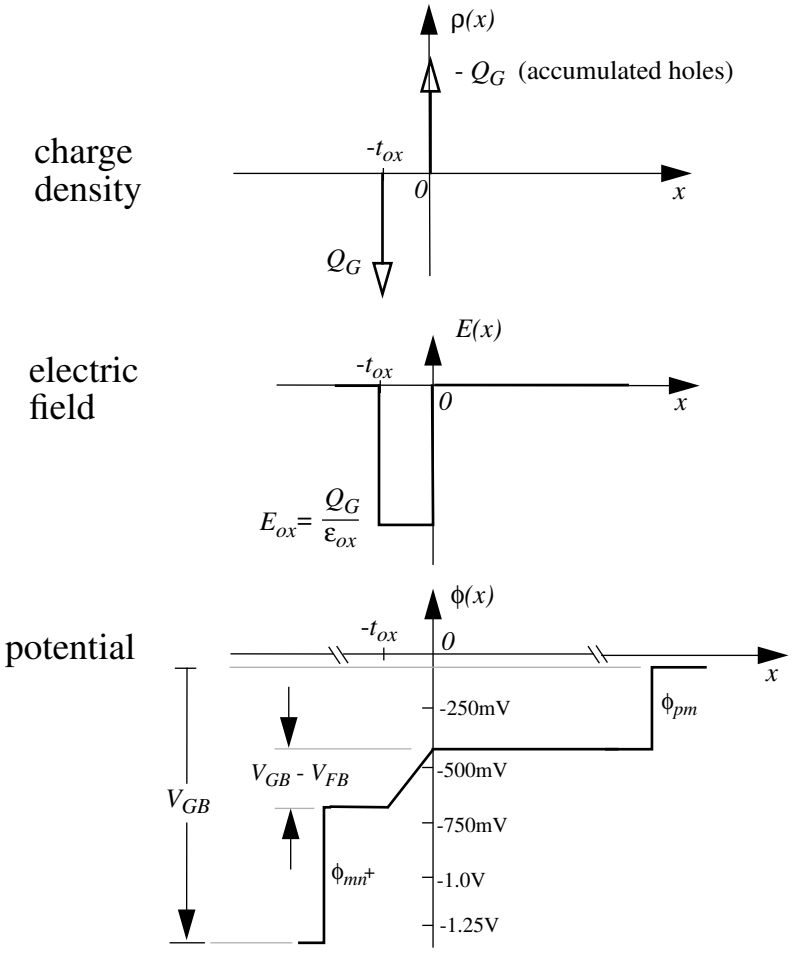
B. MOS Capacitor in Flatband



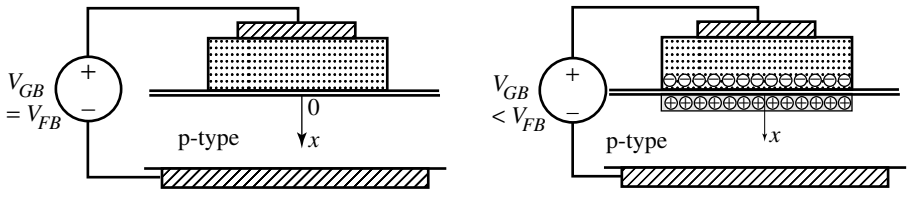
- When $V_{GB} = V_{FB}$, the gate is at the same potential as the p-type bulk -- there is no charge on the gate or in the silicon..
- When $V_{GB} < V_{FB}$, the gate is more negative than the p-type bulk and takes on a negative charge; holes are piled up (accumulated) under the oxide.

C. MOS Capacitor in Accumulation

- Charge density, electric field, and potential in accumulation: $V_{GB} < V_{FB} = -0.97$ V.



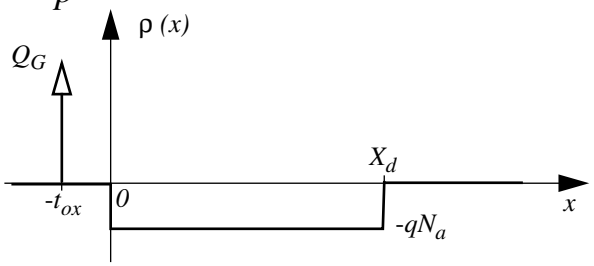
- The MOS capacitor in accumulation is a parallel plate capacitor, with the (negative) gate charge given by $Q_G = C_{ox} (V_{GB} - V_{FB})$



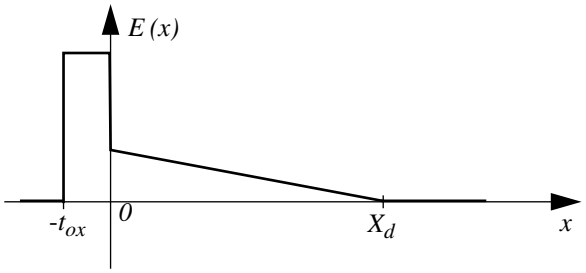
D. MOS Capacitor in Depletion

- Now we make $V_{GB} > V_{FB}$. The p-type substrate depletes--in order to have negative charge available to terminate the electric field from the positive charge on the gate.
- Analysis is the same as in TE except replace$V_{GB} - V_{FB}$ for the quantity ($\phi_n + - \phi_p$)

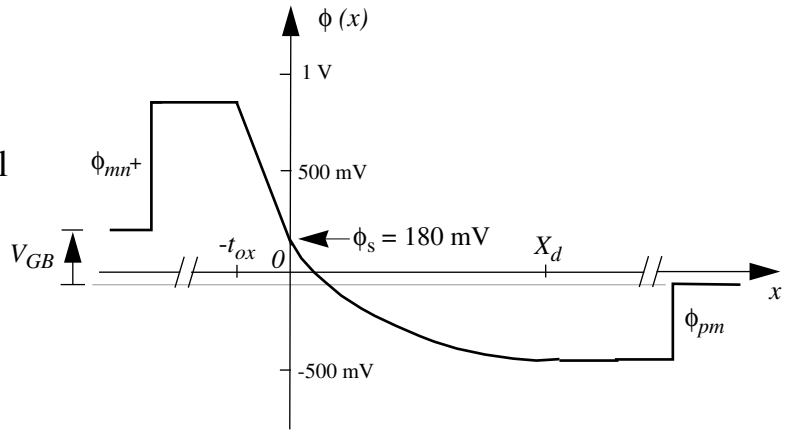
charge density



electric field



potential

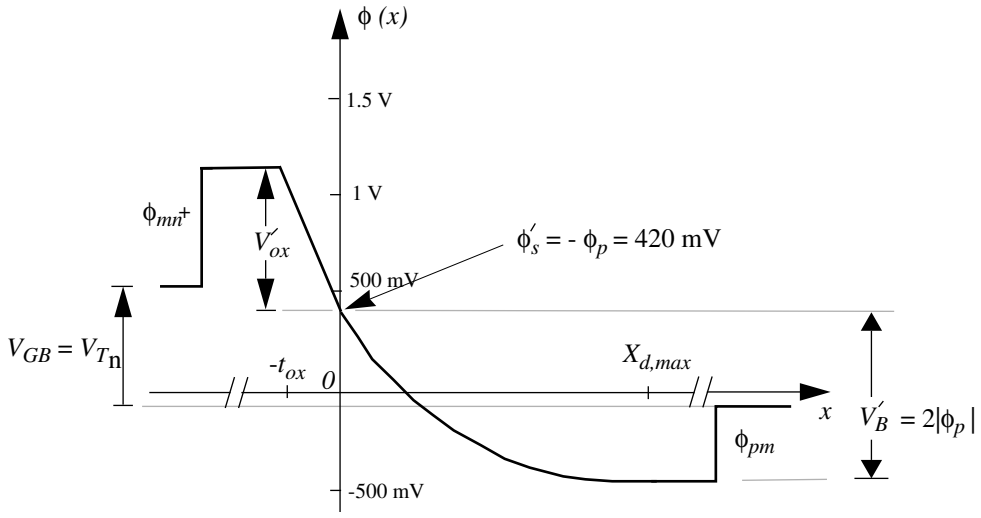


- Surface potential at oxide/silicon interface is now positive --> n-type (slightly, $n_s = 10^{13}$ cm^{-3}).

E. The Threshold Voltage

- Keep increasing V_{GB} --> surface potential keeps increasing. At some point, the surface is n-type (i.e., we say that it is *inverted*) and the electron charge is a significant contribution to the charge density.
- We approximate that onset of inversion as the point where the electron concentration n_s at the surface is the same as the hole concentration N_a in the bulk.
- The gate-bulk potential at the onset of inversion is called the *threshold voltage*, V_{Tn}.
- To find the threshold voltage, we consider the electrostatics in depletion (no electrons at the surface at the onset of inversion) -- with the surface potential equal to the opposite of the bulk potential:

$$\phi_s = -\phi_p$$



F. Threshold Voltage Expression

- We can solve for the threshold voltage:

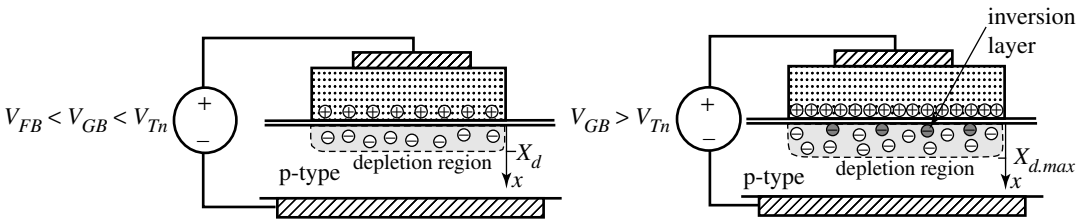
$$V_{Tn} - V_{FB} = V_{ox} + V_B$$

- The drop across the depletion region is

$$V_B = \phi_s - \phi_p = -\phi_p - \phi_p = -2\phi_p$$

- The drop across the oxide is

$$V_{ox} = E_{ox} t_{ox} = \left(\frac{-Q_B}{\varepsilon_{ox}} \right) t_{ox} = \frac{-Q_B}{C_{ox}}$$



- The bulk charge in inversion is

$$Q_B = -qN_a X_d = -qN_a \sqrt{\frac{-2\phi_p}{((1/2)\, qN_a)/\varepsilon_s}} = -\sqrt{2q\varepsilon_s N_a (-2\phi_p)}$$

- Adding up the terms, the threshold voltage is:

$$\boxed{V_{Tn} = V_{FB} - 2\phi_p + \frac{1}{C_{ox}} \sqrt{2q\varepsilon_s N_a (-2\phi_p)}}$$