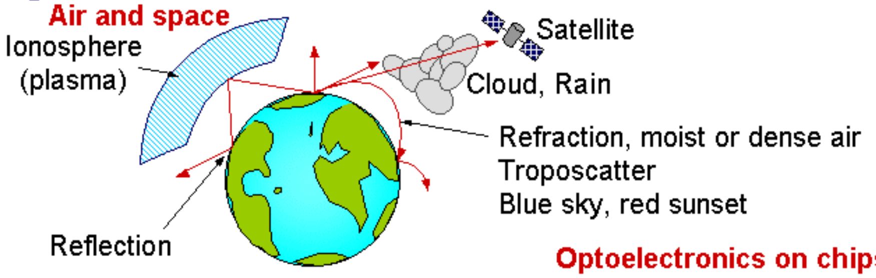
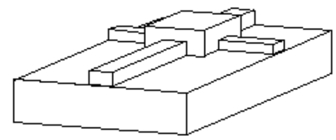
WAVES IN MEDIA

Significance to Communications

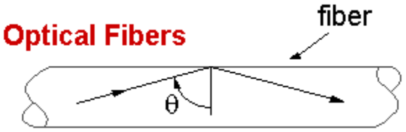
Air and space



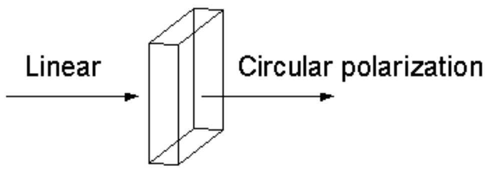
Optoelectronics on chips



Optical Fibers



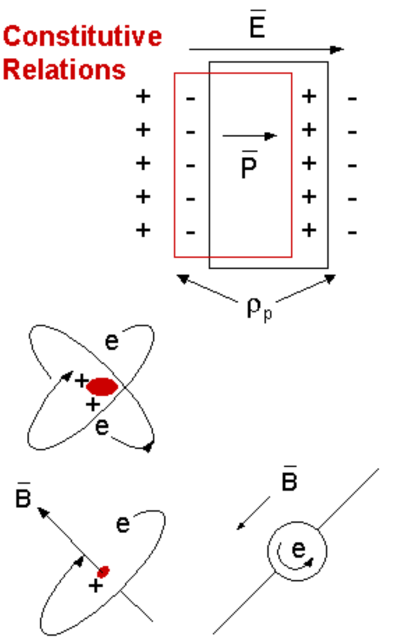
Polarization-based optoelectronic devices



L5-1

WAVES IN MEDIA

Constitutive Relations



Vacuum: $\underline{\bar{D}} = \varepsilon_o \underline{\bar{E}}$ $\nabla \bullet \underline{\bar{D}} = \rho_f$

ρ_f = free charge density

Dielectric Materials:

$\underline{\bar{D}} = \varepsilon \underline{\bar{E}} = \varepsilon_o \underline{\bar{E}} + \underline{\bar{P}}$

$\nabla \bullet \varepsilon_o \underline{\bar{E}} = \rho_f + \rho_p$

$\nabla \bullet \underline{\bar{P}} = -\rho_p$ polarization charge density

$\underline{\bar{P}}$ = "Polarization Vector"

Magnetic Materials:

$\nabla \bullet \underline{\bar{B}} = 0$

$\underline{\bar{B}} = \mu_o \underline{\bar{H}}$ in vacuum

$\underline{\bar{B}} = \mu \underline{\bar{H}} = \mu_o (\underline{\bar{H}} + \underline{\bar{M}})$

$\underline{\bar{M}}$ = "Magnetization Vector"

L5-2

TYPES OF MEDIA

Properties are a function of:	Designation:
Field direction	Anisotropic $\bar{D} = \underline{\bar{\bar{\varepsilon}}}\bar{E}$, $\bar{B} = \underline{\bar{\bar{\mu}}}\bar{H}$
Position	Inhomogeneous
Time: $\neq f(t)$	Stationary
$\neq f(\text{history})$	Amnesic
Frequency	Dispersive
$\bar{E}$ or $\bar{H}$	Non-linear
Temperature	Temperature dependent
Pressure	Compressive

L5-3

ANISOTROPIC DIELECTRICS

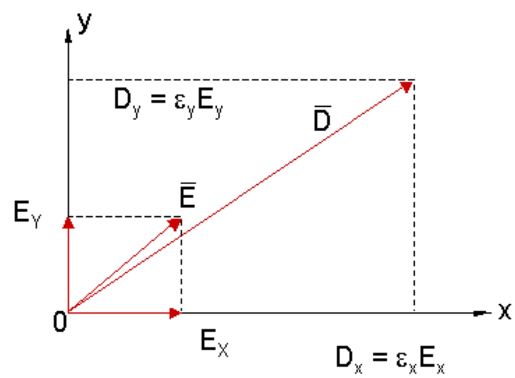
$$\underline{\bar{D}} = \underline{\bar{\bar{\varepsilon}}}\,\underline{\bar{E}}$$

$$\underline{D}_x = \varepsilon_{xx}\underline{E}_x + \varepsilon_{xy}\underline{E}_y + \varepsilon_{xz}\underline{E}_z$$

$$\underline{D}_y = \varepsilon_{yx}\underline{E}_x + \varepsilon_{yy}\underline{E}_y + \varepsilon_{yz}\underline{E}_z$$

$$\underline{D}_z = \varepsilon_{zx}\underline{E}_x + \varepsilon_{zy}\underline{E}_y + \varepsilon_{zz}\underline{E}_z$$

Let $\underline{\bar{\bar{\varepsilon}}} = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$



x,y,z are "principal axes"

Note: $\bar{D} // \bar{E}$ iff $\bar{E} // \hat{x}, \hat{y}$, or $\hat{z}$ for $\varepsilon_x \neq \varepsilon_y \neq \varepsilon_z$

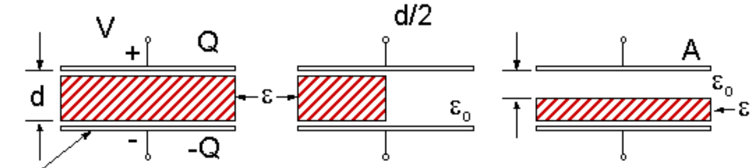
Real $\underline{\bar{\bar{\varepsilon}}}, \underline{\bar{\bar{\mu}}} \Rightarrow$ Lossless medium

L5-4

HOW TO MAKE ANISOTROPIC MATERIALS

Consider:
$\varepsilon >> \varepsilon_0$

(capacitors)
$Q = CV$

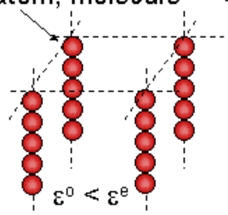


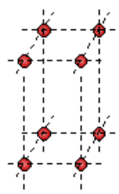
Area A (m^2)

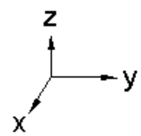
$C = \frac{\varepsilon_{eff} A}{d}$ $\qquad C \cong \frac{\varepsilon (A/2)}{d}$ $\qquad C \cong \frac{\varepsilon_o A}{(d/2)}$

$\varepsilon_{eff} = \varepsilon$ $\qquad \varepsilon_{eff} \cong \varepsilon/2$ $\qquad \varepsilon_{eff} \cong 2\varepsilon_o$

atom, molecule ⇨ "uniaxial medium" $\varepsilon_x = \varepsilon_y = \varepsilon^o$ "ordinary"
$\varepsilon_z = \varepsilon^e$ "extraordinary"







L5-5

WAVE BEHAVIOR IN UNIAXIAL MEDIUM

Assume wave in $+\hat{z}$ direction,

Derive wave equation:

$\nabla \times \underline{\bar{E}} = -j\omega \underline{\bar{B}} \qquad \nabla \cdot \underline{\bar{D}} = \rho_f = 0$

$\nabla \times \underline{\bar{H}} = j\omega \underline{\bar{D}} \qquad \nabla \cdot \underline{\bar{B}} = 0$

$$\underline{\bar{D}} = \underline{\bar{\bar{\varepsilon}}}\underline{\bar{E}} \qquad \bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon^e & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{bmatrix}, \quad \underline{\bar{\bar{\mu}}} = \mu$$

Therefore $\nabla \times (\nabla \times \underline{\bar{E}}) = \nabla(\nabla \cdot \underline{\bar{E}}) - \nabla^2 \underline{\bar{E}} = -j\omega\mu\nabla \times \bar{H} = \omega^2 \mu \underline{\bar{\bar{\varepsilon}}}\underline{\bar{E}}$

Does $\nabla \cdot \underline{\bar{E}} = 0$ here? Yes, (let's skip proof) can test final solution

Therefore $\nabla^2 \underline{\bar{E}} + \omega^2 \mu \underline{\bar{\bar{\varepsilon}}}\underline{\bar{E}} = 0 \Rightarrow$ 3 equations (x,y,z components)

$$\underbrace{\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right.}_{} \left. + \frac{\partial^2}{\partial z^2}\right]\left[\hat{x}\underline{E}_x + \hat{y}\underline{E}_y + \hat{z}\underline{E}_z\right] + \omega^2 \mu \underline{\bar{\bar{\varepsilon}}}\underline{\bar{E}} = 0$$

Assume = 0 (UPW in z direction)
This leads to 2 decoupled equations for x and y polarization

L5-6

BIREFRINGENT MEDIA

Decoupled wave equations:

$$\left[\frac{\partial^2}{\partial z^2} + \underbrace{\omega^2 \mu \varepsilon^e}_{\triangleq (k^e)^2}\right] \underline{E}_x = 0\,,\quad k^e = \omega\sqrt{\mu\varepsilon^e}\,, \qquad \left[\frac{\partial^2}{\partial z^2} + \underbrace{\omega^2 \mu \varepsilon}_{\triangleq (k^o)^2}\right] \underline{E}_y = 0\,,\quad k^o = \omega\sqrt{\mu\varepsilon}$$

(x-pol equation) (y-pol equation)

Where $\underline{E}_x \propto e^{-jk^e z} = e^{-j(\omega/v^e)z} \Rightarrow \begin{cases} v^e = 1/\sqrt{\mu\varepsilon^e} \\ v^o = 1/\sqrt{\mu\varepsilon} \end{cases}$

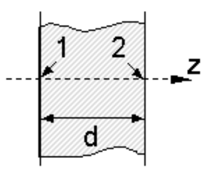
Thus the x- and y-polarized waves propagate independently at different velocities

If $v^e < v^o$ then $v^e \rightarrow$ "slow-axis velocity"

L5-7

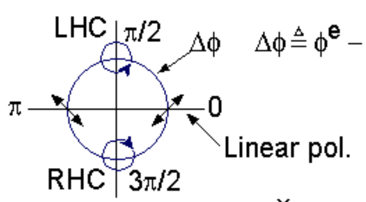
BIREFRINGENT MEDIA

Example:

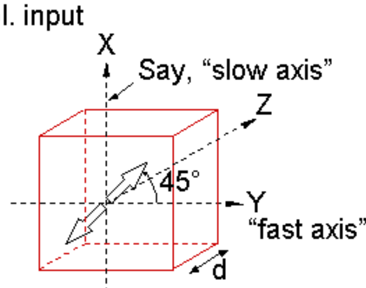


$\overline{E}_1 = E_o(\hat{x} + \hat{y})$ 45° linear pol. input

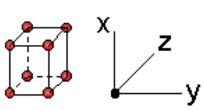
$$\overline{E}_2 = E_o\,\underbrace{\hat{x}e^{-j\phi^e} + \hat{y}e^{-j\phi^o}}_{\text{What pol.?}},\quad \phi^e = k^e d,\ \phi^o = k^o d$$

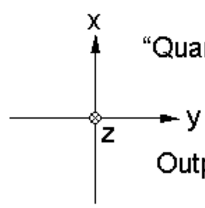


$\Delta\phi \triangleq \phi^e - \phi^o = (k^e - k^o)d$



"Quarter wave plate"

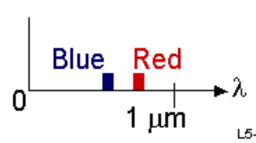




Output: d ∋ $\Delta\phi = \pi/2$

Demo; Polaroids

1) ⇕ ⊕ ⇕ ⇒
2) ⇕ ⊕ ⇔ ⇒
3) ⇕ ⊞ ⇔ ⇒
4) ⇕ ⊠ ⇔ ⇒
5) ⇕ MICA ⇔ ⇒
6) GEARS



L5-8