MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

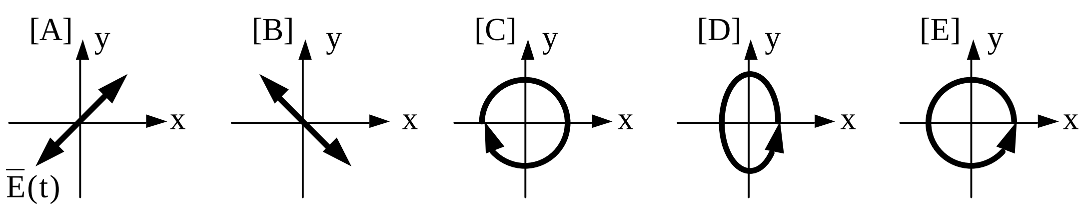
6.014 Electrodynamics—Sample quiz to illustrate type (not coverage or length)

<u>Problem 1</u>

A uniform electromagnetic wave is propagating in the +z direction, and is characterized by $\overline{E} = (\hat{x} + j\hat{y})E_o e^{jkz}$.

(a) What polarization (A-E) sketched below corresponds to this wave?

(b) A quarter-wave plate is appropriately inserted in the path of this wave with its "fast" axis aligned in the x direction. Which sketched polarization (A-E) emerges from this plate? Briefly explain your logic.



<u>Problem 2</u>

If the magnetic field in vacuum is $\overline{\underline{H}} = (x, y, z) = \hat{z}H_o e^{+jy}$,

(a) In what direction is this wave propagating?

(b) What is the numerical value of the frequency ω (radians per second)?

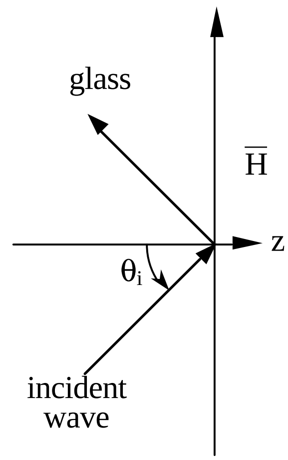
(c) What is $\overline{\underline{E}}(x, y, z)$?

<u>Problem 3</u>

A circular conducting resistive loop (R ohms) lies in the plane of this paper with an area of one square centimeter, while coming out of the paper toward the reader through the loop there is magnetic flux density $B_o \cos \omega t$ (Tesla). What current i(t) flows in this loop in a clockwise direction?

Problem 4

A plane wave inside glass is incident at z = 0 on its surface beyond the critical angle, and is perfectly reflected. Outside the glass the resulting evanescent wave has magnetic field $\overline{H} = \hat{x}H_o e^{-\alpha z}\cos(\omega t - k_y y)$.

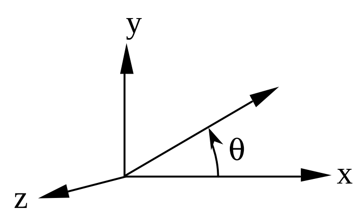


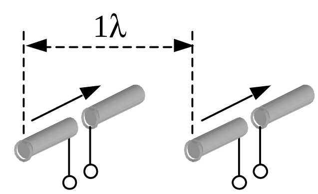
(a) Is the incident plane wave TE or TM? Briefly explain your reasons.

(b) Express the frequency ω(r s^{-1}) as a function of parameters given above.

Problem 5

An antenna array consists of two parallel z-oriented half-wavelength dipole antennas spaced one wavelength apart in the x direction; they are excited 180° out of phase at 100 MHz, as illustrated





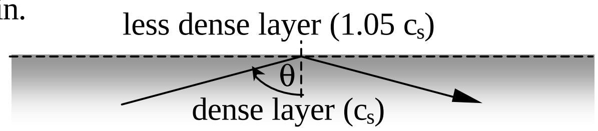
(a) Sketch the antenna gain G(θ) in the x-y plane, labeling quantitatively the angles of any gain maxima or minima.

(b) Assume the maximum gain of this antenna array is 3, and that it radiates P_t watts. What then is the maximum intensity I (Wm^{-2}) radiated by this antenna array which can be observed by a receiver R km away?

(c) If the receiver of part B has effective area A_e (m^2), and 10^{-16} watts is required for an acceptable signal-to-noise ratio, what is the maximum range R(m) at which this system can communicate?

Problem 6

The wave velocity in a certain layer is 0.5 percent faster than the wave velocity c_s in the denser layer below it, as illustrated. What is the critical angle θ_c of a wave in this case (if any)? Briefly explain.



Quiz Solutions:

Problem 1 $\quad \overline{E} = (\hat{x} + j\hat{y})e^{-jkz}$

(a) $\rightarrow E_x \sim \cos \omega t$; $E_y \sim \sin \omega t \Rightarrow \boxed{C}$ LH

(b) x = "fast axis" $\rightarrow E_x$ slides forward to $-\sin \omega t \Rightarrow \boxed{A}$ LP $\overline{E} = (j\hat{x} + j\hat{y})e^{-jkz}$

Problem 2

$$\overline{H}(x,y,z,t) = \text{Re}\left\{\hat{z}H_o e^{jy} e^{j\omega t}\right\}$$
$$= \hat{z}H_o \cos(\omega t + ky) \quad \text{with } k = 1 \text{ rad/m}$$

(a) $\boxed{\text{Propagating in } -\hat{y} \text{ direction.}}$

(b) $\omega = kc = (1 \text{ rad/m})(3 \times 10^8 \text{ m/s}) = \boxed{3 \times 10^8 \text{ rad/s}}$

(c) $|E| = \eta_o |H|$ and $\overline{E} \times \overline{H}$ in $-\hat{y}$ direction $\Rightarrow \boxed{\overline{E} = \hat{x}\eta_o H_o e^{jy}}$

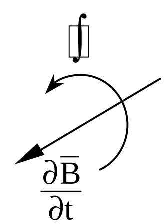
Problem 3

clockwise $\quad$ counterclockwise

$$i(t) = -V(t)/R, \qquad V(t) = \oint \overline{E} \bullet d\overline{\ell} = -\frac{\partial}{\partial t}\int_A \overline{B} \bullet d\overline{a}$$
$$= -\frac{\partial}{\partial t} B_o \cos \omega t A$$
$$= B_o A \omega \sin \omega t$$

$$\boxed{i(t) = \frac{-B_o 10^{-4} \omega}{R} \sin \omega t}$$

$A = 10^{-4} m^2$



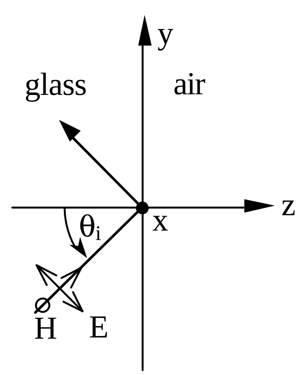
Problem 4

In air

(a) $\overline{H} = \hat{x}H_o e^{-\alpha z} \cos(\omega t - k_y y) \Rightarrow k_{tz} = -j\alpha$ (in air)

$\overline{k}_t = k_y \hat{y} - j\alpha\hat{z} \Rightarrow$ plane of incidence is x-y plane

$\overline{H}$ oscillates $\perp$ to plane of incidence $\Rightarrow \boxed{\text{TM wave}}$



(b) $k_t = \omega\sqrt{\varepsilon_o \mu_o}$ in air $\Rightarrow \omega = ck_t$

$k_t = \sqrt{k_y{}^2 - \alpha^2} \Rightarrow \omega = c\sqrt{k_y{}^2 - \alpha}$