MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.014 Electrodynamics

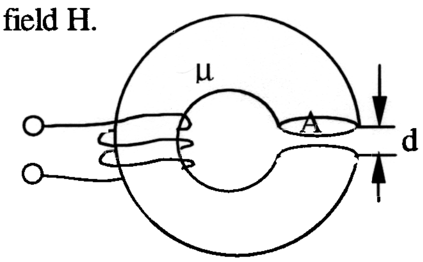
SAMPLE FINAL EXAMINATION

Closed book 3-hr test, no calculators

Answers usually are compact equations or sketches expressed only in terms of basic parameters provided in the problem statement. Note that most parts of these problems can be answered independently of the other parts.

Problem 1. (25/200pts)

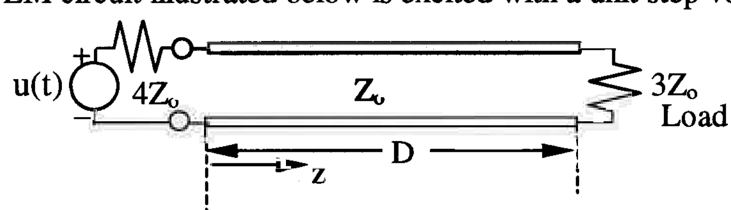
A high-permeability torus of circumference D has a narrow gap of width d, which separates pole pieces of area A. The permeability is μ_o in the gap, and inside the torus $\mu >> D\mu_o/d$. A three-turn coil carrying 5 amperes wraps around the torus, magnetizing it with field H.



a) Approximately what is H_g (a m^{-1}) in the gap?
b) In terms of H_g, approximately what is the inductance L (henries) of this toroid?

Problem 2. (35/200pts)

The air-filled TEM circuit illustrated below is excited with a unit step voltage at t = 0.



a) Sketch and dimension the voltage distribution v(z) on the line at t = D/2c.
b) Sketch and dimension the voltage distribution v(z) on the line at t = 3D/2c.
c) Sketch and dimension the voltage distribution v(z) on the line at t = 3D/2c for the case where the load resistor $3Z_o$ is in series with a capacitor $C = D/8cZ_o$ farads. Evaluate any time constants τ(sec).

Problem 3.

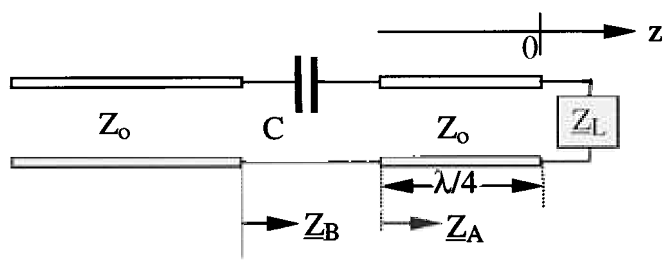
Two parallel metal plates of area A are separated by a short distance d in air and have a potential difference of V volts.

a) What is the electric energy w_e [J] stored between these two plates?

b) Using energy derivatives, derive the force f attracting these two capacitor plates.

Problem 4. (30/200pts)

A TEM transmission line of characteristic impedance Z_o is connected to a capacitor C, a quarter-wavelength TEM line, and a load impedance $\underline{Z}_L$, as illustrated.

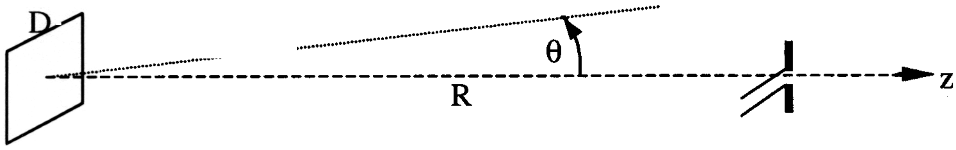


a) If the load impedance $\underline{Z}_L = -j2Z_o$ ohms, what is the impedance $\underline{Z}_A$ seen looking toward the load from the position $z = -\lambda/4$?

b) A different load impedance $\underline{Z}_L$' is installed and $\underline{Z}_B$ is then found to be matched for $C = 1/\omega Z_o$ farads. What is the new load impedance $\underline{Z}_L$'?

Problem 5. (40/200pts.)

A uniformly illuminated square aperture antenna of side length D (meters) radiates 100 watts at wavelength λ.



a) What is the maximum radiated flux density $I(\mathrm{w\ m^{-2}})$ observed at a receiver located R meters away in the main beam of the aperture antenna?

b) What is the maximum power P_r received by a matched short dipole (G = 1.5) at this distance R in the absence of any reflections?

c) At what angle θ_n (radians) off axis does the first null in the antenna pattern occur?

d) What approximate total force F(Newtons) is exerted on the aperture antenna by the radiated power P_r(watts), and in what direction?

Problem 6. (20/200pts.

A certain exotic medium is characterized by the vectors $\vec{Q}$ and $\vec{R}$, which are governed by three equations: $\nabla \times \vec{Q} = a(\partial^2/\partial t^2)\vec{R}$ $\qquad \nabla \times \vec{R} = -b\vec{Q}$ $\qquad \nabla \bullet \vec{Q} = 0$

a) Derive the wave equation for the vector $\vec{Q}$ in this medium.
Recall $\nabla \times (\nabla \times \vec{Q}) = \nabla(\nabla \bullet \vec{Q}) - \nabla^2 \vec{Q}$.

b) What is the velocity of propagation c for this vector field $\vec{Q}$?

Problem 7. (10/200pts.)

A certain rock band in a level pasture in Vermont has two isotropic loudspeakers placed one wavelength apart and driven with pressures p(t) exactly 180° out of phase. Sketch the polar acoustic power radiation pattern P(θ) relative to the loudspeakers, quantitatively indicating the angles of each maximum and each null.