MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.014 Electrodynamics

QUIZ 1

Closed Book, no calculators March 19, 2002

Note the formulas presented on the reverse side. Please present numerical answers to the extent practical without a calculator or tedious computation, and circle your answers.

Problem 1. (15/100 points)

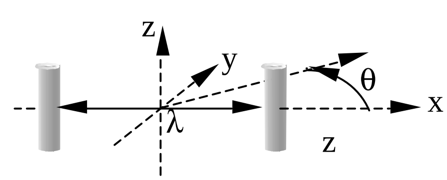
Show that the electric field $\overline{\underline{E}} = 5\hat{z}e^{-jkz}$ does not satisfy $\nabla \bullet \overline{\underline{D}} = 0$, which means that this field cannot exist in a charge-free vacuum.

Problem 2. (15/100 points)

A certain matched antenna radiates one watt isotropically (i.e. in all directions) when its terminals are excited with $\underline{V}$ = 10 volts. What is the radiation resistance R_r of this antenna?

Problem 3. (30/100 points)

A certain transmitting array antenna has two z-oriented dipoles excited in phase and spaced λ apart along the x axis.



a) Give the direction θ from the x axis of any one null in the x-y plane. (Recalling radiation-pattern formulas is an awkward way to solve this problem)

b) Does this antenna array have a null along the x, y, or z axes? Explain briefly.

Problem 4. (30/100 points)

How much power P_r can be received across a 10-km link in free space at λ = 4 meters using a 1-Watt transmitter and two matched antennas, each with gain G = π?

Problem 5. (10/100 points)

An evanescent wave in vacuum varies as $e^{-j5z + 3x}$. What is its frequency ω [rs^{-1}]?

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QUIZ 1 SOLUTIONS

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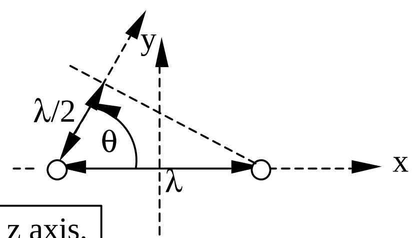
Problem 1.

$\nabla \bullet \overline{D} = (\hat{x}\,\partial/\partial x + \hat{y}\,\partial/\partial y + \hat{z}\,\partial/\partial z)\bullet 5\,\hat{z}\,e^{-jkz} = \partial/\partial z(5e^{-jkz}) = \boxed{-jk5e^{-jkz} \neq 0,\ \text{QED}}$

Problem 2.

$P_{rad} = |V|^2/2R_r \Rightarrow R_r = |V|^2/2P_{rad} = 10^2/2 = \boxed{50\ \text{ohms}}$

Problem 3.



a) Nulls occur here when two phasors cancel; see figure. This happens for $\theta = \cos^{-1}0.5 = \boxed{60^o}$.

b) There are additional nulls along the dipole axes, i.e. $\boxed{\text{the z axis.}}$

Problem 4.

For maximum power the two antennas face each other. $P_{rec} = (P_{trans}G_t/4\pi r^2)A_e$, where the effective area A_e of the receiver is $A_e = G_{rec}\lambda^2/4\pi$. So we have $P_{rec} = P_{trans}G_tG_{rec}(\lambda/4\pi r)^2 = 1\times\pi^2\times(4/4\pi 10^4)^2 = \boxed{10^{-8}\ \text{watts.}}$

Problem 5.

$k_o^2 = k_x^2 + k_z^2 = (\omega/c)^2 = 5^2 - 3^2 = 4^2$, so $\omega = 4c = \boxed{1.2\times 10^9\ \text{rs}^{-1}}$