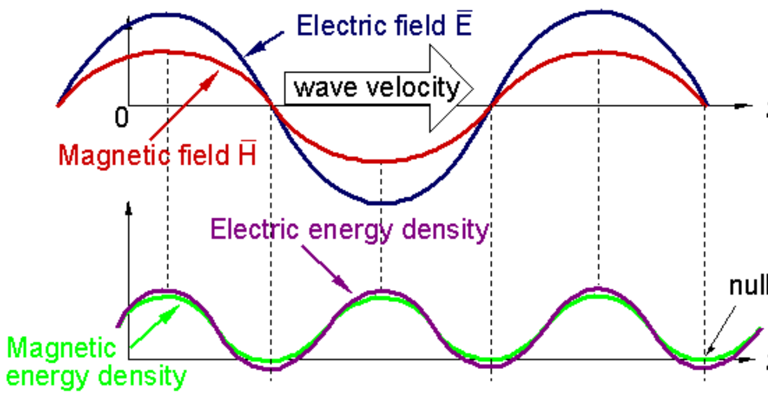
WHAT ARE ELECTROMAGNETIC WAVES?

A "wave" is a periodic disturbance propagating through a medium

EM Waves Convey Undulations in EM Fields:



Electric and Magnetic Fields are Useful Fictions:

Explain all classical electrical experiments with simple equations

4 Maxwell's equations plus the Lorentz force law

(quantum effects are separate)

L2-1

WHAT ARE ELECTRIC AND MAGNETIC FIELDS?

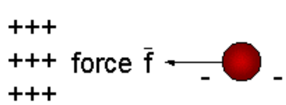
Lorentz Force Law: $\bar{f} = q(\bar{E} + \bar{v} \times \mu_o \bar{H})$[Newtons]

$\bar{E}$	Electric field	[volts/meter; V/m]
$\bar{H}$	Magnetic field	[amperes/meter; A/m]
$\bar{f}$	Mechanical force	[newtons; N]
q	Charge on a particle	[coulombs; C]
$\bar{v}$	Particle velocity vector	[meters/second; m/sec]
μ_o	Vacuum permeability	[1.26×10^{-6} Henries]

Electric and Magnetic Fields are what Produce Force $\bar{f}$:

$\bar{f} = q\bar{E}$ when $\bar{v} = 0$, defining $\bar{E}$ via an observable

$\bar{f} = q\bar{v} \times \mu_o \bar{H}$ when $\bar{E} = 0$, defining $\bar{H}$ via an observable



L2-2

Main points of L2:

- $\bar{E}$ is electric field vector [Vm^{-1}]; $\bar{H}$ is magnetic field vector [Am^{-1}]
- ε_o, μ_o are vacuum permittivity, permeability (8.854×10^{-12} [fm^{-1}], 1.257×10^{-6} [hm^{-1}])
- Faraday's law: $\nabla\times \bar{E} = -\partial \bar{B}/\partial t$ and $\int_c \bar{E}\bullet ds = -\int_A(\partial \bar{B}/\partial t)\bullet d\bar{a}$ [integral form]
- Ampere's law: $\nabla\times \bar{H} = \bar{J} + \partial \bar{D}/\partial t$ and $\int_c \bar{H}\bullet ds = \int_A(\bar{J} + \partial \bar{D}/\partial t)\bullet d\bar{a}$ [integral form]
- Gauss's laws: $\nabla\bullet \bar{D} = \rho$ [Cm^{-3}] and $\int_A \bar{D}\bullet d\bar{a} = \int_V \rho\, dv$; $\nabla\bullet \bar{B} = 0$ and $\int_A \bar{B}\bullet d\bar{a} = 0$
- "Del"$\nabla \equiv \hat{x}\partial/\partial x + \hat{y}\partial/\partial y + \hat{z}\partial/\partial z$; $\bar{A}\bullet \bar{B}=$; $\bar{A}\times \bar{B}=$; $\nabla\bullet \bar{A}=$; $\nabla\times \bar{A}=$; $\nabla\bullet(\nabla\phi)= \nabla^2\phi$?
- EM wave equation: $\nabla^2 \bar{E} - \mu_o\varepsilon_o\partial^2 \bar{E}/\partial t^2=0$; Many soln: e.g. $E_y(z,t) = E_+(t - z/c)$ [Vm^{-1}]
- WE soln: $E_+(\bullet)$ can be arbitrary function of the argument, a linear combination of z, t
- WE soln: $(\mu_o\varepsilon_o)^{-0.5} = c \cong 3\times10^8\ ms^{-1}$ in vacuum *continued…*

MAXWELL'S EQUATIONS

Differential Form:

[1]Tesla = Webers/m^2 = 10^4 Gauss

Faraday's Law: $\nabla \times \bar{E} = -\partial \bar{B}/\partial t$

Ampere's Law: $\nabla \times \bar{H} = \bar{J} + \partial \bar{D}/\partial t$

Gauss's Laws: $\nabla \bullet \bar{D} = \rho$, $\nabla \bullet \bar{B} = 0$

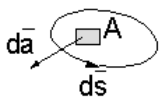
$\bar{E}$	Electric field	[volts/meter; V/m]
$\bar{H}$	Magnetic field	[amperes/meter; A/m]
$\bar{B}$	Magnetic flux density	[Tesla; T][1]
$\bar{D}$	Electric displacment	[coulombs/m^2; C/m^2]
$\bar{J}$	Electric current density	[amperes/m^2; A/m^2]
ρ	Electric charge density	[coulombs/m^3; C/m^3]

Integral Form:

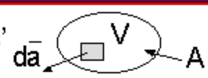
$$\int_c \bar{E} \bullet d\bar{s} = \int_A -(\partial \bar{B}/\partial t) \bullet d\bar{a} \qquad \int_A \bar{D} \bullet d\bar{a} = \int_V \rho\, dv$$

$$\int_c \bar{H} \bullet d\bar{s} = \int_A (\bar{J} + \partial \bar{D}/\partial t) \bullet d\bar{a} \qquad \int_A \bar{B} \bullet d\bar{a} = 0$$

Stoke's Theorem



Gauss' Law



L2-3

VECTOR OPERATORS ∇, $\times$, $\bullet$

"Del" (∇) Operator: Gradient of ϕ: $\nabla = \hat{x}\partial/\partial x + \hat{y}\partial/\partial y + \hat{z}\partial/\partial z$

$$\nabla\phi = \hat{x}\partial\phi/\partial x + \hat{y}\partial\phi/\partial y + \hat{z}\partial\phi/\partial z$$

"Vector Cross Product":

$\bar{A} = \hat{x}A_x + \hat{y}A_y + \hat{z}A_z$

$$\bar{A} \times \bar{B} = \det\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{x}(A_yB_z - A_zB_y) + \hat{y}(A_zB_x - A_xB_z) + \hat{z}(A_xB_y - A_yB_x)$$

"Vector Dot Product": $\bar{A} \bullet \bar{B} = A_xB_x + A_yB_y + A_zB_z$

"Divergence of $\bar{A}$": $\nabla \bullet \bar{A} = \partial A_x/\partial x + \partial A_y/\partial y + \partial A_z/\partial z$

"Curl of $\bar{A}$":

$$\nabla \times \bar{A} = \det\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

"Laplacian Operator": $\nabla \bullet (\nabla\phi) = \nabla^2\phi = (\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2)\phi$

L2-4

Main points of L2: (continued)

- Sinusoidal WE soln: e.g. $E_y(z,t) = E_+\cos(\omega t - kz)$ where $k = \omega/c = 2\pi/\lambda = \omega(\mu_o\varepsilon_o)^{0.5}$
- Can find $\bar{H}$ from $\bar{E}$ using $\nabla\times \bar{E} = -\partial \bar{B}/\partial t$; e.g. here $H_x = (-E_+/\eta_o)\cos(\omega t - kz)$

MAXWELL'S EQUATIONS: VACUUM SOLUTION

Maxwell's Equations:

Faraday's Law: $\nabla \times \bar{E} = -\partial \bar{B}/\partial t$ Gauss's Laws $\nabla \bullet \bar{D} = \rho \rightarrow 0$

Ampere's Law: $\nabla \times \bar{H} = \bar{J} + \partial \bar{D}/\partial t$, with $\bar{J} \rightarrow 0$ $\nabla \bullet \bar{B} = 0$

$\bar{B} = \mu_o \bar{H}$

$\bar{D} = \varepsilon_o \bar{E}$

EM Wave Equation:

Eliminate $\bar{H}$: $\nabla \times (\nabla \times \bar{E}) = -\mu_o (\partial/\partial t)(\nabla \times \bar{H})$

Use identity: $\nabla \times (\nabla \times \bar{A}) = \nabla(\nabla \bullet \bar{A}) - \nabla^2 \bar{A}$

Yields: $\nabla(\nabla \bullet \bar{E}) - \nabla^2 \bar{E} = -\mu_o (\partial/\partial t)(\nabla \times \bar{H}) = -\mu_o \varepsilon_o \partial^2 \bar{E}/\partial t^2$, with $\nabla(\nabla \bullet \bar{E}) \rightarrow 0$

EM Wave Equation[1] $\nabla^2 \bar{E} - \mu_o \varepsilon_o \partial^2 \bar{E}/\partial t^2 = 0$

Since:

Second derivative in space = const × second derivative in time,

Solution is any $f(\bar{r},t)$ with identical space and time dependences

[1]Homogeneous Vector Helmholz Equation

L2-5

WAVE EQUATION SOLUTION

Wave Equation has Many Solutions!

$$\nabla^2 \bar{E} - \mu_o \varepsilon_o \partial^2 \bar{E}/\partial t^2 = 0$$

Where $\nabla^2 \phi = \left(\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2\right)\phi$

Example:

Try: $\bar{E} = \hat{y} E_y(z,t)$ [no x,y dependence, "UPW"]

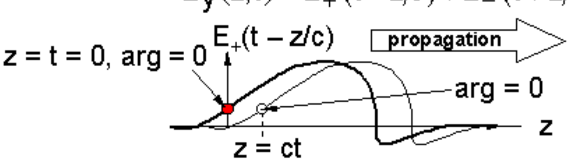
$$\left(\partial^2/\partial z^2\right) E_y - \mu_o \varepsilon_o \partial^2 E_y/\partial t^2 = 0$$

$E_y(z,t) = E_+(t - z/c)$, where $E_+(.)$ is an arbitrary function

Test: $(-c)^{-2} E_+''(t - z/c) = \mu_o \varepsilon_o E_+''(t - z/c) \Rightarrow$ **$c = 1/\sqrt{(\mu_o \varepsilon_o)}$**

Generally: $c \cong 3 \times 10^8$ [m/s] in vacuum

$E_y(z,t) = E_+(t - z/c) + E_-(t + z/c)$ more generally



The position ● where arg = 0 moves at velocity c

L2-6

UNIFORM PLANE WAVE IN Z-DIRECTION

Electric Fields (Example): $E_y(z,t) = E_+(t - z/c)$ [V/m]

More specifically, let: $E_y(z,t) = E_+ \cos \omega(t - z/c) = E_+ \cos(\omega t - kz)$,
where $k = \omega/c = \omega\sqrt{\mu_o \varepsilon_o}$

Magnetic Fields: Use Faraday's Law: $\nabla \times \bar{E} = -\partial \bar{B}/\partial t$

$$\nabla \times \bar{E} = \det \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x_{\,0} & \partial/\partial y_{\,0} & \partial/\partial z \\ E_{x\,0} & E_y & E_{z\,0} \end{vmatrix} = -\hat{x}\,\partial E_+ \cos(\omega t - kz)/\partial z$$

$$= -\hat{x} k E_+ \sin(\omega t - kz)$$

$$\bar{H}(z,t) = \hat{x}(1/\mu_o)\int k E_+ \sin(\omega t - kz)\,dt \quad [\text{A/m}]$$

$$= -\hat{x}(kE_+/\omega\mu_o)\cos(\omega t - kz) = -\hat{x}(E_+/\eta_o)\cos(\omega t - kz)$$

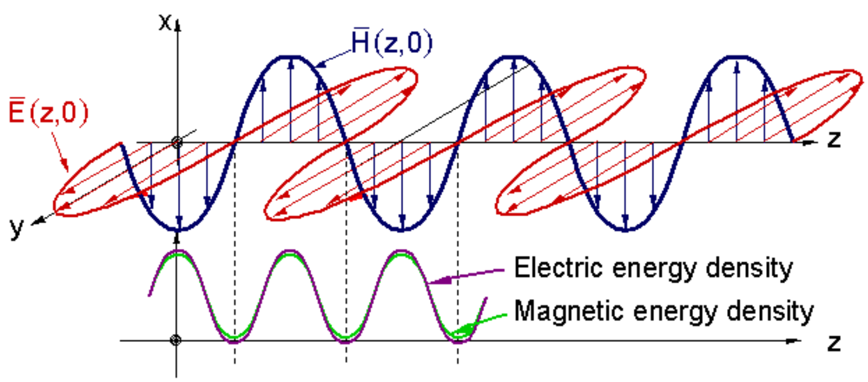
$$\left[\frac{k}{\omega} = \frac{1}{c} = \sqrt{\mu_o \varepsilon_o} \;\; ; \; \eta_o = \sqrt{\mu_o/\varepsilon_o}\right]$$

L2-7

UNIFORM PLANE WAVE EM FIELDS

EM Wave in z direction:

$$\bar{E}(z,t) = \hat{y}E_+(\omega t - kz)\ , \ \bar{H}(z,t) = -\hat{x}(E_+/\eta_o)\cos(\omega t - kz)$$



L2-8