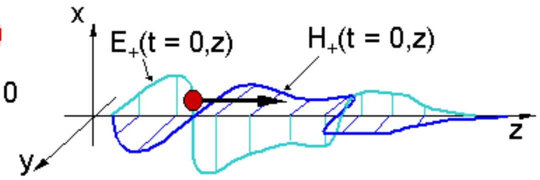
WIRELESS COMMUNICATIONS

Nature of waves: (review)

$\nabla \times \bar{E} = -\mu_0 \, \partial \bar{H}/\partial t \qquad \nabla \bullet \bar{E} = \rho/\varepsilon_0 = 0$

$\nabla \times \bar{H} = \varepsilon_0 \, \partial \bar{E}/\partial t \qquad \nabla \bullet \bar{H} = 0$



Yields Wave Equation: $\nabla^2 \bar{E} - \mu_0 \varepsilon_0 \, \partial^2 \bar{E}/\partial t^2 = 0$

Many (infinite number) solutions to Wave Equation:

e.g. $E_x(z,t) = E_+(z - ct) + E_-(z + ct)$ where $c = 1/\sqrt{\mu_0 \varepsilon_0}$
= forward + backward traveling wave
= arbitrary function $E_{+/-}$(argument)

e.g. $E_x(z,t) = E_o \cos(\omega t - kz)$ [arg = constant] marks spot ● on waveform
If $\omega t = kz$, then d/dt $(\omega t - kz)$ yields $dz/dt = \omega/k = c$
$H_y(z,t) = (E_o/\eta_o) \cos(\omega t - kz)$

In general:

$\bar{E}(\bar{r},t) = \sum_i \bar{E}_i(\bar{r},t)$ = sum of uniform plane waves in various directions i

L3-1

NATURE OF WAVES--POLARIZATION

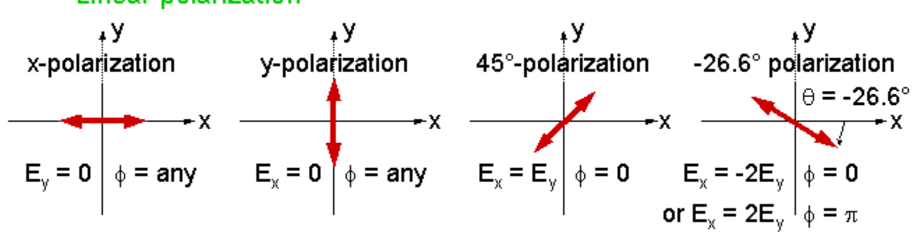
Most communications is narrowband, and therefore the signals are roughly sinusoidal.

Z-moving waves can generally have electric field components in both x and y directions.

e.g. Let $\bar{E}(r,t) = \hat{x} E_x \cos(\omega t - kz) + \hat{y} E_y \cos(\omega t - kz - \phi)$

Plots of $\bar{E}(z = 0,t)$
Cases:

Linear polarization



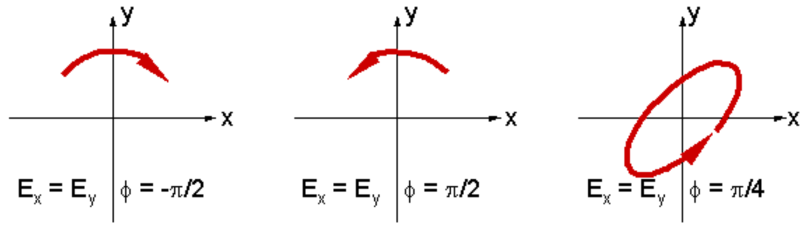
L3-2

Main points of L3: Polarized waves and phasors

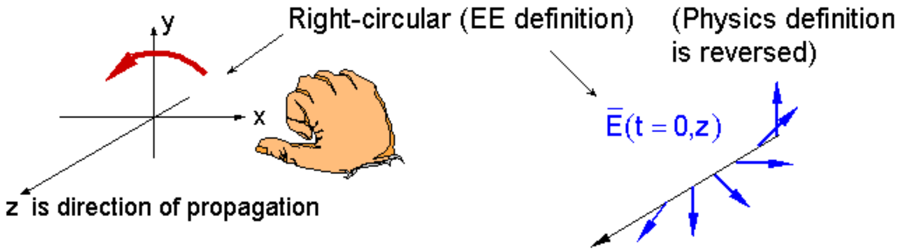
- e.g. +z wave: $\bar{E}(r,t) = \hat{x}E_x \cos(\omega t - kz) + \hat{y}E_y \cos(\omega t - kz - \phi)$, ϕ, E_x/E_y determine pol.
- If $\phi = 0$, then any linear polarization can be chosen using E_x, E_y
- If $(\phi = \pm\pi/2)$ and $(E_x = E_y)$ $\Rightarrow$ circular polarization; else $\Rightarrow$ elliptical polarization
- Convention: $\bar{E}(\bar{r},t) = R_e\{\underline{\bar{E}}(\bar{r},t) e^{j\omega t}\}$
- Example: $\underline{\bar{E}}(\bar{r}) = 5(\hat{x} + j\hat{y}) e^{-jkz} \Rightarrow \bar{E}(\bar{r},t) = 5[\hat{x}\cos(\omega t - kz) - \hat{y}\sin(\omega t - kz)]$
- Antennas are polarized and respond to $\bar{E}$ or $\bar{H}$, or to both

CIRCULAR AND ELLIPTICAL POLARIZATION

Let $\overline{E}(r,t) = \hat{x}E_x \cos(\omega t - kz) + \hat{y}E_y \cos(\omega t - kz - \phi)$

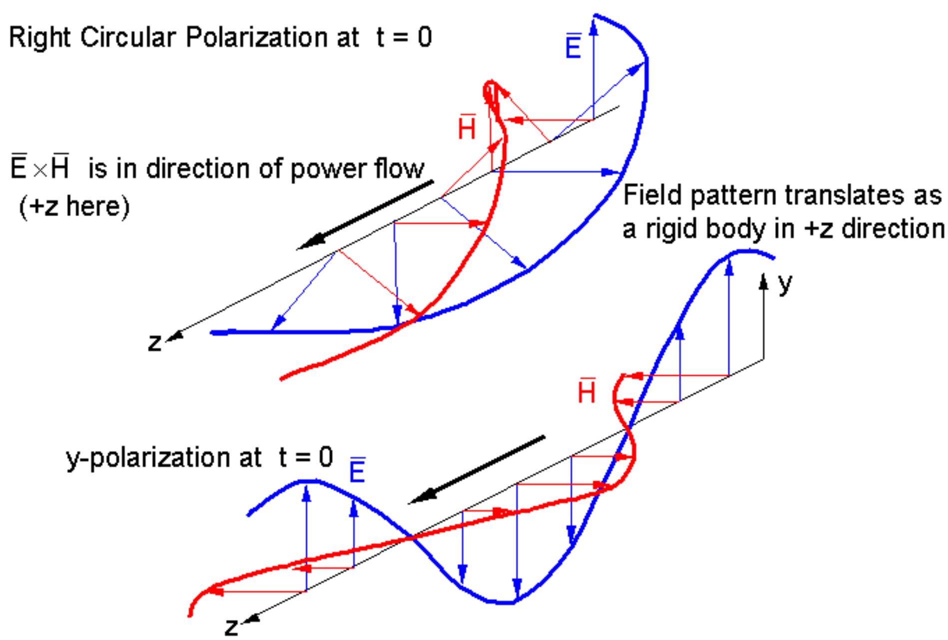


Handedness:



L3-3

EXAMPLES OF POLARIZED WAVES



L3-4

POLARIZED WAVES--REPRESENTATION

Time Domain Example:

$$\bar{E}(r,t) = \hat{x}E_x \cos(\omega t - kz - \phi_x) + \hat{y}E_y \cos(\omega t - kz - \phi_y)$$

where k = ωk = 2π/λ

Frequency Domain (Phasors):

$\underline{\bar{E}}e^{-jkz}$ represents the same wave as above! $\left(\underline{\bar{E}} = \hat{x}\underline{E}_x + \hat{y}\underline{E}_y\right)$

i.e. $\bar{E}(r,t) = \text{Re}\left\{\underline{\bar{E}}e^{-jkz}e^{j\omega t}\right\} = \text{Re}\left\{(\hat{x}\underline{E}_x + \hat{y}\underline{E}_y)e^{j(\omega t - kz)}\right\}$

where $\text{Re}\left\{\hat{x}\underline{E}_x e^{j(\omega t - kz)}\right\} = \text{Re}\left\{\hat{x}E_x e^{-j\phi_x} e^{j(\omega t - kz)}\right\}$

$$= \hat{x}E_x \text{Re}\left\{e^{j(\omega t - kz - \phi_x)}\right\} = \hat{x}E_x \cos(\omega t - kz - \phi_x)$$

[We defined $E_x = |\underline{E}_x|$]

L3-5

EXAMPLES OF PHASOR REPRESENTATION

Using $\bar{E}(r,t) = \text{Re}\left\{\underline{\bar{E}}(r)e^{j\omega t}\right\}$:

1) $\underline{\bar{E}}(r) = 5\hat{y}e^{-jkz}$ implies $\bar{E}(r,t) = 5\hat{y}\cos(\omega t - kz)$ [y-polarization]

2) $\underline{\bar{E}}(r) = 5(\hat{x} + \hat{y})e^{-jkz}$ implies $\bar{E}(r,t) = 5(\hat{x} + \hat{y})\cos(\omega t - kz)$ [45°-polarization]

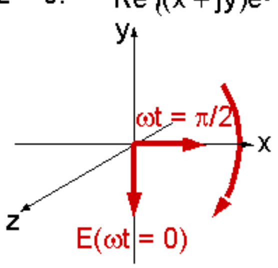
3) $\underline{\bar{E}}(r) = 5(\hat{x} + j\hat{y})e^{-jkz}$ implies $\bar{E}(r,t) = 5[\hat{x}\cos(\omega t - kz) - \hat{y}\sin(\omega t - kz)]$ [left-circular-polarization]

[Recall $e^{j\omega t} = \cos\omega t + j\sin\omega t$]

Why left-circular?

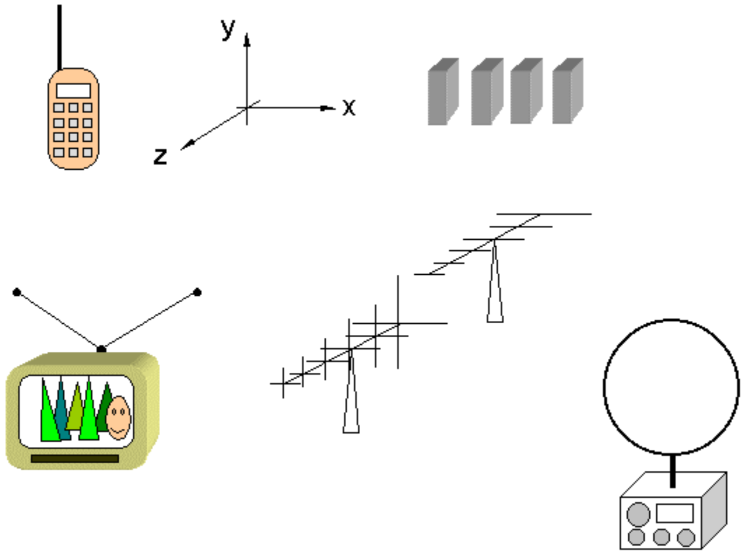
Let z = 0. $\text{Re}\left\{(\hat{x} + j\hat{y})e^{j\omega t}\right\} = \hat{x}$ at $t = 0$

$= \text{Re}\{(\hat{x} + j\hat{y})j\} = -\hat{y}$ at $\omega t = \pi/2$



L3-6

POLARIZED TRANSMITTERS AND RECEIVERS



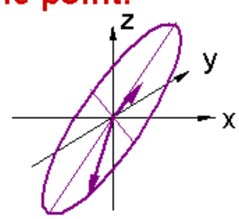
L3-7

POLARIZATION AT A POINT IN SPACE

Assume many waves superimpose at a single point:

All waves are at ω

Arrive from many directions with arbitrary polarizations



Wave #1 alone at that point produces:

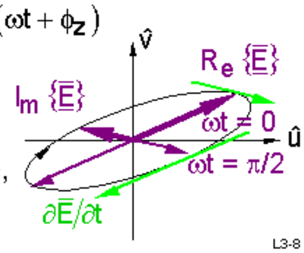
$$\bar{E}_1(t) = \hat{x}E_{x1}\cos(\omega t + \phi_{x1}) + \hat{y}E_{y1}\cos(\omega t + \phi_{y1}) + \hat{z}E_{z1}\cos(\omega t + \phi_{z1})$$

All waves together yield $\bar{E}(t) = \sum_i \bar{E}_i(t)$:

$$\bar{E}(t) = \hat{x}E_x\cos(\omega t + \phi_x) + \hat{y}E_y\cos(\omega t + \phi_y) + \hat{z}E_z\cos(\omega t + \phi_z)$$

$$\underline{\bar{E}} = \hat{x}\underline{E}_x + \hat{y}\underline{E}_y + \hat{z}\underline{E}_z = R_e\{\underline{\bar{E}}\} + jI_m\{\underline{\bar{E}}\}$$

The real and imaginary parts of $\underline{\bar{E}}$ define a plane $\hat{u},\hat{v}$
Since there is no single direction of propagation here,
We cannot say "right" or "left" elliptical



L3-8