6.014 Final Exam Solutions 5-00

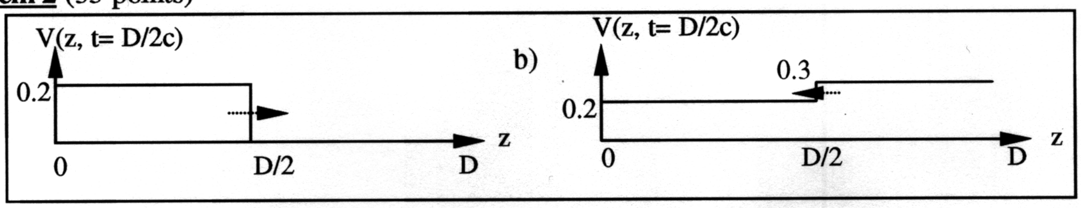
Problem 1 (25 points)

a) The integral of H around the toroid equals the current flowing through it, or 3 x 5 amperes. But this integral =~$HD + H_g d = DB/\mu + dB_g/\mu_o$, where $\mu >> D\mu_o/d$ so that the first term is negligible and the integral =~ $dB_g/\mu_o = dH_g$, and $H_g = 15/d$ (a m^{-1}). Note that B is continuous across the gap.

b) $L = \Lambda/I$ where $\Lambda = \mu_o H_g A3$ since B is continuous across the gap boundaries and is conserved within the toroid as it threads the 3I amperes. Therefore $L = \mu_o H_g A/5$ = $L = 9\ \mu_o A/d$ (henries).

Problem 2 (35 points)

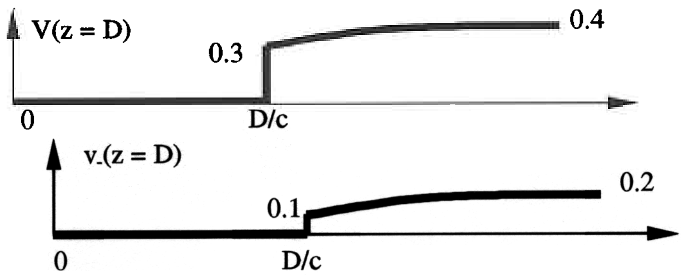
a)

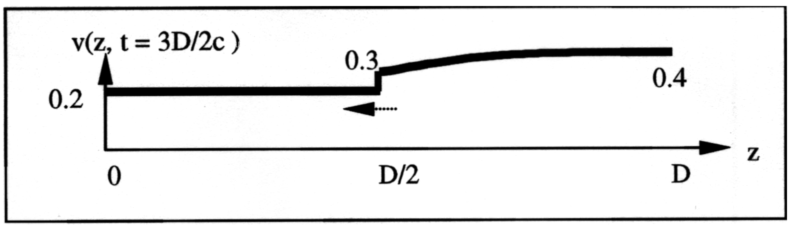


$\Gamma = (Z_n - 1)/(Z_n + 1) = (3-1)/(3+1) = 0.5$

c) The Thevenin equivalent source at the end of the TEM line is twice the forward wave, or a 0.4 volt step function, delayed by D/c seconds. This charges the capacitor with $\tau = RC = 4Z_o \times D/8cZ_o = D/2c$ (seconds). The voltage V(z=D) across the load rises instantly to ¾ of 0.4 volts, since the C voltage is initially 0. Then it rises further exponentially ($\tau = D/c$) to the open circuit value of 0.4 volts, which is:

The reflected wave, $v_-(t)$ is given by: $v_-(t) = 0.4 - v_+(t)$.





Problem 3.

a) $w_e = CV^2/2 = \boxed{\varepsilon_o AV^2/2d \text{ [J]}}$

b) $f = \partial w_e/\partial d = \partial\{CV^2/2\}/\partial d$. But $V = f(d)$, so try $f = \partial(Q^2/2C\}/\partial d$
$= \partial(dQ^2/2\varepsilon_o A\}/\partial d = Q^2/2\varepsilon_o A = C^2V^2/2\varepsilon_o A = \boxed{\varepsilon_o AV^2/2d^2 \text{ [N]}}$

Problem 4.

a) $\underline{Z}_{Ln} = -j2$, $\underline{Z}_{An} = 1/\underline{Z}_{Ln}$ (use Smith chart or eqn), $\underline{Z}_A = \underline{Z}_o/\underline{Z}_{Ln} =$
$\boxed{\underline{Z}_A = jZo/2}$

b) $\underline{Z}_B = Z_o = \underline{Z}_A + 1/j\omega C$. So $\underline{Z}_{An} = 1 + j$ since $C = 1/\omega Z_o$
$Z_{Ln}' = 1/Z_{An}$, $Z_L' = Z_o(1/(1 + j)) = \boxed{Z_o(1-j)/2}$

Problem 5. (40 points)

a) $I = G\ 100/4\pi R^2$ where $G = A4\pi/\lambda^2$, so $\boxed{I = 100(D/\lambda R)^2 \text{ (W/m}^2)}$

b) $A = G\ \lambda^2/4\pi$, $G = 3/2$, and $\boxed{P_r = AI = 37.5(D/R)^2/\pi \text{ (W)}}$

c) $\boxed{\theta_n = \lambda/D \text{ radians}}$

d) P_r/hf = #photons/sec @ momentum = hf/c. Force = momentum/sec = $P_r/c = 100/c$
$\boxed{F = 33 \times 10^{-8} \text{ Newtons in the -z direction.}}$

Problem 6. (20 points)

a)
$$\nabla\times(\nabla\times Q) = a(\partial^2/\partial t^2)\nabla\times R = -ab(\partial^2/\partial t^2) = -\nabla^2 Q$$

$$\boxed{(\nabla^2 - ab\partial^2/\partial t^2)Q = 0}$$

b) $\boxed{c_Q = (ab)^{-0.5}}$

Problem 7. (10 points)